Gravitation

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Last Time

- static equilibrium
- center of gravity

Overview

- Newton's Law of Universal Gravitation
- gravitational potential energy
- little g

Gravitation

The force that massive objects exert on one another.

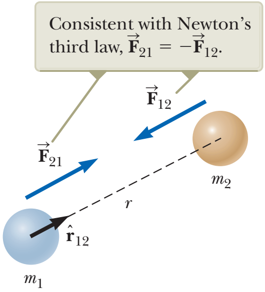
Newton's Law of Universal Gravitation

$$F_G = \frac{G m_1 m_2}{r^2}$$

for two objects, masses m_1 and m_2 at a distance r.

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Gravitation



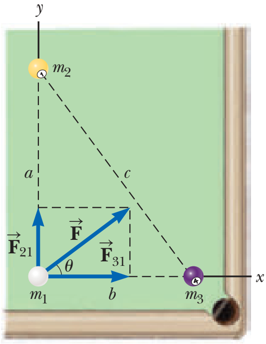
$$\vec{F}_{G,1\to 2} = -\frac{Gm_1 m_2}{r^2}\,\hat{\mathbf{r}}_{1\to 2}$$

for two objects, masses m_1 and m_2 at a distance r.
$\hat{\mathbf{r}}$ is the radial unit vector.

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Example 13.1: The Gravitational Force between Small Masses is Small

Three 0.300 kg billiard balls are placed on a table at the corners of a right triangle. The sides of the triangle are of lengths $a = 0.400$ m, $b = 0.300$ m, and $c = 0.500$ m. Calculate the gravitational force vector on the cue ball (designated m_1) resulting from the other two balls as well as the magnitude and direction of this force.



Example 13.1

Remember, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}:

$$\vec{\mathbf{F}}_{G,1\to 2} = -\frac{Gm_1 m_2}{r^2}\,\hat{\mathbf{r}}_{1\to 2}$$

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$$\vec{\mathbf{F}}_{G,1\to 2} = -\frac{Gm_1 m_2}{r^2}\,\hat{\mathbf{r}}_{1\to 2}$$

$$\begin{aligned}
\vec{\mathbf{F}}_{2\to 1} &= -\frac{Gm_2 m_1}{r^2}\,\hat{\mathbf{r}}_{2\to 1} \\
&= \frac{Gm_2 m_1}{a^2}\,\hat{\mathbf{j}} \\
&= 3.75 \times 10^{-11}\,\hat{\mathbf{j}}\ \mathrm{N}
\end{aligned}$$

$$\begin{aligned}
\vec{\mathbf{F}}_{3\to 1} &= -\frac{Gm_3 m_1}{r^2}\,\hat{\mathbf{r}}_{3\to 1} \\
&= \frac{Gm_3 m_1}{b^2}\,\hat{\mathbf{i}} \\
&= 6.67 \times 10^{-11}\,\hat{\mathbf{i}}\ \mathrm{N}
\end{aligned}$$

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So,

$$\begin{aligned}\vec{\mathbf{F}}_{\text{net}} &= \vec{\mathbf{F}}_{2\to 1} + \vec{\mathbf{F}}_{3\to 1}\\ &= (6.67\times 10^{-11}\,\hat{\mathbf{i}} + 3.75\times 10^{-11}\,\hat{\mathbf{j}})\text{ N}\end{aligned}$$

$$\vec{\mathbf{F}}_{\text{net}} = 7.65\times\times 10^{-11}\text{ N with }\theta = 29.3^\circ$$

Very small!

Gravitational Potential Energy

Remember from Chapter 7,

$$F_x = -\frac{\mathrm{dU}}{\mathrm{dx}} \quad ; \quad \Delta U = -\int F(x)\,\mathrm{dx}$$

Since $W = \int F(r)\,\mathrm{dr}$, this tells us that the work done *by gravity* on an object is equal to minus the change in potential energy.

$$\Delta U \;\;=\;\; -\int_{r_i}^{r_f} \vec{\mathbf{F}}_G(\vec{\mathbf{r}}) \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

(You solve this on the homework!)

Gravitational Potential Energy

It is useful to pick a reference point to set the scale for gravitational potential energy. What would be a good point?

Gravitational Potential Energy

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Infinite distance! For $r_i = \infty$, $U(r_i) = 0$.

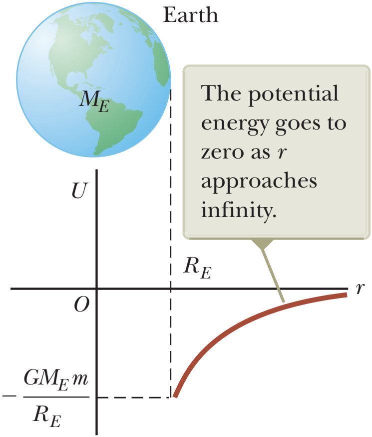
Then we can define:

$$U(r) = -\frac{Gm_1 m_2}{r}$$

This will *always* be a negative number.

Gravitational Potential Energy

$$U(r) = -\frac{Gm_1 m_2}{r}$$



Acceleration due to Gravity

This force in that it gives objects weight, F_g.

