Electricity and Magnetism
Capacitance
Capacitors in Series and Parallel

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Last time

- Van de Graaff generator
- capacitors
- capacitance
- capacitors of different shapes

Warm Up Question

True or false: A component (fixed) capacitor has a capacitance, even when storing no charge.

(A) true
(B) false

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Overview

- Parallel plate capacitors
- capacitors of different shapes
- Circuits and circuit diagrams
- Capacitors in series and parallel

Capacitance

Capacitors with different construction will have different values of C.

For example,
for a parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$.

for a **cylinderical** capacitor of length L, inner radius a and outer radius b:

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$$

for a **spherical** capacitor of inner radius a and outer radius b:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

for an **isolated charged sphere** of radius R:

$$C = 4\pi\epsilon_0 R$$

Parallel Plate Capacitor

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$$C = \frac{Q}{|\Delta V|}$$

Assuming the field inside the capacitor is uniform throughout, it is given by the expression for the field inside infinite planes of charge:

$$E = \frac{\sigma}{\epsilon_0}$$

Replace $Q = \sigma A$ and using we have:

$$C = \frac{\sigma A}{|\Delta V|}$$

Parallel Plate Capacitor

Also:

$$\Delta V = -\int \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -Ed$$

so:

$$C = \frac{Q}{|\Delta V|} = \frac{\sigma A}{Ed}$$

Using our value for E:

$$C = \frac{\sigma A}{(\sigma/\epsilon_0)d}$$

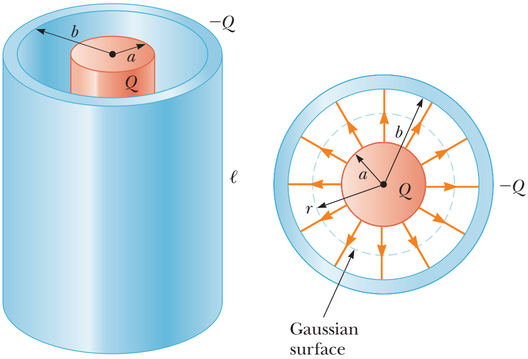
Confirms that

$$C = \frac{\epsilon_0 A}{d}$$

Cylindrical Capacitor

For a **cylinderical** capacitor of length ℓ, much greater than inner radius a and outer radius b:

$$C = 2\pi\epsilon_0 \frac{\ell}{\ln(b/a)} = \frac{\ell}{2k_e \ln(b/a)}$$



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Idea: First find ΔV across the plates, assuming charge Q, then evaluate $Q/|\Delta V|$.

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Let $\lambda = Q/\ell$ be the charge per unit length.

$$\begin{aligned} |\Delta V| = V_a - V_b &= -\int_b^a \mathbf{E} \cdot \mathrm{d}\mathbf{s} \\ &= -\int_b^a \left(\frac{2k_e\lambda}{r} \right) \mathrm{d}r \\ &= 2k_e\lambda \ln\left(\frac{b}{a}\right) \end{aligned}$$

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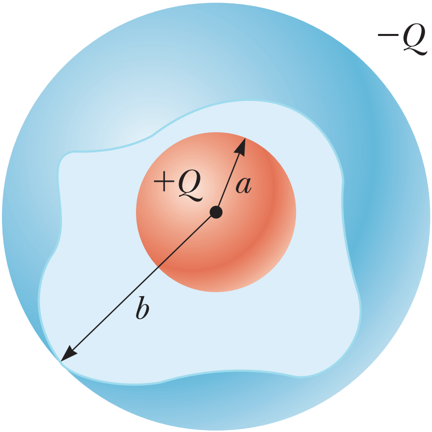
Capacitance,

$$\begin{aligned} C &= \frac{Q}{\Delta V} = \frac{Q}{2k_e\lambda \ln\left(\frac{b}{a}\right)} \\ C &= \frac{2\pi\epsilon_0\ell}{\ln\left(\frac{b}{a}\right)} \end{aligned}$$

Spherical Capacitor

For a **spherical** capacitor of inner radius a and outer radius b:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} = \frac{ab}{k_e\,(b-a)}$$



Spherical Capacitor

$$
\begin{aligned}
|\Delta V| = V_a - V_b &= -\int_b^a \mathbf{E} \cdot \mathrm{d}\mathbf{s} \\
&= -\int_b^a \left(\frac{k_e Q}{r^2}\right) \mathrm{d}r \\
&= k_e Q \left(\frac{1}{a} - \frac{1}{b}\right) \\
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\end{aligned}
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Capacitance,

$$
\begin{aligned}
C &= \frac{Q}{\Delta V} \\
C &= \frac{ab}{k_e \left(b - a \right)}
\end{aligned}
$$

Capacitance of Isolated Conductors

Isolated conductors on their own (not part of a pair) can also be said to have a capacitance.

The "other plate" is taken to be infinitely far away.

