Physics 4B Fall 2015 Final Test

Name: ______________________________

Dec 9, 2015

Please show your work! Answer as many questions as you can, in any order. Calculators are allowed. Books, notes, and internet connections are not allowed. Use any blank space to answer questions, but please make sure it is clear which question your answer refers to.
DO NOT OPEN TEST BOOKLET UNTIL TOLD TO DO SO.

Maxwell's Equations

$$\oint \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0} \qquad \oint \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{dt}} \qquad \oint \mathbf{B}\cdot \mathrm{d}\mathbf{s} = \mu_0\, I_{\text{enc}} + \mu_0\epsilon_0\, \frac{\mathrm{d}\Phi_{\mathrm{E}}}{\mathrm{dt}}$$

Constants

$k = \frac{1}{4\pi\epsilon_0} = 8.99\times 10^{9}\ \mathrm{Nm^2C^{-2}}$	$\epsilon_0 = 8.854\times 10^{-12}\ \mathrm{C^2\,N^{-1}m^{-2}}$	$\mu_0 = 4\pi\times 10^{-7}\ \mathrm{H/m}$
$g = 9.807\ \mathrm{m\,s^{-2}}$	$e = 1.602\times 10^{-19}\ \mathrm{C}$	$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.998\times 10^{8}\ \mathrm{m/s}$
$m_e = 9.109\times 10^{-31}\ \mathrm{kg}$	$m_p = 1.673\times 10^{-27}\ \mathrm{kg}$	$m_n = 1.675\times 10^{-27}\ \mathrm{kg}$

Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\sin\left(\theta+\tfrac{\pi}{2}\right) = \cos\theta$$

$$\cos\left(\theta+\tfrac{\pi}{2}\right) = -\sin\theta$$

$$\sec\theta := \frac{1}{\cos\theta}$$

$$\csc\theta := \frac{1}{\sin\theta}$$

$$\cot\theta := \frac{1}{\tan\theta}$$

$\mathbf{F} = q\mathbf{E} + q\,\mathbf{v}\times\mathbf{B}$ $\quad$ $\mathbf{F} = \frac{k\,q_1 q_2}{r^2}\,\hat{\mathbf{r}}$

$E = \frac{\sigma}{2\epsilon_0}$ $\quad$ $E = \frac{\sigma}{\epsilon_0}$ $\quad$ $E = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$

$\Phi_E = \int \mathbf{E}\cdot \mathrm{d}\mathbf{A}$ $\quad$ $\Phi_B = \int \mathbf{B}\cdot \mathrm{d}\mathbf{A}$

$U = q\,V$ $\quad$ $U = \frac{kq_1q_2}{r}$ $\quad$ $\Delta V = -\int \mathbf{E}\cdot \mathrm{d}\mathbf{s}$

$\mathbf{p} = q\mathbf{d}$ $\quad$ $\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$ $\quad$ $U = -\mathbf{p}\cdot\mathbf{E}$

$\boldsymbol{\mu} = NI\mathbf{A}$ $\quad$ $\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$ $\quad$ $U = -\boldsymbol{\mu}\cdot\mathbf{B}$

$\mathbf{E} = \frac{1}{2\pi\epsilon_0}\frac{\mathbf{p}}{z^3}$ $\quad$ $\mathbf{B} = \frac{\mu_0}{2\pi}\frac{\boldsymbol{\mu}}{z^3}$

$E = \frac{k\,q\,z}{(R^2+z^2)^{3/2}}$ $\quad$ $B = \frac{\mu_0\,I\,R^2}{2(R^2+z^2)^{3/2}}$

$\mathbf{F} = I\,\mathbf{L}\times\mathbf{B}$ $\quad$ $\frac{F_B}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\,\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$ $\quad$ $\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$ $\quad$ $\mathbf{B} = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$

$B = \frac{\mu_0 I}{2\pi r}$ $\quad$ $B = \frac{\mu_0 I\,r}{2\pi R^2}$ $\quad$ $B = \mu_0 I n$

