2D Kinematics
Projectiles

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Last time

- varying acceleration example
- vectors
- addition of vectors

Overview

- subtraction of vectors
- motion in 2 dimesions
- projectiles
- height of a projectile
- range of a projectile

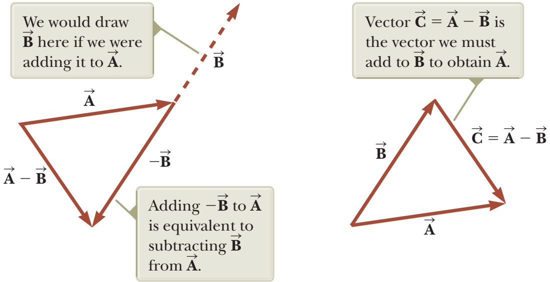
Vectors Properties and Operations

Negation

If $\vec{\mathbf{u}} = -\vec{\mathbf{v}}$ then $\vec{\mathbf{u}}$ has the same magnitude as $\vec{\mathbf{v}}$ but points in the **opposite** direction.

Subtraction

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$

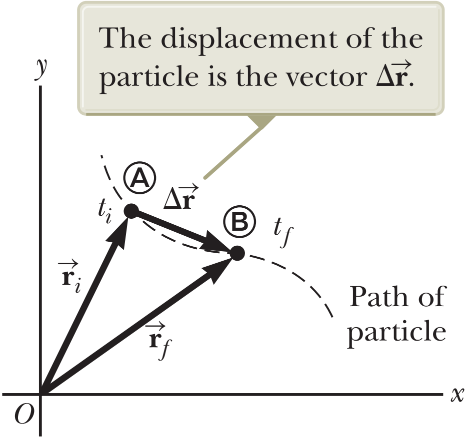


Motion in 2 Dimensions

All the same kinematics definitions and equations apply in 2 dimensions.

We can use our knowledge of vectors to solve separately the motion in the x and y directions.

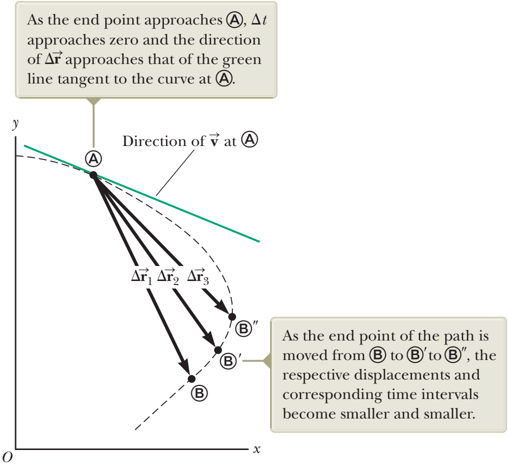
Motion in 2 Dimensions



$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\overrightarrow{\Delta r} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

Motion in 2 Dimensions



$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t+\Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

Velocity in 2 Dimensions

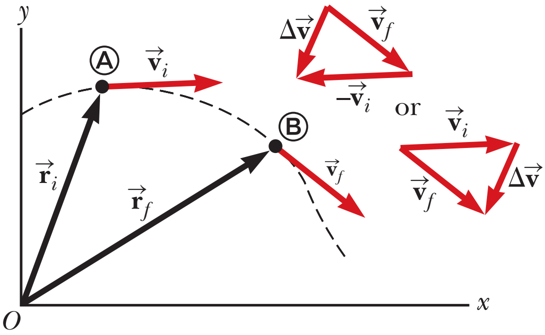
Different directions are independent $\Rightarrow$ differentiate separately!

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\begin{aligned}
\vec{\mathbf{v}} &= \frac{\mathrm{d}\,\vec{\mathbf{r}}}{\mathrm{dt}} \\
&= \frac{\mathrm{dx}}{\mathrm{dt}}\,\hat{\mathbf{i}} + \frac{\mathrm{dy}}{\mathrm{dt}}\,\hat{\mathbf{j}} \\
\vec{\mathbf{v}} &= v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}
\end{aligned}$$

(Differentiation is a linear operation.)

