Electricity and Magnetism
Electric Potential Energy
Electric Potential

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De Anza College

Jan 23, 2018

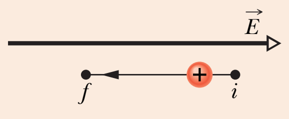
Last time

- implications of Gauss's law
- introduced electric potential energy

Warm Up Questions

In the figure, a proton moves from point i to point f in a uniform electric field directed as shown.

(a) Does the force of the electric field do positive or negative work on the proton?



(A) positive

(B) negative

[1] Halliday, Resnick, Walker, page 628.

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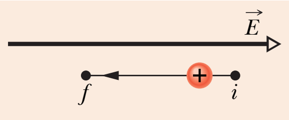

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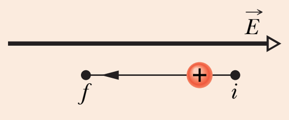
Warm Up Questions

In the figure, a proton moves from point i to point f in a uniform electric field directed as shown.

(b) Does the electric potential energy of the proton increase or decrease?



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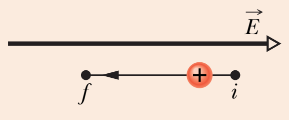

(A) increase $\longleftarrow$

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Warm Up Questions

Now we move the proton from point i to point f in a uniform electric field directed as shown with an applied force.

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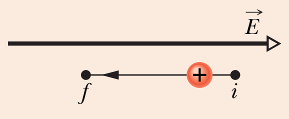

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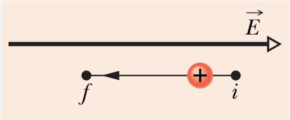

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Overview

- electric potential energy
- potential energy of a pair of charges
- potential energy of a configuration of many charges
- force vector fields and potential energy
- Electric potential

Potential Energy and the Electrostatic Force

The electrostatic force is a conservative force.

We can ask what is the stored energy (potential energy) of some particular configuration of charge.

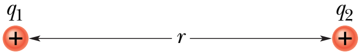
electric potential energy

The electric potential energy of a system of fixed point charges is equal to the work that must be done on the system by an external agent to assemble the system, bringing each charge in from an infinite distance.

Potential Energy of two point charges

Consider two charges q_1 and q_2 at a distance r.

They repel each other. Bringing them to that configuration requires work.

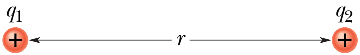


Define $U(\infty) = 0$ so that $U(r) = \Delta U = U(r) - U(\infty)$

Then, the potential energy of two point charges is:

$$U(r) = \frac{kq_1 q_2}{r}$$

Potential Energy of two point charges



Define $U(\infty) = 0$ so that $U(r) = \Delta U = U(r) - U(\infty)$

Then, the potential energy of this charge configuration is:

$$\begin{aligned} U(r) &= -\int_{\infty}^{r} \mathbf{F} \cdot \mathrm{d}\mathbf{s} \\ &= -\int_{\infty}^{r} \frac{k q_1 q_2}{(r')^2} \, \mathrm{d}r' \\ &= k\, q_1 q_2 \left[\frac{1}{r'} \right]_{\infty}^{r} \\ &= \frac{k\, q_1 q_2}{r} \end{aligned}$$

Potential Energy of many point charges

Suppose we have three point charges.

Let

$$U_{12} = \frac{k\, q_1 q_2}{r_{12}}$$

Then the total potential energy of the configuration is:

$$U_{\mathrm{net}} = U_{12} + U_{13} + U_{23}$$

Just add up all the pairwise potential energies!

