2D Motion
Vectors

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Last time

- falling objects
- varying acceleration

Overview

- vectors
- addition and subtraction of vectors

1-D Kinematics with varying acceleration

An asteroid falls in a straight line toward the Sun, starting from rest when it is 1.00 million km from the Sun. Its acceleration is given by, $a = -\frac{k}{x^2}$ where x is the distance from the Sun to the asteroid, and $k = 1.33 \times 10^{20}$ m^3/s^2 is a constant.

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\begin{aligned}
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v^2 &= 2\left[\frac{k}{x_f} - \frac{k}{x_i}\right] \\
v &= \underline{5.16 \times 10^5\ \mathrm{m/s}}
\end{aligned}
$$

Math you will need for 2-Dimensions

Before going into motion in 2 dimensions, we will review some things about vectors.

Vectors

scalar

A scalar quantity indicates an amount. It is represented by a real number. (Assuming it is a physical quantity.)

vector

A vector quantity indicates both an amount and a direction. It is represented more than one real number. (Assuming it is a physical quantity.)

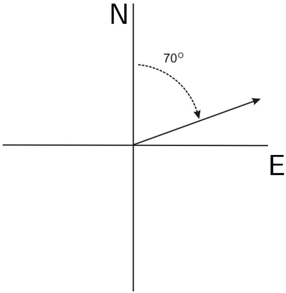
There are many ways to represent a vector.

- a magnitude and (an) angle(s)
- magnitudes in several perpendicular directions

Representing Vectors: Angles

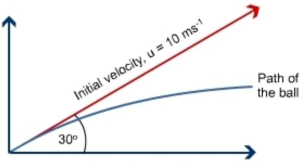
Bearing angles

Example, a plane flies 750 km h^{-1} at a bearing of 70°



Generic reference angles

A baseball is thrown at 10 m s^{-1}, 30° above the horizontal.



Representing Vectors: Unit Vectors

Magnitudes in several perpendicular directions: using *unit vectors.*

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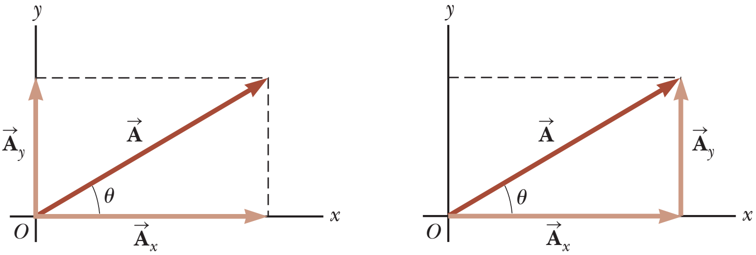
In two dimensions, a pair of perpendicular unit vectors are usually denoted $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ (or sometimes $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$).

Components

Consider the 2 dimensional vector $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, where A_x and A_y are numbers.

We then say that A_x is the i-component (or x-component) of $\vec{\mathbf{A}}$ and A_y is the j-component (or y-component) of $\vec{\mathbf{A}}$.



Notice that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Components vs Magnitude-and-Angle Notation

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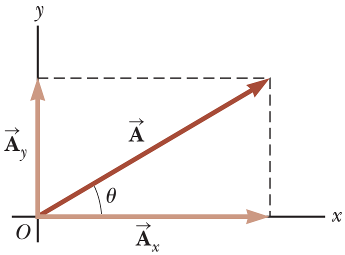

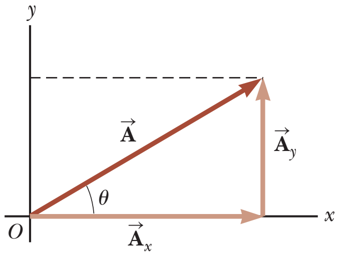

Also notice,

$$A = |\vec{\mathbf{A}}| = \sqrt{A_x^2 + A_y^2}$$

and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

if the angle is given as shown.

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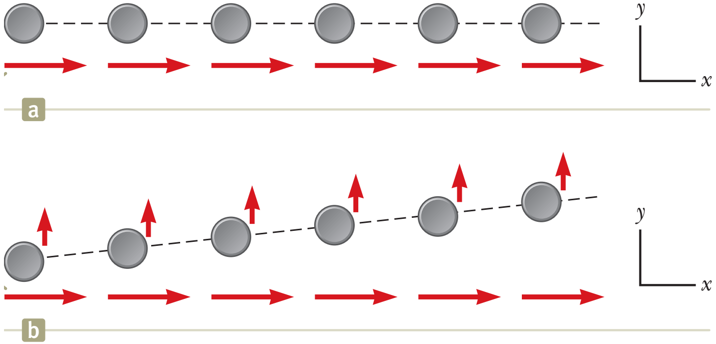
This makes life much easier: we will be able to solve for motion in the x direction separately from motion in the y direction.