Acceleration in 2 Dimensions

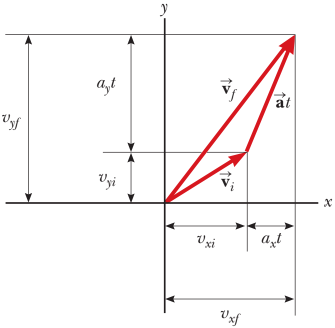


$$\overrightarrow{\Delta \mathbf{v}} = \vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$$

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t)}{\Delta t} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}}$$

Kinematic Equations in 2 Dimensions

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\, t$$



Kinematic Equations in 2 Dimensions

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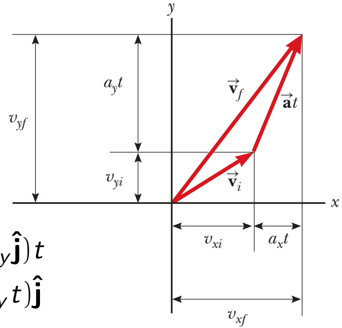

$$\begin{aligned} \vec{\mathbf{v}}_f &= (v_{x,i}\hat{\mathbf{i}} + v_{y,i}\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})\,t \\ v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} &= (v_{x,i} + a_x t)\hat{\mathbf{i}} + (v_{y,i} + a_y t)\hat{\mathbf{j}} \end{aligned}$$

Equating *x*-components (**i**-components):

$$v_x = v_{x,i} + a_x t$$

Equating *y*-components (**j**-components):

$$v_y = v_{y,i} + a_y t$$

Kinematic Equations in 2 Dimensions

The other kinematics equations work basically the same way as $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t$.

For the scalar equation, it holds for each component:

$$v_{f,x}^2 = v_{i,x}^2 + 2a_x\Delta x$$

$$v_{f,y}^2 = v_{i,y}^2 + 2a_y\Delta y$$

Projectiles

projectile

Any object that is thrown. We will use this word specifically to refer to thrown objects that experience a vertical acceleration g.

Assumption

Air resistance is negligible.

Why do we care?

Projectiles

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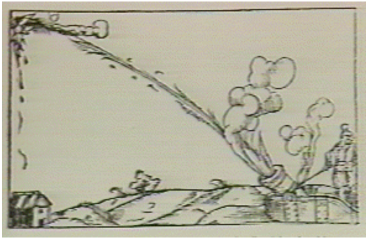
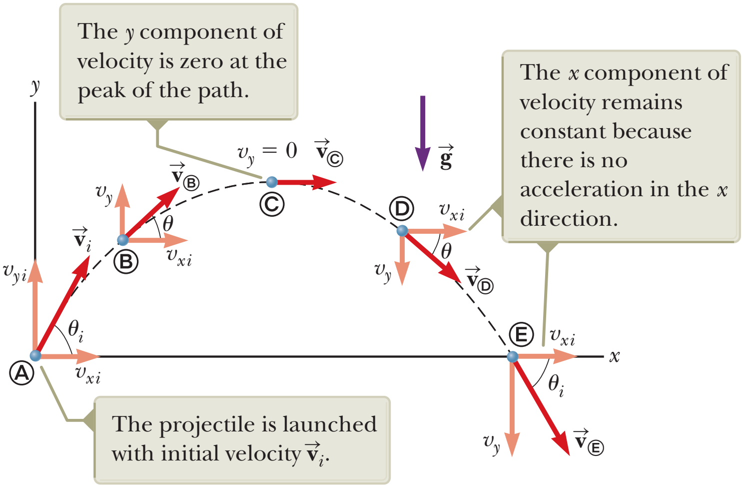
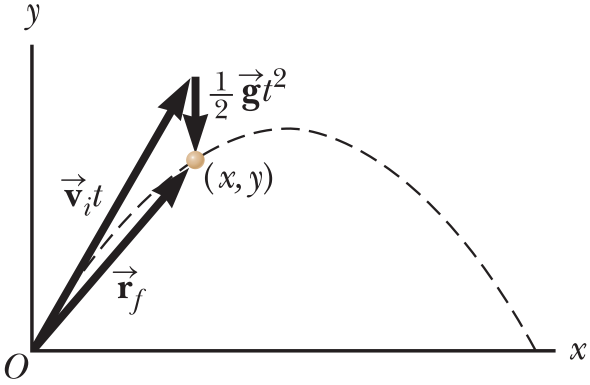
Historically...



