Electricity and Magnetism
Understanding Electric Potential
Potential around charge distributions

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De Anza College

Jan 24, 2018

Last time

- electric potential energy and force
- electric potential definition
- electric potential of point charge

Warm Up Question

Recalling that $W = \int \mathbf{F} \cdot \mathrm{d}\mathbf{s}$, which is a formula for the change in the potential energy of a charge q moved through an electric field $\mathbf{E}$?

(A) $\Delta U = -\int \mathbf{E} \cdot \mathrm{d}\mathbf{s}$

(B) $\Delta U = q_0 \int \mathbf{E} \cdot \mathrm{d}\mathbf{s}$

(C) $\Delta U = -q_0 \int \mathbf{E} \cdot \mathrm{d}\mathbf{s}$

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(C) $\Delta U = -q_0 \int \mathbf{E} \cdot d\mathbf{s}$ $\longleftarrow$

Overview

- electric field and potential
- equipotentials
- potential from many charges or charge distributions
- Electric potential difference of charged plates (?)

Work and Potential

Recall, since the electrostatic force is a conservative force:

$$W_E = -\Delta U_E$$

where W_E is the work done by the internal electrostatic force.

So, we can relate this work to potential difference:

$$W_E = -q\ \Delta V$$

If a charge moves along an equipotential surface, $\Delta V = 0$ so $W_E = 0$.

Work and Potential

For conservative forces:

$$-\Delta U = W_{\text{int}} = \int \mathbf{F} \cdot d\mathbf{s}$$

Considering the potential energy of the electrostatic force:

$$\begin{aligned} \Delta U_E &= -\int \mathbf{F} \cdot d\mathbf{s} \\ q_0 \, \Delta V &= -\int q_0 \mathbf{E} \cdot d\mathbf{s} \end{aligned}$$

giving:

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$$

(This is the integral form.)

Work and Potential

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$$

The change in potential energy can also be deduced from the field:

$$\Delta U = -q \int \mathbf{E} \cdot d\mathbf{s}$$

This is also the work done by an external applied force moving a charge along a path **s**:

$$W_{\text{app}} = -q \int \mathbf{E} \cdot d\mathbf{s}$$

Relation between Electric Potential and Electric Field

Remember **F** is related to **E**:

$$\begin{aligned} \mathbf{F} &= q_0\mathbf{E} \\ \frac{\mathbf{F}}{q_0} &= \mathbf{E} \\ \frac{1}{q_0}(-\boldsymbol{\nabla} U) &= \mathbf{E} \end{aligned}$$

So,

$$\mathbf{E} = -\boldsymbol{\nabla} V$$

Meaning of "Electrostatics"

For the first part of this course we are considering **electrostatic** situations.

In words, electrostatic means that all charges are either

- **stationary** or
- part of a **current that is not changing with time**,

and that all the electromagnetic fields can be treated as constant.

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and that all the electromagnetic fields can be treated as constant.

Formally, it means we can express the electric field as:

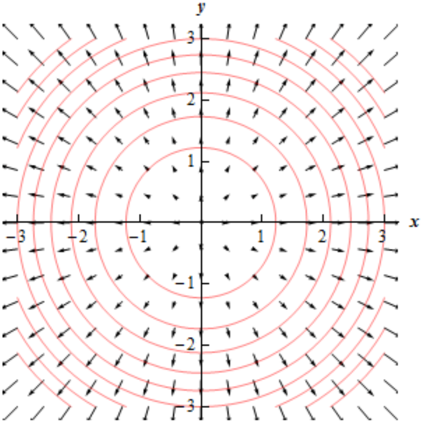
$$\mathbf{E} = -\nabla V$$

Or "the electric field has no rotation".

(Later we will see that this is not always the case.)

Relation to Vector Fields

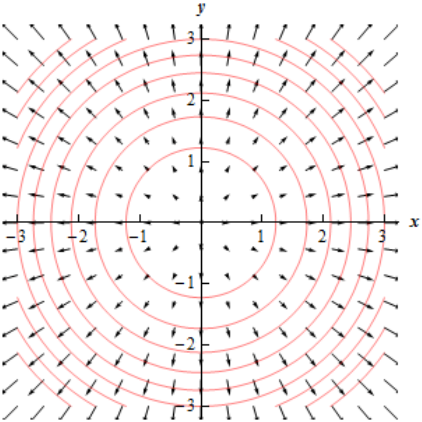
Earlier we represented the electrostatic force as a vector field:



However, by dividing out the test charge value q_0 we get the electric field. (Just re-scaling the vectors.)

