Static Equilibrium

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Last time

- simple harmonic motion

Overview

- static equilibrium
- center of gravity
- static equilibrium problems

Static Equilibrium: System in Equilibrium

Knowing that an object is in equilibrium can give a lot of information about the forces on the object.

Previously: a system was in *equilibrium* if the net force was zero.

equilibrium $\iff$ constant velocity or at rest

Also, acceleration is zero.

Static Equilibrium: Extended System in Equilibrium

Now we consider extended rigid objects.

Forces can cause rotations (torques).

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Force Equilibrium

$$\vec{\mathbf{F}}_{\text{net}} = \sum_i \vec{\mathbf{F}}_i = 0$$

Rotational Equilibrium

$$\vec{\boldsymbol{\tau}}_{\text{net},O} = \sum_i \vec{\boldsymbol{\tau}}_i = 0$$

about *any* axis O.

Static Equilibrium: Extended System in Equilibrium

Rigid Object in Equilibrium

A rigid object is said to be in equilibrium if

$\vec{\mathbf{F}}_{\text{net}} = \sum_i \vec{\mathbf{F}}_i = 0$ and

$\vec{\boldsymbol{\tau}}_{\text{net},O} = \sum_i \vec{\boldsymbol{\tau}}_{i,O} = 0$ about *any* axis O.

equilibrium $\iff$ $v =$ const. *and* $\omega =$ const.

$\Rightarrow \quad a = 0$ and $\alpha = 0$

Static Equilibrium

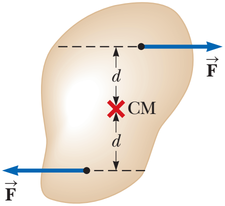
Static Equilibrium is the special case that the object is also at rest:

$$v_{\text{CM}} = 0$$

$$\omega = 0$$

Question

Quick Quiz 12.1 [1] Consider the object subject to the two forces of equal magnitude. Choose the correct statement for this situation.



(A) The object is in force equilibrium but not torque equilibrium.

(B) The object is in torque equilibrium but not force equilibrium.

(C) The object is in both force equilibrium and torque equilibrium.

(D) The object is in neither force equilibrium nor torque equilibrium.

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[1] Serway & Jewett, page 364.

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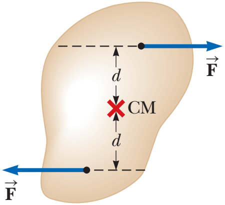

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↑

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[1] Serway & Jewett, page 364.

Rotational Equilibrium?

A system is in rotational equilibrium if $\vec{\boldsymbol{\tau}}_{\text{net}} = 0$ for any possible axis.

Suppose the system is in force equilibrium. Then $\vec{\mathbf{F}}_{\text{net}} = 0$.

If $\vec{\mathbf{F}}_{\text{net}} = 0$ and $\vec{\boldsymbol{\tau}}_{\text{net},O} = 0$ for a particular axis O, then $\vec{\boldsymbol{\tau}}_{\text{net},O'} = 0$ for any axis O'.

Rotational Equilibrium from Force Equilibrium

If an object is in force equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis.

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Rotational Equilibrium from Force Equilibrium

If an object is in force equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis.

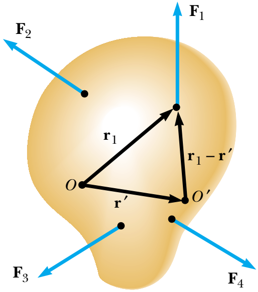
If the system is not in force equilibrium, this does not follow!

Rotational Equilibrium?

Rotational Equilibrium from Force Equilibrium

If an object is in force equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis.

Suppose $\vec{\boldsymbol{\tau}}_{\text{net},O} = 0$ and $\vec{\mathbf{F}}_{\text{net}} = 0$.



$$\begin{aligned}
\vec{\boldsymbol{\tau}}_{\text{net},O'} &= \sum_i \vec{\mathbf{r}}\,'_i \times \vec{\mathbf{F}}_i \\
&= \sum_i (\vec{\mathbf{r}}_i - \vec{\mathbf{r}}\,') \times \vec{\mathbf{F}}_i \\
&= \sum_i (\vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i) - \vec{\mathbf{r}}\,' \times \cancelto{0}{\sum_i \vec{\mathbf{F}}_i} \\
&= \vec{\boldsymbol{\tau}}_{\text{net},O} \\
&= 0
\end{aligned}$$

Center of Gravity vs. Center of Mass

Center of gravity – the point in an extended object at which a single gravitational force acting is equivalent to the combination of all the individual gravitational forces acting on each mass element in the object.