Projectile Velocity



Vector Addition can give a Projectile's Trajectory



$$\Delta \boldsymbol{r} = \vec{\mathbf{r}}_f - 0 = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}} t^2$$

Principle Equations of Projectile Motion

(Notice, these are just special cases of the kinematics equations!)

$$\Delta x = v_{ix} t$$

$$\Delta y = v_{iy} t - \frac{1}{2} g t^2$$

$$v_x = v_{ix}$$

$$v_y = v_{iy} - g t$$

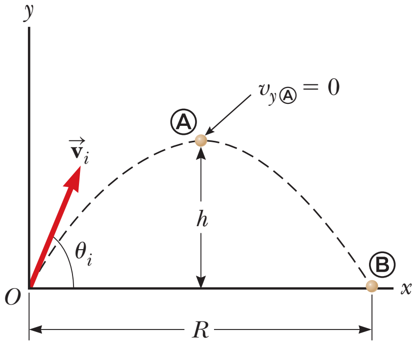
$$v_x^2 = v_{ix}^2$$

$$v_y^2 = v_{iy}^2 - 2g(\Delta y)$$

Height of a Projectile

How can we find the maximum height that a projectile reaches?

c



Height of a Projectile

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Find the height when $v_y = 0$.

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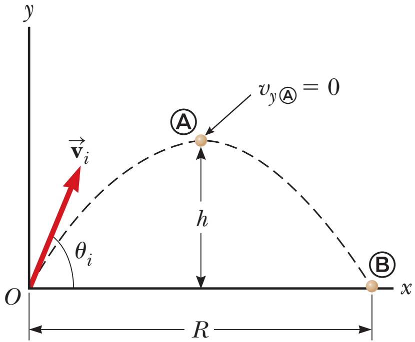

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$$v_{f,y}^2 = v_{i,y}^2 - 2g\Delta y$$

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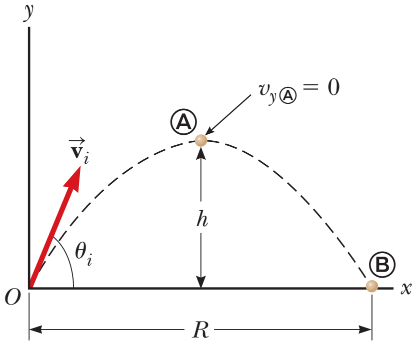

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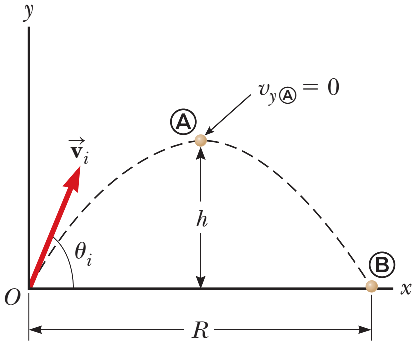


Find the height when $v_y = 0$.

$$v_{f,y}^2 = v_{i,y}^2 - 2g\Delta y$$

$$0 = v_{y,i}^2 - 2gh$$

$$h = \frac{v_{y,i}^2}{2g}$$

In the diagram, $v_{y,i} = v_i \sin\theta$.

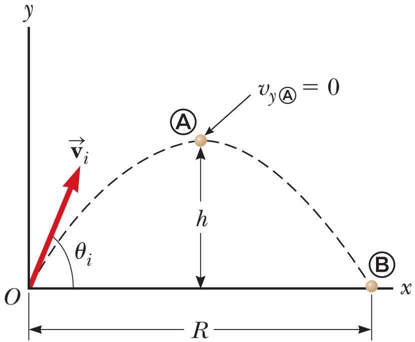
$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

Time of Flight of a Projectile

time of flight

The time from launch to when projectile hits the ground.

How can we find the time of flight of a projectile?



Assuming that it is launched from the ground and lands on the ground at the same height...

Time of Flight of a Projectile

There are several ways to find an expression for this time.

One way: Notice that just when striking the ground, $\Delta y = 0$.

$$\begin{aligned} \Delta y &= v_{y,i} t + \frac{1}{2} a_y t^2 \\ 0 &= v_i \sin\theta\, t - \frac{1}{2} g t^2 \end{aligned}$$

Now cancel a factor of t. Warning! This will remove one solution to this equation in t. What is it?