$r = \frac{mv_\perp}{|q|B}$ $\quad$ $T = \frac{2\pi m}{|q|B}$ $\quad$ $p = v_\parallel T$

$C = \frac{Q}{\Delta V}$ $\quad$ $C = \frac{\kappa\,\epsilon_0 A}{d}$

$C = 2\pi\epsilon_0\,\frac{L}{\ln(b/a)}$ $\quad$ $C = 4\pi\epsilon_0\,\frac{ab}{b-a}$

$L = \frac{N\Phi_B}{i}$ $\quad$ $L = \mu_0 n^2 A\ell$ $\quad$ $\mathcal{E}_L = -L\,\frac{\mathrm{di}}{\mathrm{dt}}$

$M = \frac{N_1\Phi_{B,12}}{i_2} = \frac{N_2\Phi_{B,21}}{i_1}$ $\quad$ $\mathcal{E}_{M,1} = -M\,\frac{\mathrm{di}_2}{\mathrm{dt}}$ $\quad$ $\Delta V_s = \Delta V_p\,\frac{N_s}{N_p}$

$U = \frac{1}{2}\frac{Q^2}{C}$ $\quad$ $u = \frac{1}{2}\epsilon_0 E^2$

$U_B = \frac{1}{2}Li^2$ $\quad$ $u_B = \frac{B^2}{2\mu_0}$

$\mathcal{E} = \frac{\mathrm{dW}}{\mathrm{dq}}$ $\quad$ $P = \frac{\mathrm{dW}}{\mathrm{dt}}$ $\quad$ $P = I\,\Delta V$

$R = \frac{\Delta V}{I}$ $\quad$ $J = \frac{I}{A}$ $\quad$ $\rho = \frac{E}{J} = \frac{1}{\sigma}$

$R = \frac{\rho L}{A}$ $\quad$ $\rho - \rho_0 = \rho_0\alpha(T - T_0)$

$v_d = \frac{J}{ne}$ $\quad$ $\Delta V_H = \frac{B\,I}{e\,n\,t}$

$q(t) = C\mathcal{E}(1 - e^{-t/\tau_C})$ $\quad$ $i(t) = \frac{\mathcal{E}}{R}e^{-t/\tau_C}$ $\quad$ $\Delta v_C(t) = \mathcal{E}(1 - e^{-t/\tau_C})$

$q(t) = Q_0 e^{-t/\tau_C}$ $\quad$ $i(t) = I_0 e^{-t/\tau_C}$ $\quad$ $\Delta v_C(t) = (\Delta V_0)e^{-t/\tau_C}$

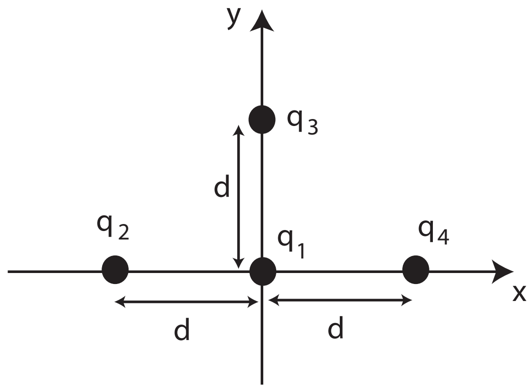
$\tau_C = RC$ $\quad$ $i(t) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L})$ $\quad$ $\Delta v_L(t) = \mathcal{E}e^{-t/\tau_L}$

$\tau_L = \frac{L}{R}$ $\quad$ $i(t) = I_0 e^{-t/\tau_L}$ $\quad$ $\Delta v_L(t) = (\Delta V_0)e^{-t/\tau_L}$

$I_d = \epsilon_0\,\frac{\mathrm{d}\Phi_\mathrm{E}}{\mathrm{dt}}$ $\quad$ $c = f\lambda$ $\quad$ $c = \frac{E}{B}$

