Rotation
Nonisolated Systems & Angular Momentum
Oscillations
Simple Harmonic Motion

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Mar 17, 2020

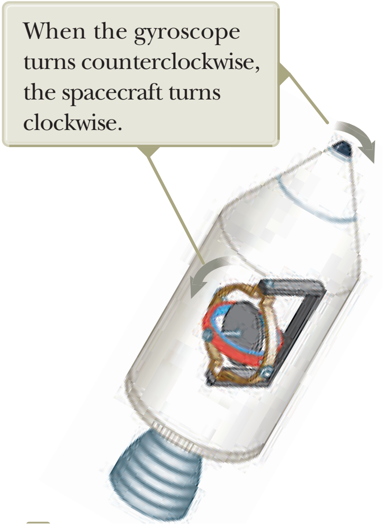
Last Time

- angular momentum of an object moving in a straight line
- conservation of angular momentum

Overview

- torque and changes in angular momentum

Angular Momentum Conservation Application



A massive flywheel is driven to cause rotations in the entire rocket.

Angular Momentum Conservation Application Example

#42, page 359

42. A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0\ \text{kg} \cdot \text{m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \times 10^5\ \text{kg} \cdot \text{m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s. If the orientation of the spacecraft is to be changed by 30.0°, for what time interval should the gyroscope be operated?

Angular Momentum Conservation Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

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$\theta = \frac{\pi}{6}$

$$t = \frac{\theta}{\omega_s}$$

Angular Momentum Conservation Application Example

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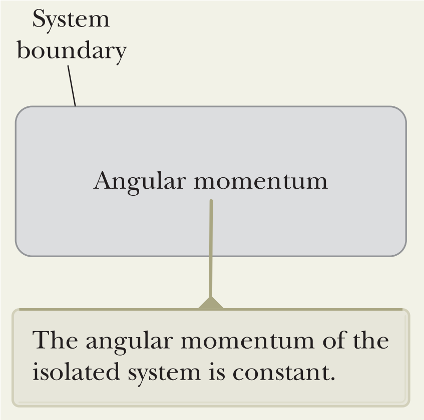
$$t = 131 \text{ s}$$

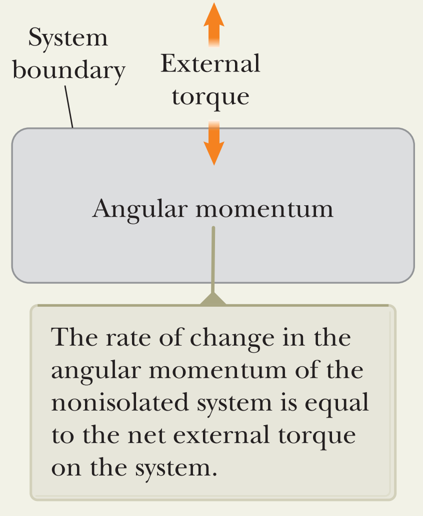
Conservation of Angular Momentum

For an *isolated* system, *ie.* a system with no external torques, total angular momentum is *conserved*.



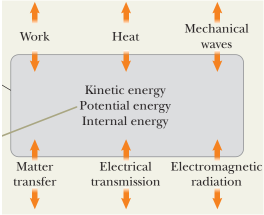
[1]Figures from Serway & Jewett.

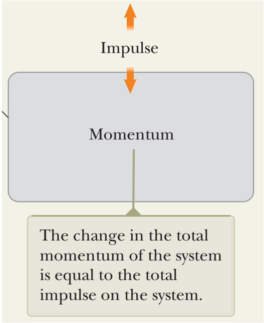
Non-isolated System

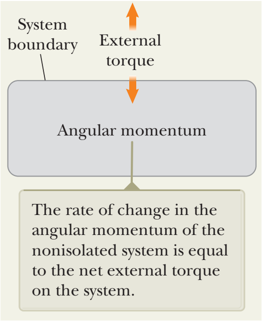


$$\vec{\boldsymbol{\tau}}_{\text{ext}} = \frac{d\vec{\mathbf{L}}}{dt}$$

Recap of Non-isolated Systems

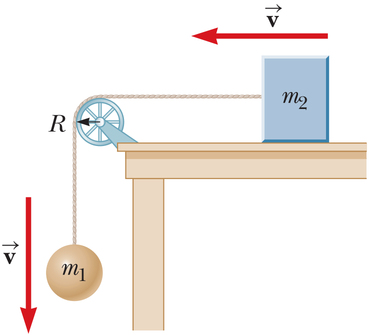






Non-isolated Example

A sphere of mass m_1 and a block of mass m_2 are connected by a light cord that passes over a pulley as shown. The radius of the pulley is R, and the mass of the thin rim is M. The spokes of the pulley have negligible mass. The block slides on a frictionless, horizontal surface. Find an expression for the linear acceleration of the two objects.



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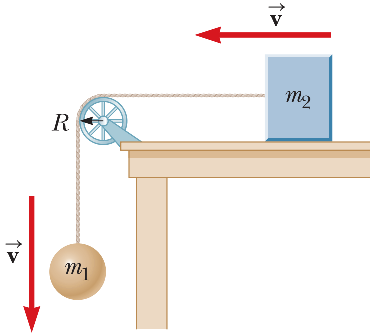
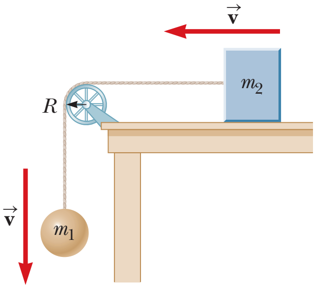


Non-isolated Example (11.4)

The system: block, sphere, and pulley.



$$\vec{\boldsymbol{\tau}}_{\text{net,ext}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$

Consider the angular momentum and torques about the axis of the pulley.

