Kinematics
Varying Acceleration
Vectors

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Last time

- using the kinematics equations

Overview

- falling objects and g
- varying acceleration
- vectors and components (?)

Falling Objects

One common scenario of interest where acceleration is constant is falling objects.

Objects (with mass) near the Earth's surface accelerate at a constant rate of $\boldsymbol{g = 9.8\ \mathrm{ms}^{-2}}$. (Or, about 10 ms^{-2}) The kinematics equations for constant acceleration all apply.

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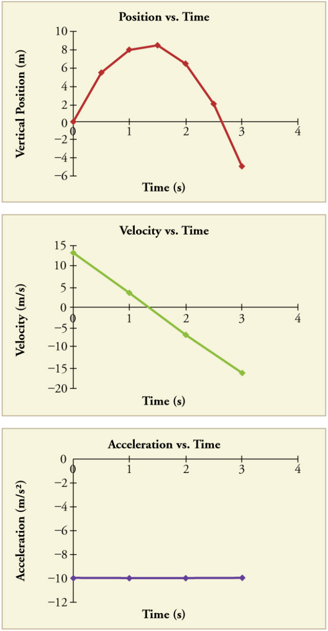
What if we threw it upwards with a speed of 10.0 m/s? Downwards at 10.0 m/s?

Falling Objects



[1]OpenStax Physics

Acceleration in terms of g

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Consider the drag race car in the earlier example. For the car the maximum acceleration was $a = 26.0$ m s^{-2}. How many gs is that?

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B roughly 1

C roughly 2 and a half

D 26

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Acceleration of a Falling Object

Question

A baseball is thrown straight up. It reaches a peak height of 15 m, measured from the ground, in a time 1.7 s. Treating "up" as the positive direction, what is the acceleration of the ball when it reached its peak height?

A 0 m/s^2

B -8.8 m/s^2

C $+8.8 \text{ m/s}^2$

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[1] Princeton Review: Cracking the AP Physics Exam

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1-D Kinematics with varying acceleration

If we have a varying acceleration $\vec{\mathbf{a}}(t)$, we should use calculus **not** the kinematics equations.

In the homework (question 57), there is a situation where the jerk, $J = \frac{\mathrm{da}}{\mathrm{dt}}$ is constant, but not zero.

In that case the acceleration is not constant:

$$\Delta a = \int_0^t J \, \mathrm{dt}'$$

so,

$$a(t) = a_i + Jt$$

The velocity cannot be found by using $v(t) = v_i + at$!

Instead, integrate again.

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1-D Kinematics with varying acceleration

Suppose you have a particle with a varying acceleration, but you don't know a as a function of t.

Instead, you know a as a function of position, x: $a(x)$

Can you use that knowledge to find changes in velocity, displacement, etc?

Yes!

Consider the integral:

$$\int a(x)\,\mathrm{dx}$$

It's not immediately obvious what it's physical meaning is, but let's try to investigate...

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Rearranging:

$$\int a \, \mathrm{dx} = \int \frac{\mathrm{dx}}{\mathrm{dt}} \left(\frac{\mathrm{dv}}{\mathrm{dx}} \, \mathrm{dx} \right)$$

Chain rule again, and using $v = \frac{\mathrm{dx}}{\mathrm{dt}}$:

$$\int a \, \mathrm{dx} = \int v \, \mathrm{dv}$$

1-D Kinematics with varying acceleration

$$\int_{x_i}^{x_f} a\,\mathrm{dx} = \int_{v_i}^{v_f} v\,\mathrm{dv} = \left.\frac{v^2}{2}\right|_{v_i}^{v_f}$$

So, we find that $\int a(x)\,\mathrm{dx}$ does mean something physically:

$$\frac{1}{2}(v_f^2 - v_i^2) = \int a\,\mathrm{dx}$$

IF a is a constant:

$$\frac{1}{2}(v_f^2 - v_i^2) = a\,\Delta x \quad \Rightarrow \quad v_f^2 = v_i^2 + 2a\Delta x.$$

If a varies with x, all we need to do is evaluate the integral $\int a(x)\,\mathrm{dx}$.

1-D Kinematics with varying acceleration

An asteroid falls in a straight line toward the Sun, starting from rest when it is 1.00 million km from the Sun. Its acceleration is given by, $a = -\frac{k}{x^2}$ where x is the distance from the Sun to the asteroid, and $k = 1.33 \times 10^{20}$ m^3/s^2 is a constant.

After it has fallen through half-a-million km, what is its speed?

Summary

- falling objects
- varying acceleration
- vectors and components

Assignment due Thursday, Jan 16.

(Uncollected) Homework Serway & Jewett,

- Ch 2, onward from page 49. Probs: 53, 56, 57, 59