Rotation
Nonisolated Systems & Angular Momentum
Oscillations
Simple Harmonic Motion

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Last Time

- angular momentum of an object moving in a straight line
- conservation of angular momentum
Overview

- torque and changes in angular momentum
Angular Momentum Conservation Application

A massive flywheel is driven to cause rotations in the entire rocket.

When the gyroscope turns counterclockwise, the spacecraft turns clockwise.

A massive flywheel is driven to cause rotations in the entire rocket.
#42, page 359

42. A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0 \text{ kg} \cdot \text{m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \times 10^5 \text{ kg} \cdot \text{m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s. If the orientation of the spacecraft is to be changed by 30.0°, for what time interval should the gyroscope be operated?
Angular Momentum Conservation Application

Example

Time of gyroscope operation to achieve $30.0^\circ$ rotation of craft?
Angular Momentum Conservation Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Conservation of angular momentum:

\[ 0 = I_g \omega_g + I_s \omega_s \]
Angular Momentum Conservation Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

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\[ \omega_s = 0.004 \text{ rad s}^{-1} \]
Angular Momentum Conservation Application

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Conservation of angular momentum:

\[ 0 = I_g \omega_g + I_s \omega_s \]

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\[ \theta = \frac{\pi}{6} \]

\[ t = \frac{\theta}{\omega_s} \]
Angular Momentum Conservation Application

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Conservation of angular momentum:

\[ 0 = I_g \omega_g + I_s \omega_s \]

\[ \omega_s = 0.004 \text{ rad s}^{-1} \]

\[ \theta = \frac{\pi}{6} \]

\[ t = \frac{\theta}{\omega_s} \]

\[ t = 131 \text{ s} \]
Conservation of Angular Momentum

For an isolated system, i.e., a system with no external torques, total angular momentum is conserved.

1 Figures from Serway & Jewett.
Non-isolated System

The rate of change in the angular momentum of the nonisolated system is equal to the net external torque on the system.

\[ \vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt} \]
Recap of Non-isolated Systems

**Impulse**

The change in the total momentum of the system is equal to the total impulse on the system.

**Momentum**

**Angular momentum**

The rate of change in the angular momentum of the nonisolated system is equal to the net external torque on the system.

- Work
- Heat
- Mechanical waves
- Kinetic energy
- Potential energy
- Internal energy
- Matter transfer
- Electrical transmission
- Electromagnetic radiation
- System boundary
- External torque

**Figure 9.4**
Non-isolated Example

A sphere of mass $m_1$ and a block of mass $m_2$ are connected by a light cord that passes over a pulley as shown. The radius of the pulley is $R$, and the mass of the thin rim is $M$. The spokes of the pulley have negligible mass. The block slides on a frictionless, horizontal surface. Find an expression for the linear acceleration of the two objects.
Non-isolated Example

A sphere of mass $m_1$ and a block of mass $m_2$ are connected by a light cord that passes over a pulley as shown. The radius of the pulley is $R$, and the mass of the thin rim is $M$. The spokes of the pulley have negligible mass. The block slides on a frictionless, horizontal surface. Find an expression for the linear acceleration of the two objects, using the concepts of angular momentum and torque.
Non-isolated Example (11.4)

The system: block, sphere, and pulley.

Consider the angular momentum and torques about the axis of the pulley.

The only net external torque on this system is from the force of gravity on $m_1$. 

\[
\vec{\tau}_{\text{net, ext}} = \frac{d\vec{L}}{dt}
\]
Non-isolated Example (11.4)

\[ \vec{\tau}_{\text{net,ext}} = \frac{d \vec{L}}{dt} \]

\[ m_1 g R = \frac{d}{dt} (m_1 v R + m_2 v R + I \omega) \]

\[ m_1 g R = \frac{d}{dt} (m_1 v R + m_2 v R + (M R^2) \frac{v}{R}) \]

\[ m_1 g R = \frac{dv}{dt} (m_1 + m_2 + M) R \]

\[ a = \frac{m_1 g}{(m_1 + m_2 + M)} \]
Gyroscopic Motion

Tops, gyroscopes, and other spinning objects with a fixed point on their axis of rotation exhibit interesting behavior.
Gyroscopic Motion

Tops, gyroscopes, and other spinning objects with a fixed point on their axis of rotation exhibit interesting behavior.

In particular, they are stable against an external gravitational force.

The effect of gravity is to cause precession.

Precessional motion is an additional rotation of the object’s rotation axis. A smooth change in the angular momentum!
Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?
Gyroscope Motion

Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?

Why doesn’t a top fall over?
Gyroscopic Motion

Why does the angular momentum vector rotate about an axis perpendicular to the force of gravity?

Why doesn’t a top fall over?

\[ \tau_{\text{ext}} = \frac{d\vec{L}}{dt} \]

The change in angular momentum must be in the direction of the external torque!
The torque supplied by the gravitational force is perpendicular to the force itself.

The magnitude of the angular momentum doesn’t change - only the direction.
We can find the value for the precession angular speed \( \omega_p \) if we make an approximation.

Suppose that the top spins with angular speed \( \omega \), and let \( \omega \) be very large. Then the top’s angular momentum \( L = I\omega \) without considering the precessional motion is almost the same as its total angular momentum \( \text{with} \) the precession:

\[
\vec{L} \approx \vec{L}_{\text{tot}}
\]

This is a reasonable approximation as long as \( \omega \gg \omega_p \).
Angular speed of precession:

\[ \omega_p = \frac{d\phi}{dt} \]
11.5 The Motion of Gyroscopes and Tops

The angular momentum vector rotates through an angle $\theta$ in a time interval $\Delta t$. The change in angular momentum is

$$dL = (L \sin \theta) \omega_p$$

Therefore, like the torque vector, $\tau_{\text{net}} = \frac{dL}{dt}$.

Because the top undergoes precessional motion, we can take the total angular momentum to be simply the gyroscope. In practice, $\omega_p$ is made very large.

The right-hand rule indicates that $\tau_{\text{net}}$ is in the $xy$ direction about the pivot.

The direction of $\tau_{\text{net}}$ is parallel to the direction of $dL$.

The magnitude of $\tau_{\text{net}}$ is in the direction of $dL$.

The rate at which $\tau_{\text{net}}$ is in the $xy$ direction about the pivot.

The change in angular momentum is $dL = (L \sin \theta) \omega_p$.

$$\omega_p = \frac{Mg r_{CM}}{L}$$

$$\omega_p = \frac{M g r_{CM}}{L}$$

$$\omega_p = \frac{M g r_{CM}}{I \omega}$$
Summary

- conservation of angular momentum

4th Assignment due Friday, Mar 20.

Final Exam Tuesday, Mar 24, via Canvas & Zoom, be ready at 9am.

(Uncollected) Homework

- Play with tops, bicycle wheels, gyroscopes, etc., whatever you have in your house

Serway & Jewett:

- Ch 11, onward from page 357. Problems: 30, 51, 45, 53, 55
- Look at example 11.9.