Kinematics
Kinematic Equations

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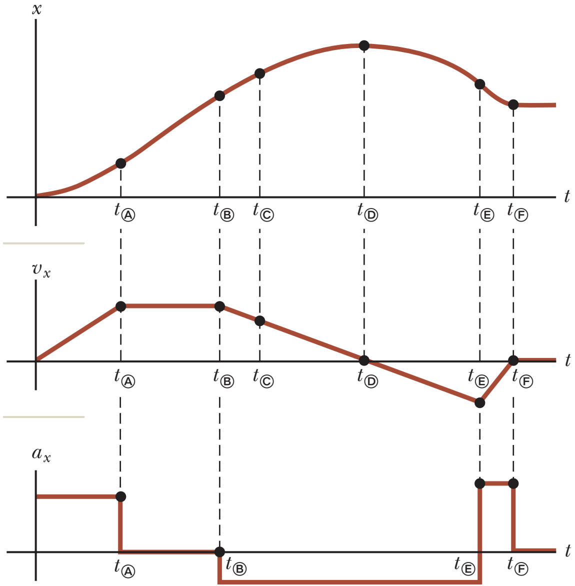
Last time

- kinematic quantities
- graphs of kinematic quantities

Overview

- relating graphs
- derivation of kinematics equations

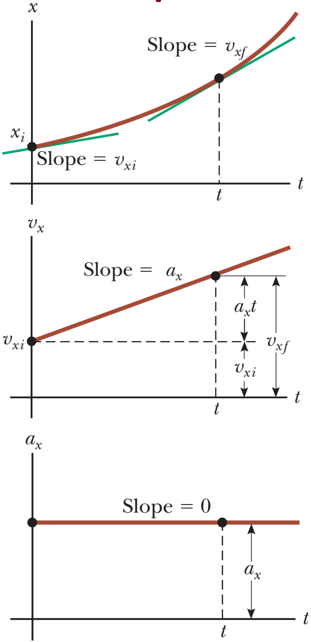
Acceleration vs. Time Graphs



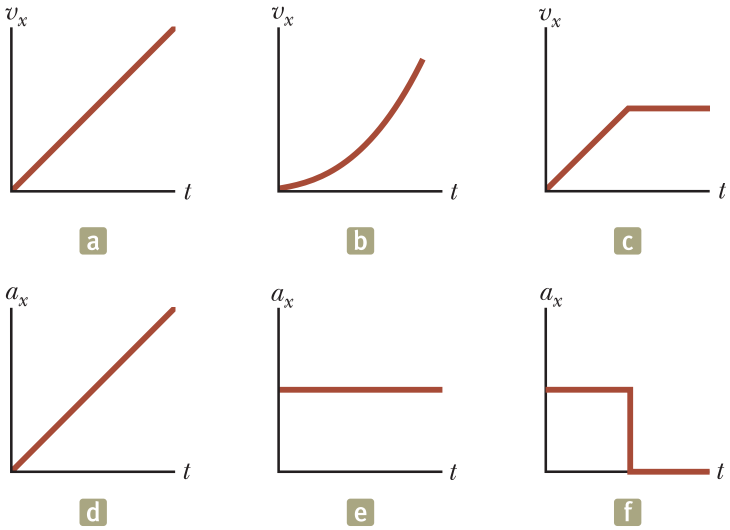
Question

What does the area under an **acceleration-time** graph represent?

Constant Acceleration Graphs



Matching Velocity to Acceleration Graphs



The Kinematics Equations

The kinematic equations describe motion in terms of position, velocity, acceleration, and time. There are 5 that apply specifically to the case of **constant acceleration**.

To solve kinematics problems: identify what equations apply, then do some algebra to find the quantity you need.

The Kinematics Equations

For zero acceleration:

$$\overrightarrow{\Delta r} = \vec{\mathbf{v}}\, t$$

Always:

$$\overrightarrow{\Delta r} = \vec{\mathbf{v}}_{avg}\, t$$

For constant acceleration:

$$\vec{\mathbf{v}}_{\mathbf{f}} = \vec{\mathbf{v}}_{\mathbf{i}} + \vec{\mathbf{a}}\, t$$

$$\overrightarrow{\Delta r} = \vec{\mathbf{v}}_{\mathbf{i}}\, t + \frac{1}{2}\vec{\mathbf{a}}\, t^2$$

$$\overrightarrow{\Delta r} = \vec{\mathbf{v}}_{\mathbf{f}}\, t - \frac{1}{2}\vec{\mathbf{a}}\, t^2$$

$$\overrightarrow{\Delta r} = \frac{\vec{\mathbf{v}}_{\mathbf{i}} + \vec{\mathbf{v}}_{\mathbf{f}}}{2}\, t$$

$$v_{f,x}^2 = v_{i,x}^2 + 2\, a_x \Delta x$$

The Kinematics Equations

For constant velocity ($\vec{\mathbf{a}} = 0$):

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}\, t \tag{1}$$

The Kinematics Equations

For constant velocity ($\vec{\mathbf{a}} = 0$):

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}\,t \qquad (1)$$

In general, when velocity is not constant, this is always true:

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}_{\text{avg}}\,t$$

The Kinematics Equations

Always true:

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}_{\text{avg}} t \tag{2}$$

This is just a rearrangement of the definition of average velocity:

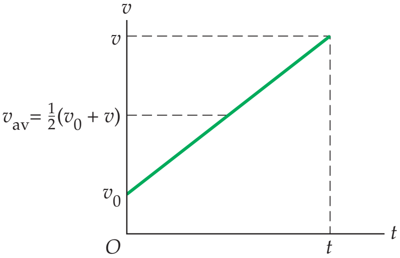
average velocity, $\vec{\mathbf{v}}_{\text{avg}}$

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\overrightarrow{\Delta \boldsymbol{r}}}{t}$$

where $\overrightarrow{\Delta \boldsymbol{r}}$ is the displacement that occurs in a time interval t.