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$ $\quad$ $P_\mathrm{rad} = \frac{S}{c}$ $\quad$ $P_\mathrm{rad} = \frac{2S}{c}$

1. In the diagram charges $q_1 = +50$ nC, $q_2 = -30$ nC, $q_3 = +10$ nC, and $q_4 = +30$ nC, and $d = 10$ cm.



(a) What is the electric field at the location of charge q_3 due to the other three charges? [5 pts]

(b) What is the force on charge q_3? [3 pts]

(c) What is the electric potential at the location of charge q_3 due to the other three charges? [3 pts]

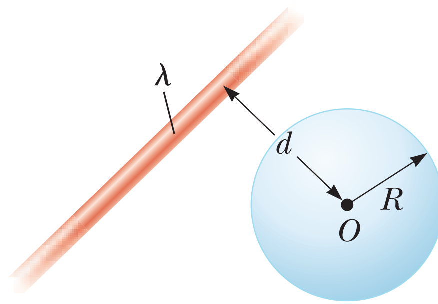
(d) How much work must be done to move q_3 from very, very far away from the other three charges up to its location in the diagram? [3 pts]

2. An infinitely long line charge having a uniform charge per unit length λ lies a perpendicular distance d from point O. Determine (and give your reasoning) the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge when

   (a) $R < d$ and [2 pts]

   (b) $R > d$. [4 pts]

   (c) How does the flux through the surface depend on R when as $R \gg d$, so that $\frac{d}{R} \to 0$? [3 pts]



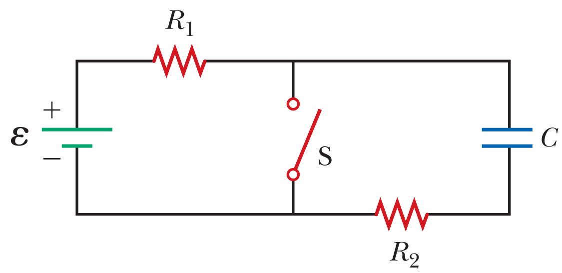
3. A proton of kinetic energy 4.80 MeV travels head-on toward a stationary lead nucleus. The atomic number of lead is 82. Assuming that the proton does not penetrate the nucleus, that the only force between proton and nucleus is the Coulomb force, and that since the lead nucleus is so much more massive than the proton its recoil will be negligible,

   (a) calculate the smallest center-to-center separation d_p between proton and nucleus when the proton momentarily stops. [6 pts]

   (b) If the proton were replaced with an alpha particle (which contains two protons) of the same initial kinetic energy, the alpha particle would stop at center-to-center separation d_a. What is d_a/d_p? [3 pts]

4. A common flashlight bulb is rated at 0.45 A and 3.6 V (the values of the current and voltage under operating conditions, in other words, when the bulb is on and warmed up). If the resistance of the tungsten bulb filament at room temperature (20°C) is 1.4 Ω, what is the temperature of the filament when the bulb is on? (Tungsten has a temperature coefficient of resistivity, $\alpha = 4.5 \times 10^{-3}$ K^{-1} and a resistivity of $\rho = 5.25 \times 10^{-8}$ Ωm at room temperature.) [6 pts]

5. In the circuit shown, the switch S has been open for a long time. It is then suddenly closed. The resistors are of equal value $R_1 = R_2 = R$, the emf of the battery is $\mathcal{E}$, and the capacitor has capacitance C. Determine the time constant



(a) before the switch is closed τ_1 and [2 pts]

(b) after the switch is closed, τ_2. [2 pts]

(c) Let capacitor be initially charged and the switch be closed at t = 0. Determine the current in the switch as a function of time. [5 pts]

(d) Suppose the switch is closed at time $t = 0$ and opened at $5\tau_1$, then closed again at $10\tau_1$, then opened at time $15\tau_1$. Sketch a plot of the potential difference across the capacitor with time. Your time axis should go to $20\tau_1$. [4 pts]