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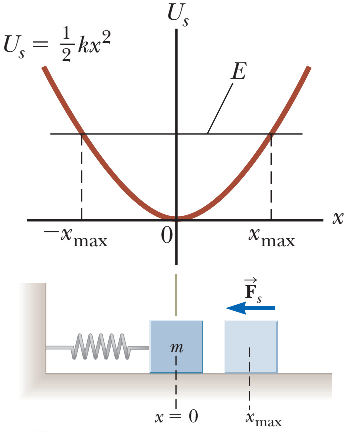
Just add up all the pairwise potential energies!

$$U_{\text{net}} = \sum_{ij} U_{ij}$$

Energy Diagrams

Potential energy can be plotted as a function of position. *eg.* potential energy of a spring:



$$F_{s,x} = -\frac{\mathrm{d}U_s}{\mathrm{d}x} = -kx$$

Fields and Potentials: Reminder Conservative Forces & Potential Energy

For conservative forces:

$$W_{\text{int}} = -\Delta U$$

This means that for a displacement along the x-axis:

$$\Delta U = -\int_{x_i}^{x_f} F_x \, \mathrm{dx}$$

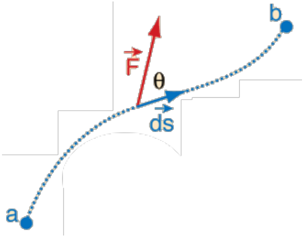
and

$$F_x = -\frac{\mathrm{dU}}{\mathrm{dx}}$$

Fields and Potentials: Reminder Conservative Forces & Potential Energy

In general, potential energy can be found by a path integral

$$\Delta U = -\int \mathbf{F} \cdot d\mathbf{s}$$



And the conservative force is the gradient of the potential energy function:

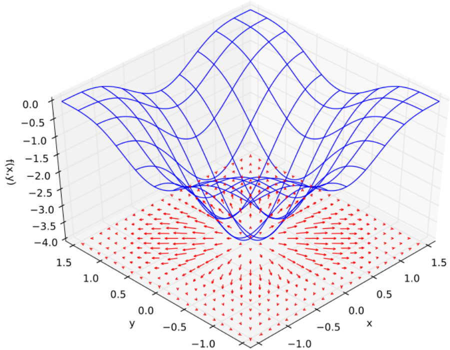
$$\mathbf{F} = -\boldsymbol{\nabla} U$$

Vector Differential Operations

Gradient of a scalar field at a point, f:

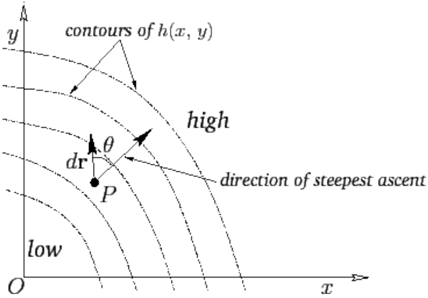
$$\nabla f = \frac{\partial}{\partial x} f \; \mathbf{i} + \frac{\partial}{\partial y} f \; \mathbf{j} + \frac{\partial}{\partial z} f \; \mathbf{k}$$

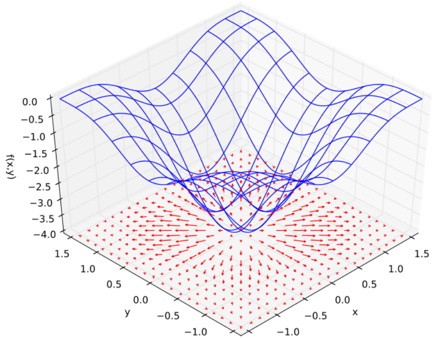
Measures the rate and direction of change in a *scalar* field.



[1]Figure from Wikipedia by IkamusumeFan.

Vector Differential Operations



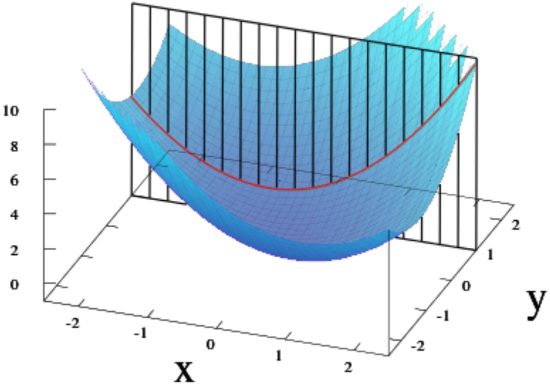


[1]Figure from http://farside.ph.utexas.edu/teaching (left); Wikipedia by IkamusumeFan (right)

Partial Derivatives

Consider a scalar function $f(x, y)$:

$$f(x, y) = x^2 + xy + y^2$$



$$\frac{\partial}{\partial x} f(x, y) = 2x + y \qquad \frac{\partial}{\partial y} f(x, y) = x + 2y$$

[1]Figure from Wikipedia by IkamusumeFan.