The Kinematics Equations

If acceleration is constant ($\vec{\mathbf{a}} = \text{const}$), the velocity-time graph is a straight line



The area under the graph is equal to $\overrightarrow{\Delta \boldsymbol{r}}$, and using the area of a trapezoid:

$$\overrightarrow{\Delta \boldsymbol{r}} = \left(\frac{\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f}{2} \right) t \tag{3}$$

[1]Figure from James S. Walker "Physics".

The Kinematics Equations

For constant acceleration:

$$\overrightarrow{\Delta r} = \left(\frac{\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f}{2} \right) t$$

From $\vec{\mathbf{v}}_{\text{avg}} = \frac{\overrightarrow{\Delta r}}{t}$, we also have:

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f}{2}$$

(constant acceleration)

The Kinematics Equations

From the definition of acceleration:

$$\vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}}$$

We can integrate this expression.

$$\overrightarrow{\Delta \boldsymbol{v}} = \int_0^t \vec{\mathbf{a}}\,\mathrm{dt}'$$

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For constant acceleration:

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v_i}} + \vec{\mathbf{a}}\,t \tag{4}$$

where $\vec{\mathbf{v_i}}$ is the velocity at $t = 0$ and $\vec{\mathbf{v}}(t)$ is the velocity at time t.

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_\mathbf{i} + \vec{\mathbf{v}}_\mathbf{i} t + \frac{1}{2} \vec{\mathbf{a}} t^2$$

Equivalently,

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}_\mathbf{i} t + \frac{1}{2} \vec{\mathbf{a}} t^2 \qquad (5)$$

How do we know this?

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_{\mathbf{i}} + \vec{\mathbf{v}}_{\mathbf{i}}\, t + \frac{1}{2}\vec{\mathbf{a}}\, t^2$$

Equivalently,

$$\overrightarrow{\Delta \boldsymbol{r}} = \vec{\mathbf{v}}_{\mathbf{i}}\, t + \frac{1}{2}\vec{\mathbf{a}}\, t^2 \tag{5}$$

How do we know this? Integrate!

$$\overrightarrow{\Delta \boldsymbol{r}} = \int_0^t \vec{\mathbf{v}}\, \mathrm{dt}'$$

The Kinematics Equations

Similarly:

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}_\mathbf{i}} + \vec{\mathbf{v}_\mathbf{f}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

Equivalently,

$$\overrightarrow{\Delta r} = \vec{\mathbf{v}_\mathbf{f}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2 \tag{6}$$

Show by substituting $\vec{\mathbf{v}_\mathbf{i}} = \vec{\mathbf{v}_\mathbf{f}} - \vec{\mathbf{a}}\,t$ into equation (5).

The Kinematics Equations

The last equation we will derive is a scaler equation.

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$$\overrightarrow{\boldsymbol{\Delta r}} = \left(\frac{\vec{\mathbf{v_i}} + \vec{\mathbf{v_f}}}{2}\right) t$$

We could also write this as:

$$(\Delta x)\,\hat{\mathbf{i}} = \left(\frac{v_{i,x} + v_{f,x}}{2} t\right) \hat{\mathbf{i}}$$

where Δx, $v_{i,x}$, and $v_{f,x}$ could each be positive or negative.

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We do the same for equation (4):

$$v_{f,x}\,\hat{\mathbf{i}} = (v_{i,x} + a_x t)\,\hat{\mathbf{i}}$$

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Rearranging for t:

$$t = \frac{v_{f,x} - v_{i,x}}{a_x}$$

The Kinematics Equations

$$t = \frac{v_{f,x} - v_{i,x}}{a} \; ; \qquad \Delta x = \left(\frac{v_{i,x} + v_{f,x}}{2}\right) t$$

Substituting for t in our Δx equation:

$$\begin{aligned} \Delta x &= \left(\frac{v_{i,x} + v_{f,x}}{2}\right)\left(\frac{v_{f,x} - v_{i,x}}{a_x}\right) \\ 2a_x \Delta x &= (v_{i,x} + v_{f,x})(v_{f,x} - v_{i,x}) \end{aligned}$$

so,

$$v_{f,x}^2 = v_{i,x}^2 + 2\, a_x\, \Delta x \qquad (7)$$

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Summary

- graphs
- the kinematic equations

Assignment (now posted on website) Due Jan 16.

Quiz Tomorrow at the start of class.

(Uncollected) Homework Serway & Jewett,

- Ch 2, onward from page 49. Conceptual Q: 7; Problems: 25, 29, 33, 39, 83