Relation to Vector Fields

Now suppose this vector field is **E**, the electrostatic E-field:



$$\mathbf{E} = -\nabla V$$

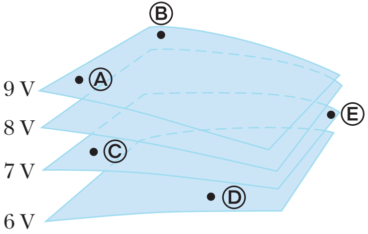
Now the red lines represent lines of equal electric potential. V is also a scalar potential.

Equipotential Surfaces

The fields from charges extend out in 3 dimensions.

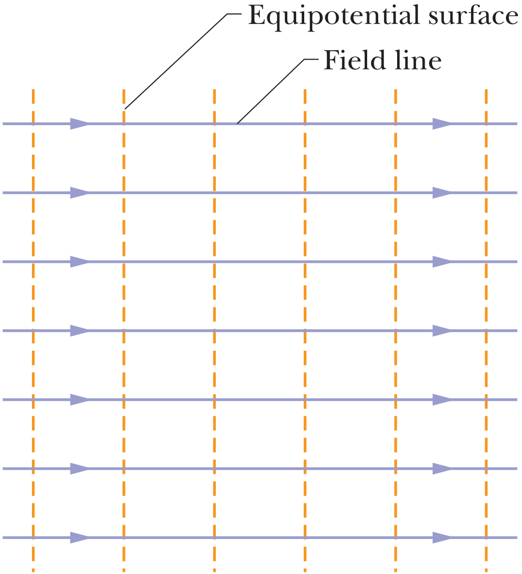
We can find 2-dimensional surfaces of constant electric potential.

These surfaces are called *equipotentials*.



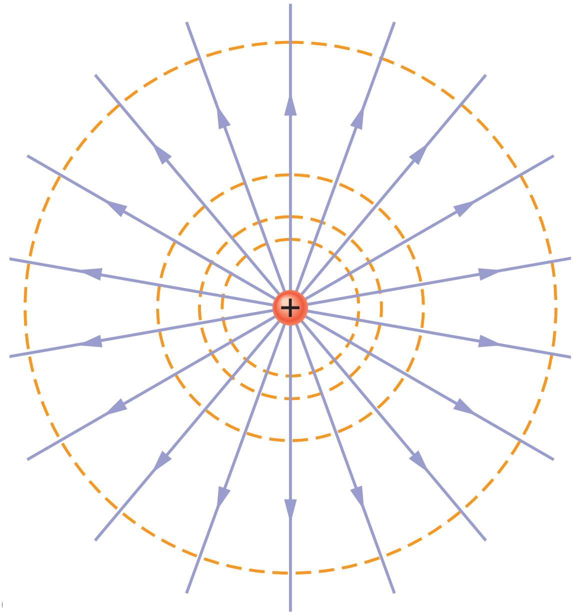
Sketching them sheds light on the potential energy a test charge would have at certain points: in particular, it is takes a particular constant value for any point on a surface.

Equipotential Surfaces: Examples



[1]Figure from Halliday, Resnick, Walker.

Equipotential Surfaces: Examples



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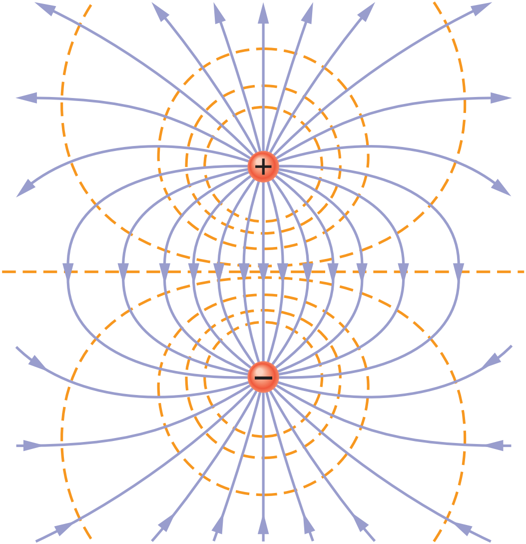
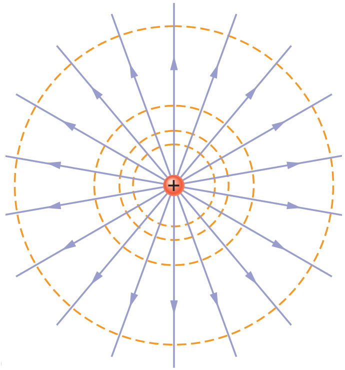