The only net external torque on this system is from the force of gravity on m_1.

Non-isolated Example (11.4)

$$\vec{\boldsymbol{\tau}}_{\text{net,ext}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{dt}}$$

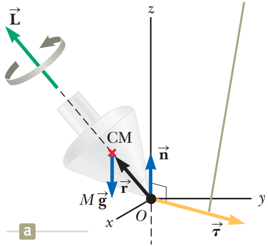
$$m_1 g R = \frac{\mathrm{d}}{\mathrm{dt}}(m_1 v R + m_2 v R + I\omega)$$

$$m_1 g R = \frac{\mathrm{d}}{\mathrm{dt}}(m_1 v R + m_2 v R + (MR^2)\frac{v}{R})$$

$$m_1 g R = \frac{\mathrm{dv}}{\mathrm{dt}}(m_1 + m_2 + M)R$$

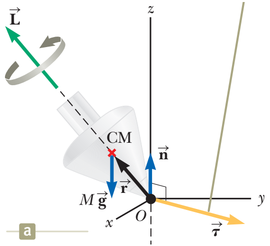
$$a = \frac{m_1 g}{(m_1 + m_2 + M)}$$

Gyroscopic Motion



Tops, gyroscopes, and other spinning objects with a fixed point on their axis of rotation exhibit interesting behavior.

Gyroscopic Motion



Tops, gyroscopes, and other spinning objects with a fixed point on their axis of rotation exhibit interesting behavior.

In particular, they are stable against an external gravitational force.

The effect of gravity is to cause *precession*.

Precessional motion is an additional rotation of the object's rotation axis. A smooth change in the angular momentum!

Gyroscopic Motion

Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?

Gyroscopic Motion

Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?

Why doesn't a top fall over?

Gyroscopic Motion

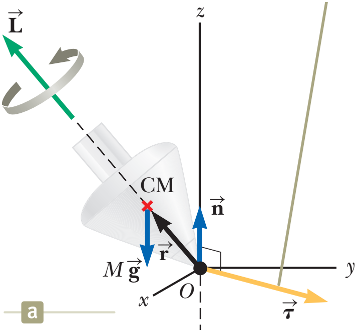
Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?

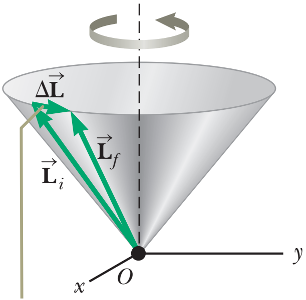
Why doesn't a top fall over?

$$\vec{\boldsymbol{\tau}}_{\text{ext}} = \frac{d\vec{\mathbf{L}}}{dt}$$

The change in angular momentum must be in the direction of the external torque!

Gyroscopic Motion - Skipping, not on Final





The torque supplied by the gravitational force is perpendicular to the force itself.

The magnitude of the angular momentum doesn't change - only the direction.

Gyroscopic Motion - Skipping, not on Final

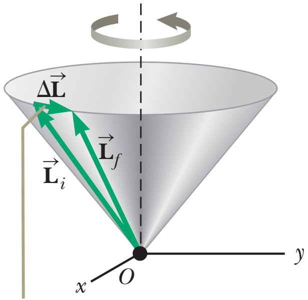
We can find the value for the precession **angular speed** ω_p if we make an approximation.

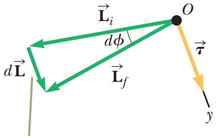
Suppose that the top spins with angular speed ω, and let ω be very large. Then the top's angular momentum $L = I\omega$ without considering the precessional motion is almost the same as its total angular momentum *with* the precession:

$$\vec{\mathbf{L}} \approx \vec{\mathbf{L}}_{\text{tot}}$$

This is a reasonable approximation as long as $\omega >> \omega_p$.

Gyroscopic Motion - Skipping, not on Final

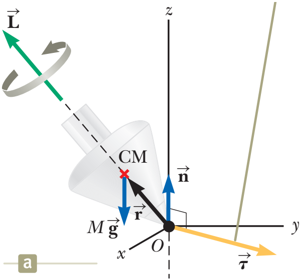


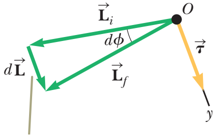


Angular speed of precession:

$$\omega_p = \frac{d\phi}{dt}$$

Gyroscopic Motion - Skipping, not on Final





$$
\begin{aligned}
\tau_{net} &= \frac{dL}{dt} \\
Mg\, r_{CM} \sin\theta &= \frac{dL}{d\phi}\frac{d\phi}{dt} \\
&= (L \sin\theta)\, \omega_p \\
\omega_p &= \frac{M\, g\, r_{CM}}{L}
\end{aligned}
$$

$$\omega_p = \frac{Mgr_{CM}}{I\omega}$$

Summary

- conservation of angular momentum

4th Assignment due Friday, Mar 20.

Final Exam Tuesday, Mar 24, via Canvas & Zoom, be ready at 9am.

(Uncollected) Homework

- Play with tops, bicycle wheels, gyroscopes, etc., whatever you have in your house

Serway & Jewett:

- Ch 11, onward from page 357. Problems: 30, 51, 45, 53, 55
- Look at example 11.9.