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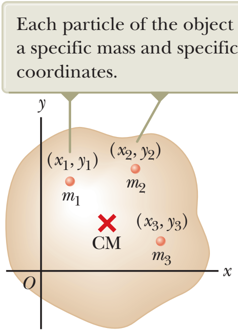
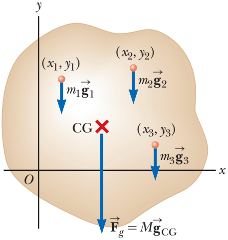
If the gravitational field is uniform (the same at all points) then the center of gravity is the same point as the center of mass.

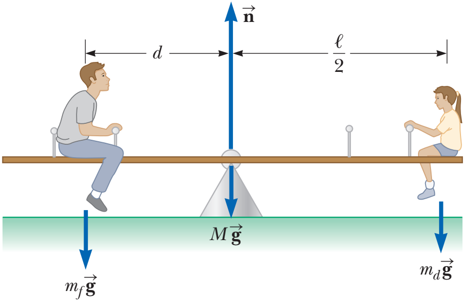




Center of Gravity

In particular, the torque caused by a single force $\vec{\mathbf{F}}_g$ through the center of gravity must be the same as the net torque of all the forces $m_i\vec{\mathbf{g}}$ on all the masses m_i in the system.

When an object is supported below its center of mass, there should be no net torque due to gravity.



Center of Gravity

In the diagram the forces $\vec{\mathbf{F}}_g$ were in the $-y$ direction.

The x-coordinate of the center of gravity satisfies:

$$x_{\mathrm{CG}}\, M\, g_{\mathrm{CG}} = \sum_i x_i m_i\, g_i$$

where $M = \sum_i m_i$ is the total mass.

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Same as center of mass expression! $x_{\mathrm{CG}} = x_{\mathrm{CM}}$

Question

Quick Quiz 12.3[2] A meterstick of uniform density is hung from a string tied at the 25-cm mark. A 0.50-kg object is hung from the zero end of the meterstick, and the meterstick is balanced horizontally. What is the mass of the meterstick?

(A) 0.25 kg

(B) 0.50 kg

(C) 1.0 kg

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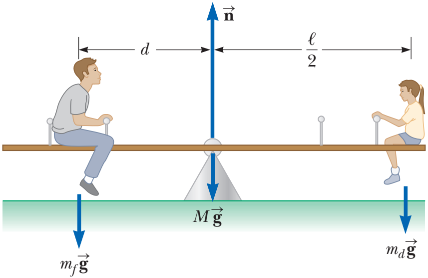
Seesaw - Example 12.1

A seesaw consisting of a uniform board of mass M and length ℓ, supports at rest a father and daughter with masses m_f and m_d, respectively. The support (called the fulcrum) is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance $\ell/2$ from the center.



Determine where the father should sit to balance the system at rest.

Seesaw - Example 12.1

Where should the father sit to balance the system?

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Net torque is zero.

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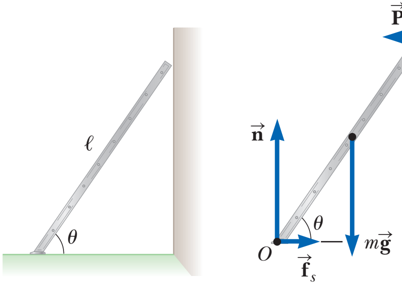
Analyzing torques about the fulcrum point:

$$\begin{aligned} \tau_{\text{net}} &= 0 \\ m_f g d - m_d g \frac{\ell}{2} &= 0 \\ m_f g d &= m_d g \frac{\ell}{2} \\ d &= \frac{m_d \ell}{2 m_f} \end{aligned}$$

Example 12.3 - Slipping Ladder

A uniform ladder of length ℓ, rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ_{min} at which the ladder does not slip.



Example 12.3 - Slipping Ladder

y-dir:

$$F_{\text{net},y} = 0 \Rightarrow n = mg$$

x-dir:

$$F_{\text{net},x} = 0 \Rightarrow P = f_s$$

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$$\tau_{\text{net},O} = 0 \Rightarrow \ell P \sin\theta = \frac{\ell}{2} mg \cos\theta$$

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$$\theta_{\text{min}} = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = 51^\circ$$

Summary

- static equilibrium
- center of gravity
- static equilibrium practice

4th Assignment! Due tomorrow.

Final Exam Tuesday, Mar 24, via Canvas & Zoom, be ready at 9am.

(Uncollected) Homework Serway & Jewett,

- Read 12.1–12.3 of Chapter 12.
- Ch 12, onward from page 400. Probs: 3, 11, 15, 23, 25, 45, 51