Equipotential Surfaces: Examples

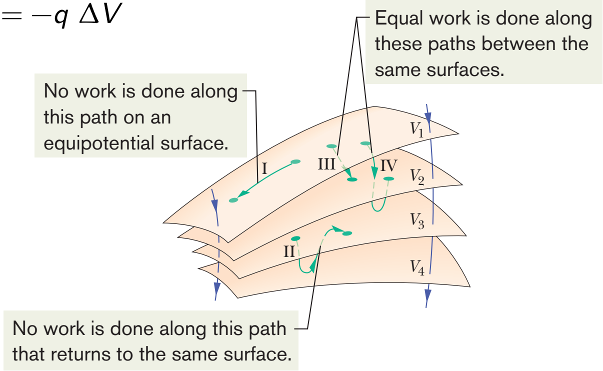


Equipotential surfaces are always perpendicular to field lines.

If a charge is moved along an equipotential surface the work done by the force of the electrostatic field is zero.

Equipotentials

$$W_E = -q\ \Delta V$$

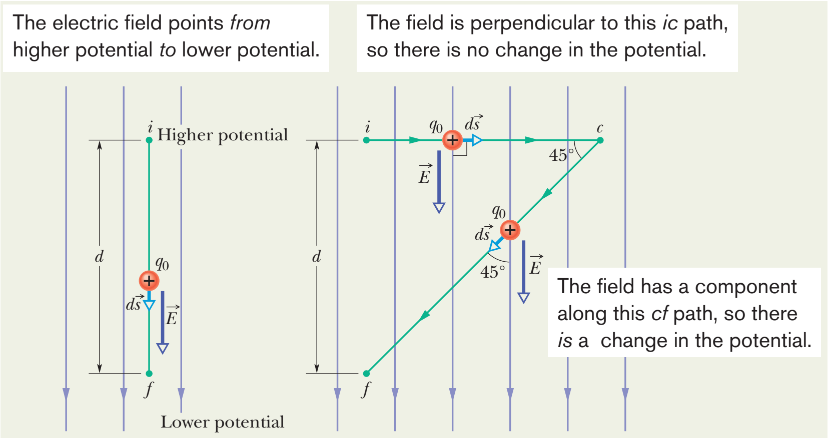


No work is done by the electrostatic force moving a charge along an equipotential.

The same work is done moving a charge from one equipotential to another, regardless of the path you move it along!

Example: Uniform E-field

$$\Delta V_{i \to f} = -\int_i^f \mathbf{E} \cdot \mathbf{ds} = -Ed \quad \text{(indep. of path)}$$

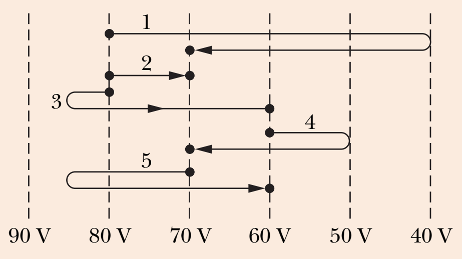


[1]Halliday, Resnick, Walker, page 634.

Question

The figure shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an **electron** from one surface to another.

Electron has a negative charge!



1-(a) What is the direction of the electric field associated with the surfaces?

(A) rightwards

(B) leftwards

(C) upwards

(D) downwards

[1] Halliday, Resnick, Walker, page 633.