Visualizing Motion in 2 Dimensions

Imagine an air hockey puck moving with horizontally constant velocity:



If it experiences a momentary upward (in the diagram) acceleration, it will have a component of velocity upwards.
The horizontal motion remains unchanged!

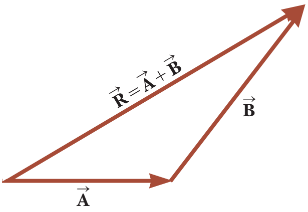
Vectors Properties and Operations

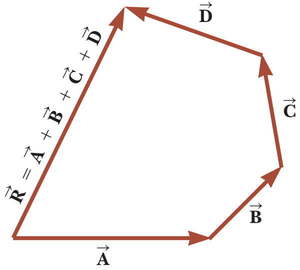
Equality

Vectors $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if and only if the magnitudes and directions are the same. (Each component is the same.)

Addition

$\vec{\mathbf{A}} + \vec{\mathbf{B}}$

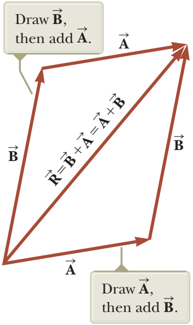




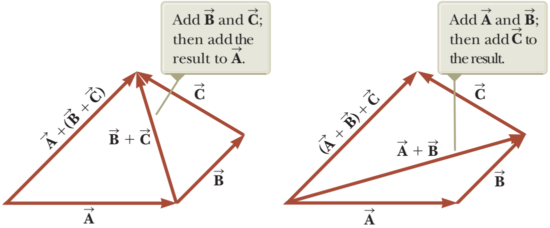
Vectors Properties and Operations

Properties of Addition

- $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$ (commutative)



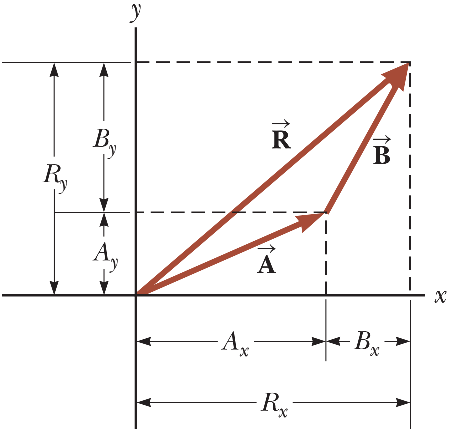
- $(\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} = \vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}})$ (associative)



Vectors Properties and Operations

Doing addition:

Almost always the right answer is to break each vector into components and sum each component independently.



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Vector Addition Example

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. What is the magnitude and direction of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip?

[0] Based on S&J Example 3.5, pg 69.

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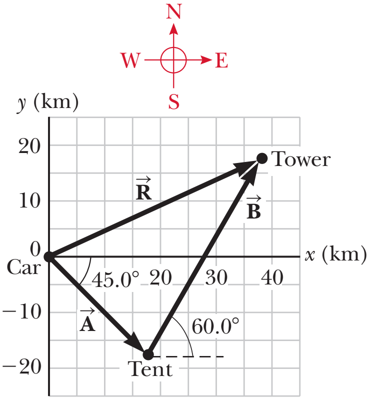

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$A_x = A\cos(-45.0^\circ) = 17.7$ km
$A_y = A\sin(-45.0^\circ) = -17.7$ km

$B_x = B\cos(60.0^\circ) = 20.0$ km
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$$\begin{aligned}
\vec{\mathbf{R}} &= (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \\
&= (17.7 + 20)\hat{\mathbf{i}} + (-17.7 + 34.6)\hat{\mathbf{j}} \text{ km} \\
&= 37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}} \text{ km} \\
&= 41.3 \text{ km at } 24.2^\circ \text{ north of east}
\end{aligned}$$

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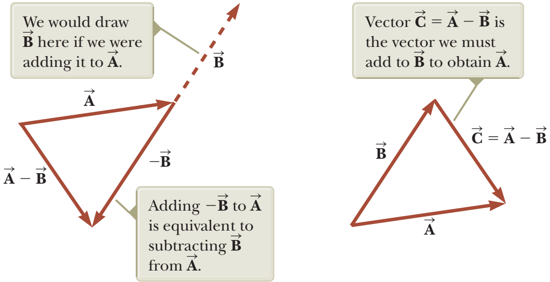
Vectors Properties and Operations

Negation

If $\vec{\mathbf{u}} = -\vec{\mathbf{v}}$ then $\vec{\mathbf{u}}$ has the same magnitude as $\vec{\mathbf{v}}$ but points in the **opposite** direction.

Subtraction

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$



Summary

- vectors
- vector addition and subtraction

Assignment due Thursday, Jan 16.

(Uncollected) Homework Serway & Jewett,

- Ch 3, onward from page 71. Objective Qs: 1 & 3; Conc Qs: 5
  Probs: 31, 55, 65