Vector Differential Operations

Gradient of a scalar field at a point, f:

$$\nabla f = \frac{\partial}{\partial x} f \ \mathbf{i} + \frac{\partial}{\partial y} f \ \mathbf{j} + \frac{\partial}{\partial z} f \ \mathbf{k}$$

Divergence of a vector field at a point $\mathbf{v} = [v_x, v_y, v_z]$:

$$\nabla \cdot \mathbf{v} = \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z$$

Curl of a vector field at a point $\mathbf{v}$:

$$\nabla \times \mathbf{v} = \left(\frac{\partial}{\partial y} v_z - \frac{\partial}{\partial z} v_y \right) \mathbf{i} + \left(\frac{\partial}{\partial z} v_x - \frac{\partial}{\partial x} v_z \right) \mathbf{j} + \left(\frac{\partial}{\partial x} v_y - \frac{\partial}{\partial y} v_x \right) \mathbf{k}$$

Vector Operations

A couple of useful identities:

For any scalar field f (with continuous 2nd derivatives),

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} f) = 0$$

$\boldsymbol{\nabla} f$ is an **irrotational (curl-free)** vector field. (Using now.)

Vector Operations

A couple of useful identities:

For any scalar field f (with continuous 2nd derivatives),

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} f) = 0$$

$\boldsymbol{\nabla} f$ is an **irrotational (curl-free)** vector field. (Using now.)

For any vector field $\mathbf{v}$ (with continuous 2nd derivatives),

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{v}) = 0$$

$\boldsymbol{\nabla} \times \mathbf{v}$ is an **solenoidal (divergence-free)** vector field. (Useful later.)

Connection to Vector Fields

Electrical forces, $\mathbf{F}$, can be represented as vector fields.[1]

Electrostatic force $\mathbf{F}$ is conservative ($\boldsymbol{\nabla} \times \mathbf{F} = 0$), we can define $\mathbf{F} = -\boldsymbol{\nabla} U$, where U is a scalar field.

For any scalar field U (with continuous 2nd derivatives),

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} U) = 0$$

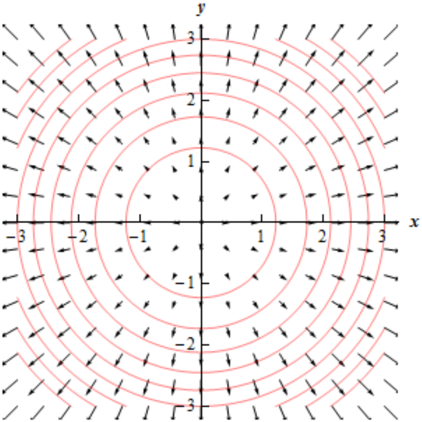
$\Rightarrow \quad \boldsymbol{\nabla} U$ is an **irrotational (curl-free)** vector field.

Conservative forces are associated with curl-free fields.

[1] Warning: some forces can not be!

Vector Field of a Conservative Force Example

2 - dimensional example: a vector force field $\mathbf{F} = 2x\,\mathbf{i} + 2y\,\mathbf{j}$:

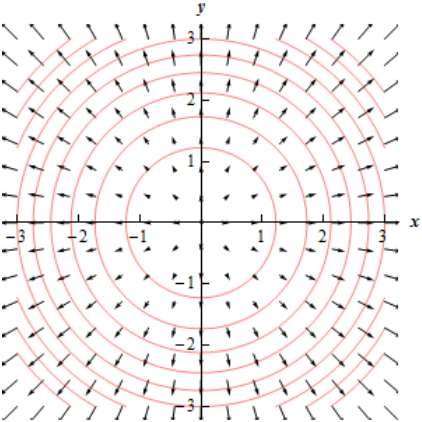


It is an irrotational field, so we can write $\mathbf{F} = -\boldsymbol{\nabla} U$, where U is a **scalar potential** function.