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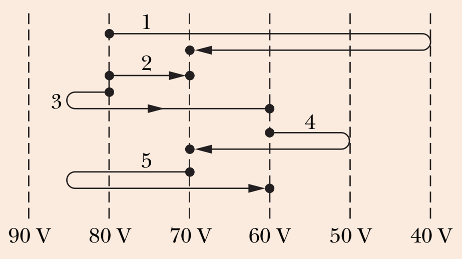

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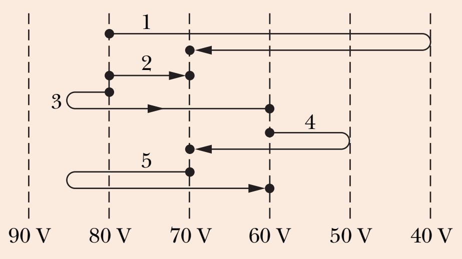

2-(c) Rank the paths according to the work *we* do, greatest first.

(A) 1, 2, 3, 4, 5

(B) 2, 4, 3, 5, 1

(C) 4, (1, 2, and 5), 3

(D) 3, (1, 2, and 5), 4

[1] Halliday, Resnick, Walker, page 633.

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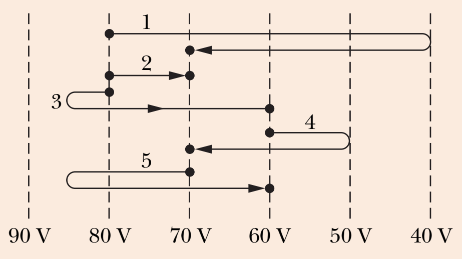

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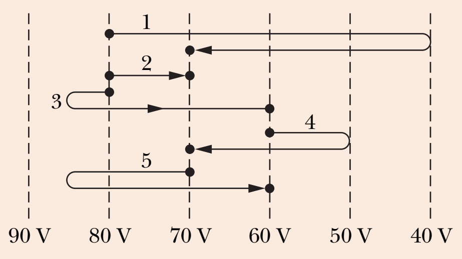

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Potential from many charges

The electric potential from many point charges could be found by adding up the potential due to each separately:

$$V_{\text{net}} = V_1 + V_2 + \ldots + V_n$$

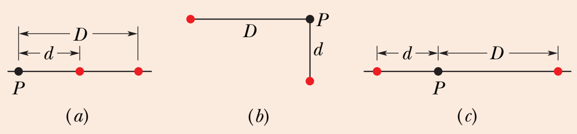
This is

$$V_{\text{net}} = \sum_i V_i$$

Notice that this is a scalar equation, not a vector equation.

Question

The figure shows three arrangements of **two protons**. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



(A) a, b, c

(B) c, b, a

(C) b, (a and c)

(D) all the same

[1] Halliday, Resnick, Walker, page 636.

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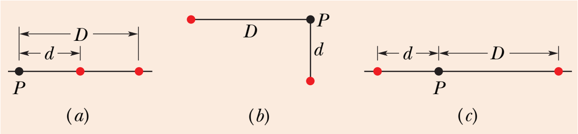

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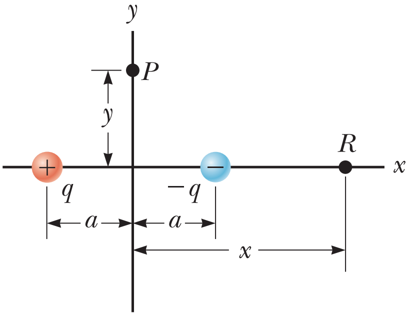
More Practice with Electric Potential, Ex 25.4

What is the electric potential, V, along an axis through the middle of a dipole at point P?



[1]Figure from Serway & Jewett, 9th ed.

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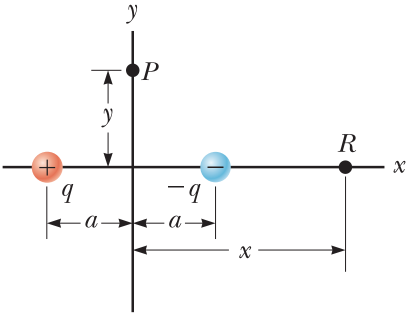
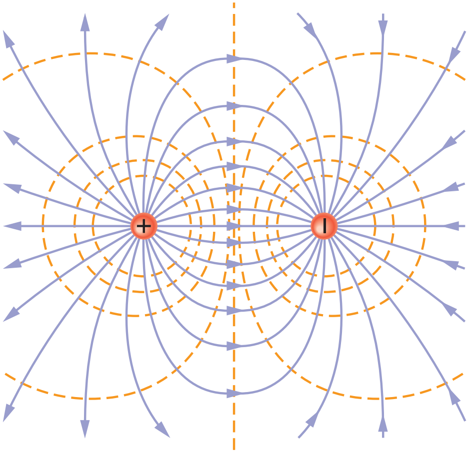