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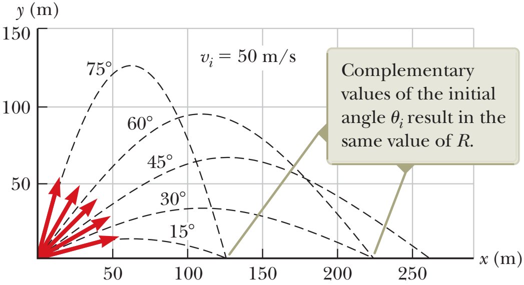
$$\frac{1}{2} g t = v_i \sin\theta$$

$$t_{\text{flight}} = \frac{2 v_i \sin\theta}{g}$$

Time of Flight of a Projectile

Quick Quiz 4.3[1] Rank the launch angles for the five paths in the figure with respect to time of flight from the shortest time of flight to the longest. (Assume the magnitude v_i remains the same.)



A 15°, 30°, 45°, 60°, 75°

B 45°, 30°, 60°, 15°, 75°

C 15°, 75°, 30°, 60°, 45°

D 75°, 60°, 45°, 30°, 15°

[1] Page 86, Serway & Jewett

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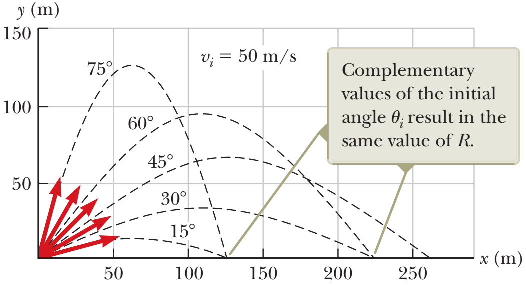

A 15°, 30°, 45°, 60°, 75° ←

B 45°, 30°, 60°, 15°, 75°

C 15°, 75°, 30°, 60°, 45°

D 75°, 60°, 45°, 30°, 15°

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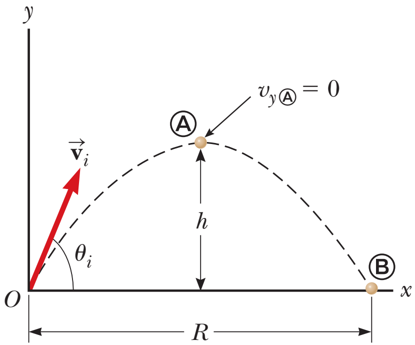
[1] Page 86, Serway & Jewett

Range of a Projectile

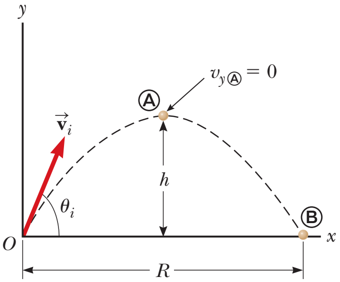
range

The distance in the horizontal direction that a projectile covers before hitting the ground.

How can we find the range of a projectile?



Range of a Projectile

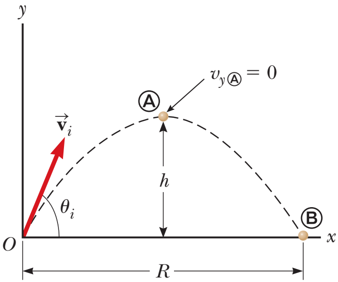


There is no acceleration in the x-direction! ($a_x = 0$)

$$\Delta x = v_x t$$

We just need t. But t is the time of flight!

Range of a Projectile



$$\Delta x = v_x t$$

$$R = v_i \cos\theta \left(\frac{2v_i \sin\theta}{g}\right)$$

$$R = \frac{2v_i^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

Range of a Projectile

A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. How far does he jump in the horizontal direction?

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$$
\begin{aligned}
R &= \frac{v_i^2 \sin(2\theta)}{g} \\
&= \frac{(11.0 \text{ m/s})^2 \sin(2 \times 20.0)}{9.8 \text{ m/s}^2} \\
&= \underline{7.94 \text{ m}}
\end{aligned}
$$

Summary

- motion in 2 dimesions
- projectile motion

Quiz start of class, Friday, Jan 17.

Assignment due tomorrow.

(Uncollected) Homework Serway & Jewett,

- Ch 4, onward from page 102. Conc Qs: 1, 3; Probs: 7
- Ch 4 Probs: 11†, 15, 13, 19, 21, 29, 56, 59

† Go Mountain Lions!