For an object of mass m near the surface of the Earth:

$$F_g = mg$$

and

$$g = \frac{GM_{\mathrm{E}}}{R_{\mathrm{E}}^2}$$

where

- $M_{\mathrm{E}} = 5.97 \times 10^{24}$ kg is the mass of the Earth and
- $R_{\mathrm{E}} = 6.37 \times 10^{6}$ m is the radius of the Earth.

The force $\vec{\mathbf{F}}_g$ acts downwards towards the center of the Earth.

Acceleration due to Gravity

The acceleration due to gravity, g, can vary with height!

$$F_G = \frac{GM_{\mathsf{E}}m}{r^2} = m\left(\frac{GM_{\mathsf{E}}}{r^2}\right) = mg$$

Depends on r the distance from the center of the Earth. Suppose an object is at height h above the surface of the Earth, then:

$$g = \frac{GM_{\mathsf{E}}}{(R_{\mathsf{E}} + h)^2}$$

g decreases as h increases.

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g is the not just the acceleration due to gravity, but also **the magnitude of the gravitational field**.

Fields

Gravitational force:

$$\vec{\mathbf{F}}_G = m(-\frac{GM\hat{\mathbf{r}}}{r^2}) = m\vec{\mathbf{g}}$$

Electrostatic force:

$$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

Gravitational field:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_G}{m}$$

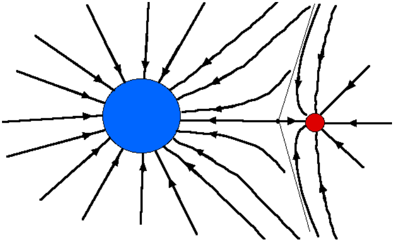
Electric field:

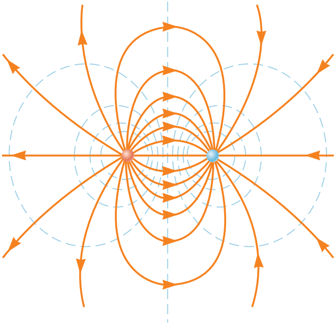
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_E}{q}$$

The field tells us what force a test particle of mass m (in the gravitational case) or charge q (in the electrostatic case) would feel at that point in space and time.

Examples of Fields

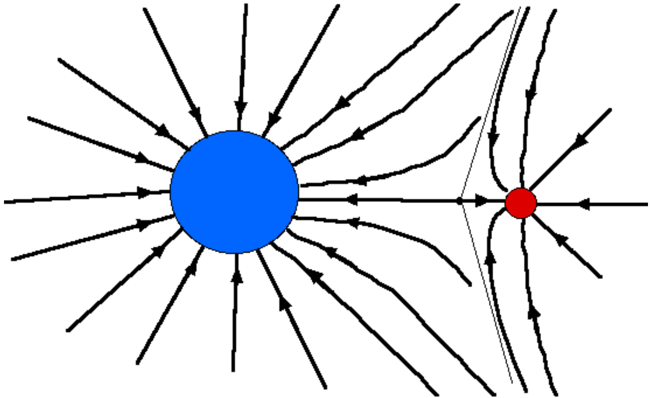
Fields are drawn with lines showing the direction of force that a test particle will feel at that point. The density of the lines at that point in the diagram indicates the approximate magnitude of the force at that point.





Examples of Fields

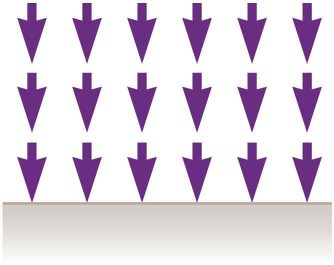
The gravitational field caused by the Sun-Earth system looks something like:



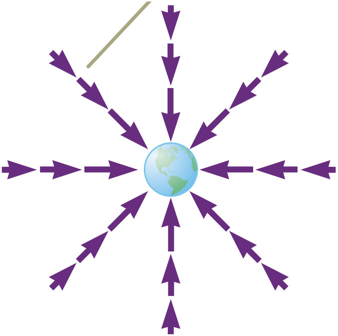
[1]Figure from http://www.launc.tased.edu.au

Gravitational Field of the Earth

Near the surface of the Earth:

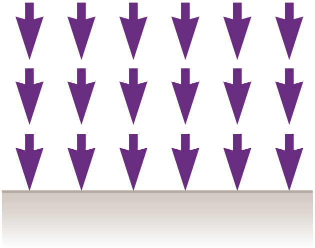


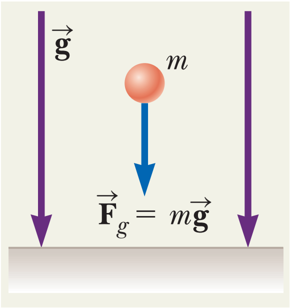
Farther out from the Earth:



Gravitational Field of the Earth

Uniform $\vec{\mathbf{g}}$:





A test mass m experiences a force $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$, where $\vec{\mathbf{g}}$ is the field vector.