$$
\begin{aligned}
V &= V_1 + V_2 \\
&= \frac{k_e q}{\sqrt{a^2 + y^2}} - \frac{k_e q}{\sqrt{a^2 + y^2}} \\
&= 0
\end{aligned}
$$

[1] Figure from Serway & Jewett, 9th ed.

More Practice with Electric Potential

Equipotentials:



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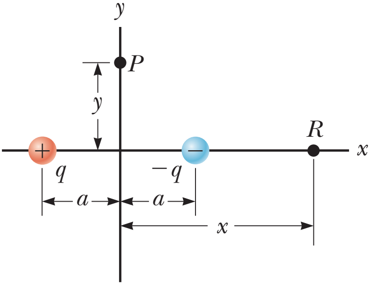

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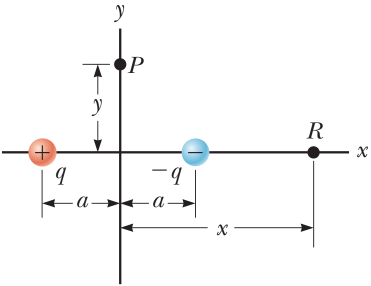
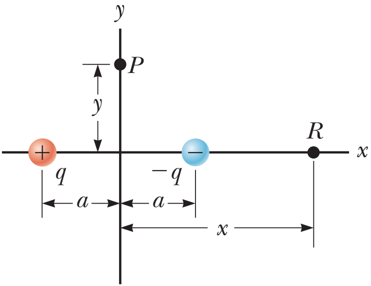


$$
\begin{aligned}
V &= V_1 + V_2 \\
&= \frac{k_e q}{(x+a)} - \frac{k_e q}{(x-a)} \\
&= -\frac{2k_e q a}{x^2 - a^2}
\end{aligned}
$$

More Practice with Electric Potential, Ex 25.4

Far away along the x axis, what is the electric potential, V, and the electric field?



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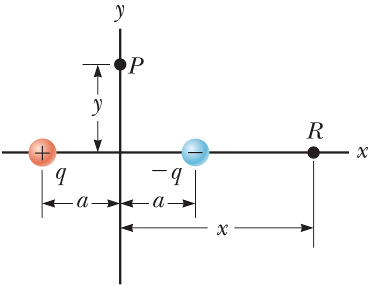

$$V = -\frac{2k_e qa}{x^2 - a^2}$$

for $x \gg a$

$$\approx -\frac{2k_e qa}{x^2}$$

More Practice with Electric Potential, Ex 25.4

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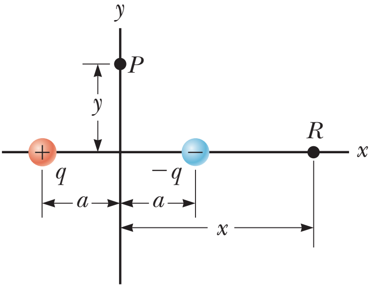
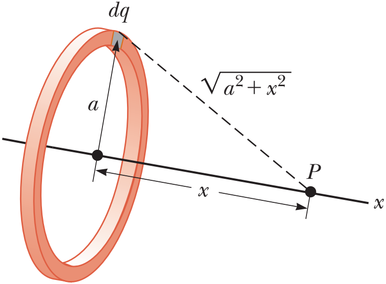


$$\begin{aligned} \mathbf{E} &= -\nabla V \\ &= -\frac{\partial}{\partial x} V \ \mathbf{i} \\ &= -\frac{4k_e qa}{x^3} \ \mathbf{i} \quad (x \gg a) \end{aligned}$$

Potential from a charge distribution example: Uniformly charged ring



Potential at point P? (Ring's total charge $= Q$.)

