Rotation
Torque
Moment of Inertia

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Last time

- rotational quantities
- rotational kinematics
- torque
Overview

- net torque
- Newton’s second law for rotation
- moment of inertia
- calculating moments of inertia
Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be (non-null) vectors.

Could this possibly be a valid equation?

$$\vec{a} = \vec{b} \cdot \vec{c}$$

(A) yes

(B) no
Quick review of Vector Expressions

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Quick review of Vector Expressions

Let \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \) be vectors. Let \( m \) and \( n \) be non-zero scalars.

Could this possibly be a valid equation?

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m = \vec{b} \times \vec{c}
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(A) yes
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Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors. Let $m$ and $n$ be non-zero scalars.

Suppose

- $\mathbf{a} \neq 0$ and
- $\mathbf{b} \neq 0$ and
- $\mathbf{a} \neq n \mathbf{b}$

for any value of $n$. Could this possibly be a true equation?

$$\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$$

(A) yes

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Quick review of Vector Expressions

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**(A)** yes

**(B)** no
Torque

*Torque* is a measure of force-causing-rotation.

It is not a force, but it is related. It depends on a force vector and its point of application relative to an axis of rotation.

Torque is given by:

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

That is: the cross product between

- a vector \( \vec{r} \), the displacement of the point of application of the force from the axis of rotation, and
- an the force vector \( \vec{F} \)
Torque

\[ \tau = \vec{r} \times \vec{F} = rF \sin \phi \hat{n} \]

where \( \phi \) is the angle between \( \vec{r} \) and \( \vec{F} \), and \( \hat{n} \) is the unit vector perpendicular to \( \vec{r} \) and \( \vec{F} \), as determined by the right-hand rule.
A torque is supplied by applying a force at point A. To produce the same torque, the force applied at point B must be:

(A) greater  
(B) less  
(C) the same

\(^1\)Image from Harbor Freight Tools, www.harborfreight.com
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Object that can rotate about an axis at $O$:

There are two forces acting, but the two torques produced, $\vec{\tau}_1$ and $\vec{\tau}_2$ point in opposite directions.

$\vec{\tau}_1$ would produce a counterclockwise rotation
$\vec{\tau}_2$ would produce a clockwise rotation
Net Torque

The net torque is the sum of the torques acting on the object:

\[ \vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i \]

In this case, with \( \hat{n} \) pointing out of the slide:

\[ \vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 = (d_1 F_1 - d_2 F_2) \hat{n} \]
Example 10.3 - Net Torque on a Cylinder

A one-piece cylinder is shaped as shown, with a core section protruding from the larger drum. The cylinder is free to rotate about the central $z$ axis shown in the drawing. A rope wrapped around the drum, which has radius $R_1$, exerts a force $T_1$ to the right on the cylinder. A rope wrapped around the core, which has radius $R_2$, exerts a force $T_2$ downward on the cylinder.

What is the net torque acting on the cylinder about the rotation axis (which is the $z$ axis)?
Example 10.3 - Net Torque on a Cylinder

First: Find an expression for the net torque acting on the cylinder about the rotation axis.

Second: Let $T_1 = 5.0 \text{ N}$, $R_1 = 1.0 \text{ m}$, $T_2 = 15 \text{ N}$, and $R_2 = 0.50 \text{ m}$. What is the net torque? Which way is the rotation?
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\[ \vec{\tau}_{\text{net}} = 2.5 \text{ Nm}, \text{ counter-clockwise (or } \hat{k}) \]
Rotational Version of Newton’s Second Law

Tangential components of forces give rise to torques. They also cause tangential accelerations. Consider the tangential component of the net force, \( F_{\text{net},t} \):

\[
F_{\text{net},t} = m a_t
\]

from Newton’s second law.

\[
\vec{\tau}_{\text{net}} = \vec{r} \times \vec{F}_{\text{net}} = r F_{\text{net},t} \hat{n}
\]

Now let’s specifically consider the case of a single particle, mass \( m \), at a fixed radius \( r \).
Rotational Version of Newton’s Second Law

A single particle, mass $m$, at a fixed radius $r$. 

For such a particle, $F_{\text{net},t} = ma_t$

$$\vec{\tau}_{\text{net}} = r F_{\text{net},t} \hat{n}$$

$$= r m a_t \hat{n}$$

$$= r m (\vec{\alpha} \cdot r)$$

$$= (mr^2) \vec{\alpha}$$
Rotational Version of Newton’s Second Law

\[(mr^2)\] is just some constant for this particle and this axis of rotation.

Let this constant be \((\text{scalar}) \ I = mr^2\).
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Let this constant be (scalar) $I = mr^2$.

Scalar $I$ is not impulse! This is just an unfortunate notation coincidence.

$I$ is called the moment of inertia of this system, for this particular axis of rotation.

Replacing the constant quantity in our expression for $\vec{\tau}_{\text{net}}$:

$$\vec{\tau}_{\text{net}} = I \vec{\alpha}$$
Rotational Version of Newton’s Second Law

Compare!

\[ \vec{\tau}_{\text{net}} = I \vec{\alpha} \]
\[ \vec{F}_{\text{net}} = m \vec{a} \]

Now the moment of inertia, \( I \), stands in for the inertial mass, \( m \).
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\[ \vec{F}_{\text{net}} = m \vec{a} \]

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The moment of inertia measures the rotational inertia of an object (how hard is it to rotate an object?), just as mass is a measure of inertia.
Moment of Inertia

We just found that for a single particle, mass $m$, radius $r$, 

$$I = mr^2$$

For an extended object with mass distributed over varying distances from the rotational axis, each particle of mass $m_i$ experiences a torque:

$$\vec{\tau}_i = m_i r_i^2 \vec{\alpha}$$
Moment of Inertia

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\]

And we sum over these torques to get the net torque.

\[
\vec{\tau}_{\text{net}} = \sum_i m_i r_i^2 \vec{\alpha}
\]

All particle in the object rotate together, so for an object made of a collection of particles, masses \( m_i \) at radiuses \( r_i \):

\[
I = \sum_i m_i r_i^2
\]
Moment of Inertia

If the object’s mass is far from the point of rotation, more torque is needed to rotate the object (with some angular acceleration).

![Diagram of easier and difficult rotation](http://biomech.byu.edu)

The barbell on the right has a greater moment of inertia.

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1Diagram from Dr. Hunter’s page at http://biomech.byu.edu (by Hewitt?)
And we sum over these torques to get the net torque. So, for a collection of particles, masses $m_i$ at radii $r_i$:

$$I = \sum_i m_i r_i^2$$

For a continuous distribution of mass, we must integrate over each small mass $\Delta m$:

$$I = \int r^2 \, dm$$
Summary

- Newton’s
- moments of inertia

Collected HW due Monday.

3rd Test Thursday.

(Uncollected) Homework Serway & Jewett,

- Ch 10, Probs: 27 (net torque)
- Look at examples 10.4, and 10.11