Gravitational Potential

(Not gravitational potential energy!)

We can define a new quantity **gravitational potential**. Usually written Φ or V.

The change in gravitational potential is equal to the integral of the field along a path (with a minus sign).

$$\Delta\Phi = -\int \vec{\mathbf{g}} \cdot \mathrm{d}\,\vec{\mathbf{s}} \tag{1}$$

Notice: this is very similar to what we had for the relation between force and potential energy:

$$\Delta U = -\int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{s}} \tag{2}$$

In fact, eq (2) $= m\times$ eq (1)

Gravitational Potential

The gravitation potential can help us figure out what the gravitational potential energy will be if we put a particle of mass m at a particular point where the gravitational potential is Φ.

$$U = m\,\Phi$$

Gravitational Potential

$$\Delta\Phi = -\int \vec{\mathbf{g}} \cdot \mathrm{d}\,\vec{\mathbf{s}}$$

and so, the radial component of $\vec{\mathbf{g}}$ can be found by:

$$g_r = -\frac{\mathrm{d}\Phi}{\mathrm{d}r}$$

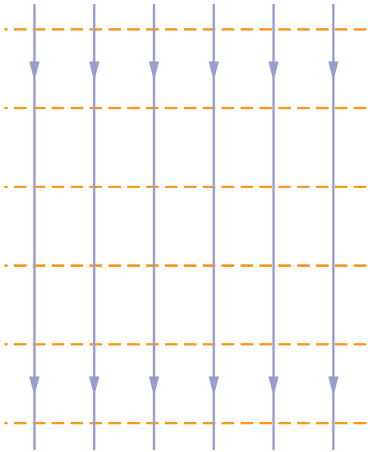
For the gravitational field around a point-like mass M,

$$\vec{\mathbf{g}} = -\frac{GM}{r^2}\hat{\mathbf{r}} \quad , \qquad \Phi = -\frac{GM}{r}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \quad , \qquad U = -\frac{GMm}{r}$$

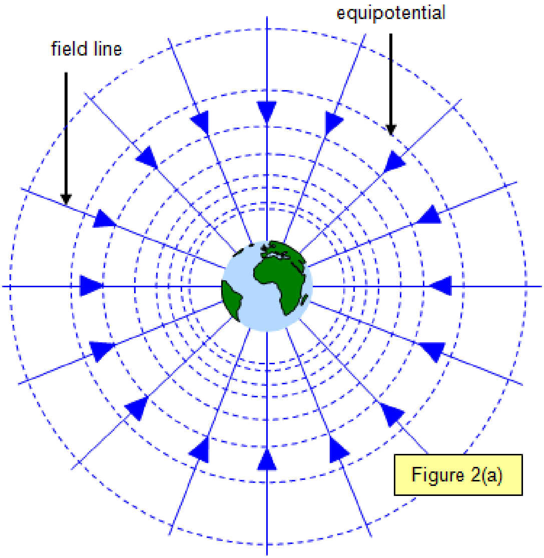
Gravitational Potential

A uniform field, as near the surface of the Earth.



The blue lines represent the gravitational field.
The orange dashed lines are surfaces of equal gravitational potential.

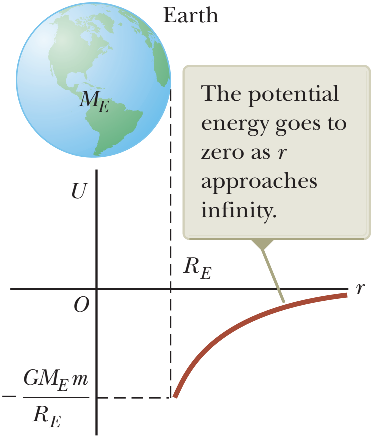
Gravitational Potential



[1] Figure from http://www.schoolphysics.co.uk/

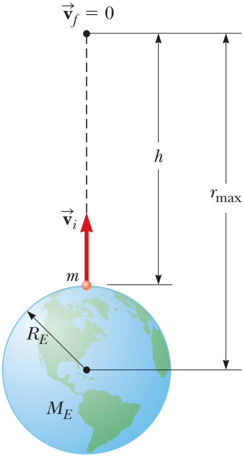
Gravitational Potential Energy

$$U(r) = -\frac{GM_E}{r}m = m\,\Phi_E$$



Escape Speed

How fast does an object need to be projected with to escape Earth's gravity?



(You solve this on the homework!)

Summary

- Newton's Law of Universal Gravitation
- gravitational potential energy
- little g

Final Exam Tuesday, Mar 24, via Canvas & Zoom, be ready at 9am.

Exam Test Run, by Canvas & Zoom. Today 2pm.

(Uncollected) Homework Serway & Jewett,

- Ch 13, onward from page 410. Probs: 3, 6[1], 11, 15, 31, 35, 44, 55 (I recommend you try these before you start 4B)

[1] Ans: $\vec{\mathbf{F}} = (-10.0\,\hat{\mathbf{i}} + 5.93\,\hat{\mathbf{j}}) \times 10^{-11}$ N