# Resistive Forces

## 1 Resistive Forces

Fluid resistance is a force that occurs when an object is moving through a fluid medium. It explains why different objects can fall at different rates through air. It also explains why airplanes need to run their engines to keep moving through air. These forces depend on the velocity of the object relative to the fluid.

## 2 Two Models

1. $R \sim v$ "Stokes Drag"

   This model is appropriate when:

   - slow moving objects
   - very viscous fluids
   - laminar (smooth) flow in the fluid

   The equation for the fluid resistance in this model is:

   $$\mathbf{R} = -b\mathbf{v}$$

   The minus sign only indicates that the force $\mathbf{R}$ is in the opposite direction to the velocity vector $\mathbf{v}$. The *magnitude* of the force is

   $$R = bv$$

2. $R \sim v^2$ "the Drag Equation"

   This model is appropriate when:

   - fast moving objects
   - low viscousity fluids
   - turbulent flow in the fluid

The equation for the fluid resistance in this model is:

$$\mathbf{R} = \frac{1}{2} D\rho A v^2 \ (-\hat{\mathbf{v}})$$

where $\hat{\mathbf{v}}$ is a unit vector in the direction of the objects velocity so the $-\hat{\mathbf{v}}$ factor simply indicates that the force is in the opposite direction to the velocity. The *magnitude* of the force is

$$R = \frac{1}{2} D\rho A v^2$$

# 3 When to use these models

Resistive forces occur whenever an object moves through a fluid (gas or liquid). This includes boats in water, submarines, airplanes, skydivers, cars, *etc.*

One specific case where these can be applied is to **falling objects** (*eg.* skydivers).

## 3.1 Falling objects

An object falling (due to the force of gravity) through a fluid will accelerate until effectively reaching a terminal velocity, but it will not go faster than that. This is a situation that only applies to falling objects, though of course, resistive forces can occur in other circumstances as well. Terminal velocities can be found by noting that at equilibrium the object no longer accelerates.

To find a terminal velocity:

$$\begin{aligned} \mathbf{F}_{\text{net}} &= 0 \\ mg - R(v_T) &= 0 \\ R(v_T) &= mg \end{aligned}$$

where $R(v_T)$ is the value of the resistive force when $v = v_T$. Since $R$ is a function of $v$, putting in the appropriate function for $R$, the terminal velocity can be found. See the lecture notes for each case.

## 3.2 Powered motion

In the case of powered motion, say a submarine moving through water, we might find the amount of force $F_{\text{thrust}}$ needed to keep the object moving at a particular constant velocity, $v$. In this case:

$$\begin{aligned} \mathbf{F}_{\text{net}} &= 0 \\ F_{\text{thrust}} - R(v) &= 0 \\ F_{\text{thrust}} &= R(v) \end{aligned}$$

Putting in the appropriate function for $R(v)$ and the desired value of the velocity gives the value of $F_{\text{thrust}}$ needed to keep the object moving at the desired speed.