6. A solenoid with N turns has a radius R and an overall length of ℓ.

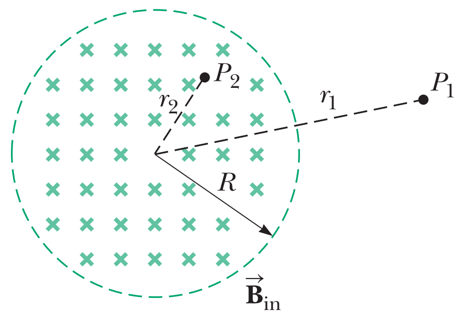
   (a) What is its inductance? [2 pts]

   (b) When a current i flows in the solenoid, what is the magnetic flux through each turn of the solenoid? [3 pts]

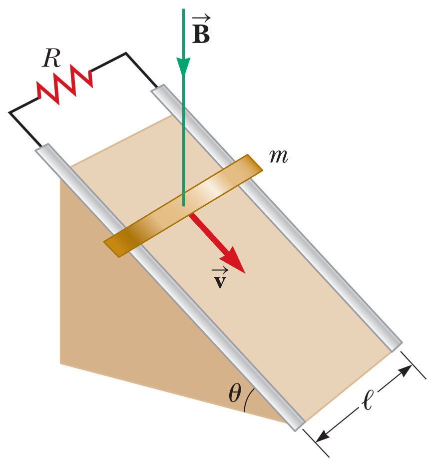
   (c) The diagram shows a cross-sectional view of the magnetic field inside the solenoid looking down its length. The current in the solenoid varies as $i(t) = at^2 + b$. Find an expression for the electric field at point P_1. [5 pts]

   (d) Find an expression for the electric field at point P_2. [4 pts]

   (e) If a and b are both positive numbers, what directions will the electric field have at points P_1 and P_2? [2 pts]



7. A bar of mass m that can slide without friction on a pair of rails separated by a distance ℓ and located on an inclined plane that makes an angle θ with respect to the ground. The resistance of the resistor is R and a uniform magnetic field of magnitude B is directed downward, perpendicular to the ground, over the entire region through which the bar moves.



(a) The rod will come to equilibrium and slide down the rails at a constant speed. When it is moving at constant speed, what current flows in the loop? [5 pts]

(b) What is the direction of the current flow? (Describe it or label it clearly on the diagram.) [1 pt]

(c) At what speed v does the bar slide along the rails? [5 pts]

8. A step-up transformer is designed to have an output voltage of 1800 V (rms) when the primary is connected across a 120-V (rms) source.

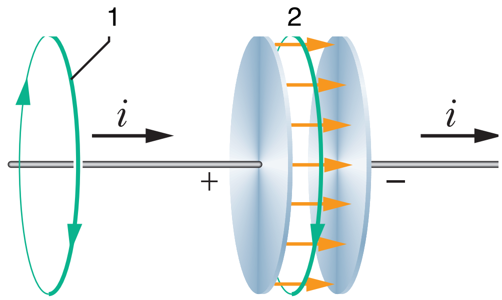
   (a) If the primary winding has exactly 60 turns, how many turns are required on the secondary? [3 pts]

   (b) If a load resistor across the secondary has a resistance 1.2 kΩ and so draws a current of 1.50 A, what is the current in the primary, assuming ideal conditions? [3 pts]

   (c) What is the effective equivalent resistance on the primary side? [3 pts]

   (d) If the transformer actually has an efficiency of 95.0%, what is the current in the primary when the secondary current is 1.50 A? [3 pts]

9. The diagram shows a capacitor composed of two circular plates of radius R connected to very long straight wires, carrying current i.



(a) Evaluate $\oint \mathbf{B} \cdot d\mathbf{s}$ for the two paths 1 and 2 (and flat circular surfaces enclosed by each path) shown, and show that it is the same for both paths. Assume that both paths have the same radius r and that $r > R$. [6 pts]

(b) Find an expression for $\oint \mathbf{B} \cdot d\mathbf{s}$ for path 2 when $r < R$. Show your work! [4 pts]

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