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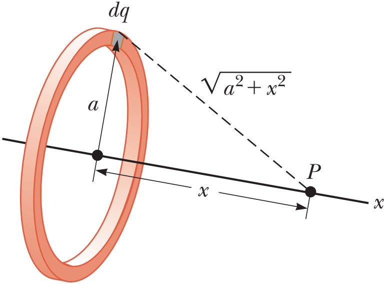

Potential at point P? (Ring's total charge $= Q$.)

Unlike the E-field case, we do not have to worry about direction (vectors). Very simple integral!

$$V = \int \frac{k_e \, \mathrm{dq}}{r} = \frac{k_e}{r} \int \mathrm{dq} = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

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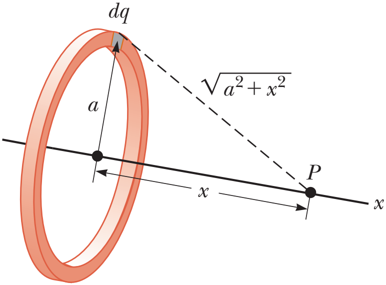

Knowing potential at point P, find $\mathbf{E}$?

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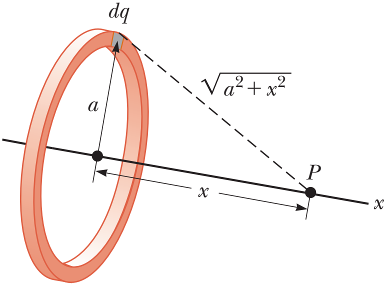

Knowing potential at point P, find $\mathbf{E}$?

$$\begin{aligned}\mathbf{E} &= -\nabla V \\ &= -\frac{\partial}{\partial x}\frac{k_e Q}{\sqrt{x^2+a^2}}\,\mathbf{i} \\ &= \frac{k_e Q\, x}{(x^2+a^2)^{3/2}}\,\mathbf{i}\end{aligned}$$

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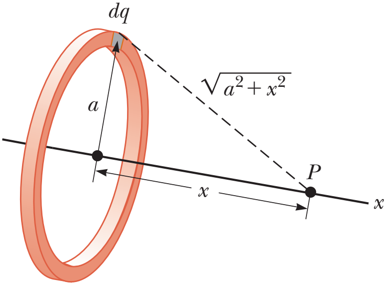


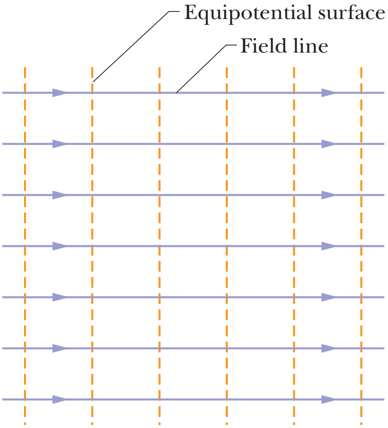
Knowing potential at point P, find $\mathbf{E}$? (Maybe easier this way!!)

$$\begin{aligned}\mathbf{E} &= -\nabla V \\ &= -\frac{\partial}{\partial x}\frac{k_e Q}{\sqrt{x^2+a^2}}\,\mathbf{i} \\ &= \frac{k_e Q\, x}{(x^2+a^2)^{3/2}}\,\mathbf{i}\end{aligned}$$

Potential Difference across a pair of charged plates

We know that the field between two charged plates is uniform.
$E = \frac{\sigma}{\epsilon_0}$.

$$\Delta V = -\int_0^d \mathbf{E} \cdot d\mathbf{s} = -E\,d$$



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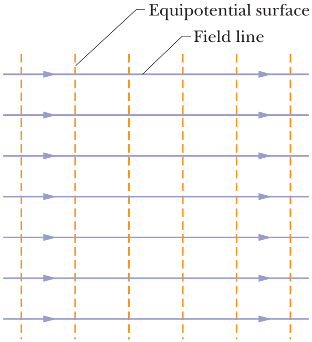

The potential difference between the two plates, separation, d:

$$|\Delta V| = E\, d$$

Summary

- electric field and potential
- equipotentials
- potential from many charges or charge distributions
- Electric potential difference of charged plates (?)

First Test this Friday, Jan 26, covering Ch 23-25.

Homework

- Study for test.

Serway & Jewett:

- Ch 25, Problems: 36, 37, 41, 43, 45, 63, 65, 67