Here $U(x, y) = -(x^2 + y^2)$.

Vector Field of a Conservative Force Example

A force field $\mathbf{F}$ that can be expressed as $\mathbf{F} = -\nabla U$:



$U(x, y) = -(x^2 + y^2)$.

The potential function U is constant along each red line.

Non-Conservative Forces

Some forces do not conserve mechanical energy ($E_{\text{mech}} = K + U$).

The work done by the force taking the system around a closed path is not zero.

These forces are *non-conservative forces*.

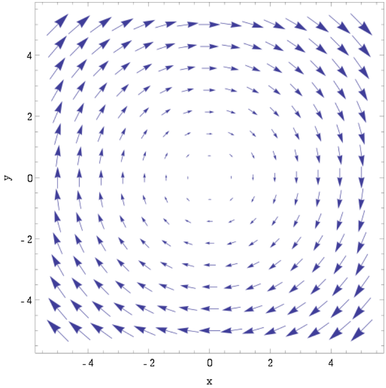
Examples of non-conservative forces:

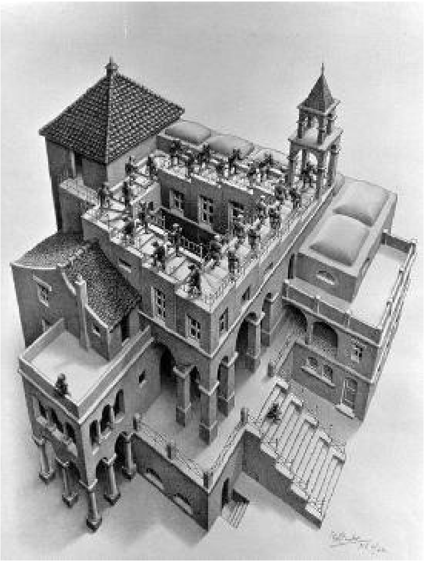
- friction
- air resistance
- external applied forces

Mechanical energy can increase, or it can decrease as it is converted to heat or other inaccessible forms.

Non-Conservative Forces

Non-conservative vector fields cannot be represented with a topological map.





[1]Lithograph in the mathematically-inspired impossible reality style, by M.C. Escher.

Electric Potential

Electric potential is a new quantity that relates the effect of a charge configuration to the potential energy that a test charge would have in that environment.

It is denoted V.

electric potential, V

the potential energy per unit charge:

$$V = \frac{U}{q}$$

V has a unique value at any point in an electric field.

It is characteristic only of the electric field, meaning it can be determined just from the field.

Electric Potential

Potential is potential energy per unit charge:

$$V = \frac{U}{q}$$

The units are Volts, V.

$1\ \text{V} = 1\ \text{J/C} = 1\ \text{A}\ \Omega = 1\,\frac{\text{kg m}^2}{\text{A s}^3}$

Volts are also the units of **potential difference**, the change in potential: ΔV.

Electric Field and Electric Potential

Potential, V, is potential energy per unit charge:

$$U = qV$$

Electric field, $\mathbf{E}$, is force per unit charge:

$$\mathbf{F} = q\,\mathbf{E}$$

Notice the relation! Both quantities are defined so that we can predict physical quantities associated with putting a charge at a certain point.

The electron volt

$$U = qV$$

This relation also gives us a unit of energy which is very convenient for particle physics.

The fundamental unit of charge is $e = 1.60 \times 10^{-19}$ C.

Energy can be measured in terms of **electron volts**, eV.

$1 \text{ eV} = 1.60 \times 10^{-19}$ J

This unit is much smaller than the Joule!

Gravitational Potential

Potential, V, is potential energy per unit charge:

$$U = qV$$

For comparison, gravitational potential, ϕ, is also a defined quantity, for a test mass m:

$$U = m\,\phi$$

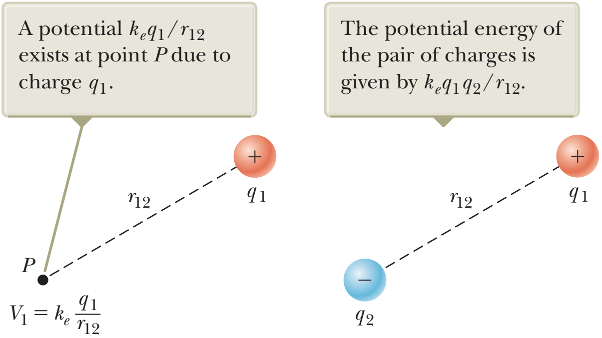
Gravitational potential at a distance r from a sphere of mass M radius $R < r$:

$$\phi_M = -\frac{GM}{r}$$

Electric Potential and Potential Energy

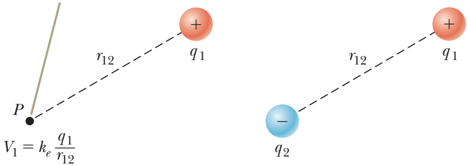
Electric potential gives the potential energy that would be associated with test charge q_0 if it were at a certain point P.

$$U_{P,q_0} = q_0 V_p$$



[1]Figure from Serway and Jewett, 9th ed.

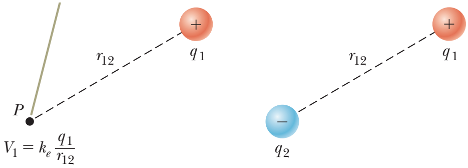
Electric Potential and Potential Energy



For a point charge q_2, its potential energy when near another point charge q_1 is

$$U = \frac{k\, q_1 q_2}{r}$$

Electric Potential and Potential Energy



For a point charge q_2, its potential energy when near another point charge q_1 is

$$U = \frac{k\, q_1 q_2}{r}$$

We say that the electric potential at point P due to q_1 is

$$V = \frac{k\, q_1}{r}$$

so that if a charge q_2 is placed there:

$$q_2 V = q_2 \left(\frac{k\, q_1}{r} \right) = U$$

gives the potential energy of the 2-charge configuration!

Electric Field and Electric Potential

Table of quantities for the field and potential of a point charge Q.

	electric field	electric potential
at point P	$E = \frac{k\,Q}{r^2}$	$V = \frac{k\,Q}{r}$
charge q_0 at P	$F_{q_0} = \frac{k\,Q\,q_0}{r^2}$	$U = \frac{k\,Q\,q_0}{r}$

Work and Potential

Recall, since the electrostatic force is a conservative force:

$$W_E = -\Delta U_E$$

where W_E is the work done by the internal electrostatic force.

So, we can relate this work to potential difference:

$$W_E = -q\ \Delta V$$

If a charge moves along an equipotential surface, $\Delta V = 0$ so $W_E = 0$.

Work and Potential

For conservative forces:

$$-\Delta U = W_{\text{int}} = \int \mathbf{F} \cdot d\mathbf{s}$$

Considering the potential energy of the electrostatic force:

$$\begin{aligned} \Delta U_E &= -\int \mathbf{F} \cdot d\mathbf{s} \\ q_0\, \Delta V &= -\int q_0 \mathbf{E} \cdot d\mathbf{s} \end{aligned}$$

giving:

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$$

(This is the integral form.)

Work and Potential

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$$

The change in potential energy can also be deduced from the field:

$$\Delta U = -q \int \mathbf{E} \cdot d\mathbf{s}$$

This is also the work done by an external applied force moving a charge along a path $\mathbf{s}$:

$$W_{\text{app}} = -q \int \mathbf{E} \cdot d\mathbf{s}$$

Summary

- potential energy and vector fields
- introduced electric potential
- related potential and work
- related potential and field

First Test this Friday, Jan 26, covering Ch 23-25.

Homework Serway & Jewett:

- Ch 25, onward from page 767. Obj. Qs: 1, 7; Concep. Qs: 5; Problems: 1, 3, 7, 13, 15, 17