The capacitance is found by dividing the charge on the conductor by it's electric potential, taking $V(\infty) = 0$

Capacitance of and Isolated Spherical Conductor

For an **isolated charged sphere** of radius R:

$$C = 4\pi\epsilon_0 R = \frac{R}{k_e}$$

Two ways to argue this,

(1) set $a = R$ and take $b \to \infty$ in $C = \frac{ab}{k_e(b-a)}$, or

(2) recall that the potential of a sphere of charge Q is $V = \frac{k_e Q}{R}$

Circuits

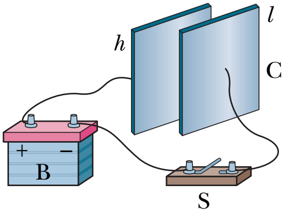
Circuits consist of electrical **components** connected by wires.

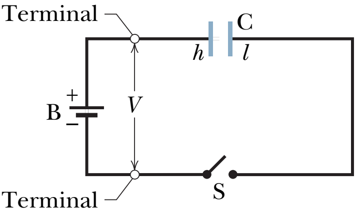
Some types of components: batteries, resistors, capacitors, lightbulbs, LEDs, diodes, inductors, transistors, chips, etc.

The wires in circuits can be thought of as channels for an electric field that distributes charge to (or charge flow through) the components.

Circuits

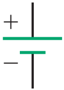
The different elements can be combined together in various ways to make complete circuits: paths for charge to flow from one terminal of a battery or power supply to the other.



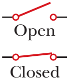


This circuit is said to be incomplete while the switch is open.

Circuit component symbols

battery ΔV 

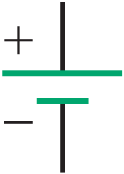
capacitor C 

switch S 

resistor R 

inductor L 

Circuits: Batteries



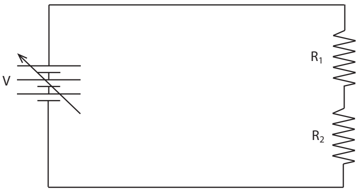
Batteries cause a potential difference between two parts of the circuit.

This can drive a charge flow. (Current is the rate of flow of charge.)

Series and Parallel

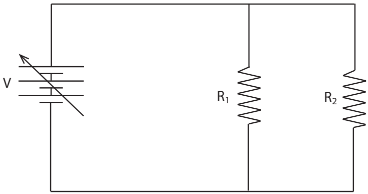
Series

When components are connected one after the other along a single path, they are connected in series.



Parallel

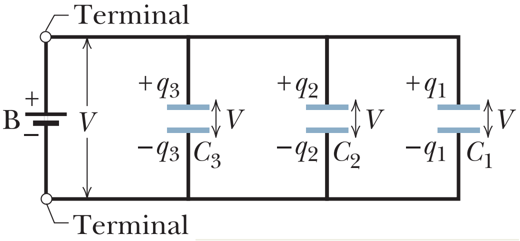
When components are connected side-by-side on different paths, they are connected in parallel.



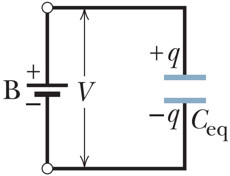
Capacitors in Parallel

Capacitors in parallel all have the **same potential difference** across them.

Three capacitors in parallel:



Equivalent circuit:



We could replace all three capacitors in the circuit with one equivalent capacitance. The current and potential difference in the rest of the circuit is unchanged by this.

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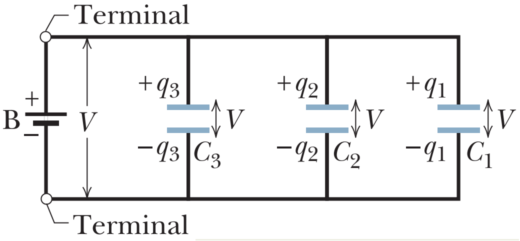

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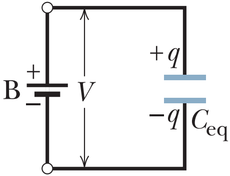

We could replace all three capacitors in the circuit with one equivalent capacitance. The current and potential difference in the rest of the circuit is unchanged by this.

What would be the capacitance of this equivalent capacitor?

Capacitors in Parallel

Capacitors in parallel all have the **same potential difference** across them.

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V$$

The total charge on the three capacitors is the **sum** of the charge on each.

$$q_{\text{net}} = q_1 + q_2 + q_3$$

where $q_1 = C_1 \, \Delta V$.

Capacitance is $C = q/(\Delta V)$:

$$C_{\text{eq}} = \frac{q_{\text{net}}}{\Delta V}$$

Capacitors in Parallel

Equivalent capacitance:

$$\begin{aligned} C_{\text{eq}} &= \frac{q_{\text{net}}}{\Delta V} \\ &= \frac{q_1}{\Delta V} + \frac{q_2}{\Delta V} + \frac{q_3}{\Delta V} \\ &= C_1 + C_2 + C_3 \end{aligned}$$

Capacitors in Parallel

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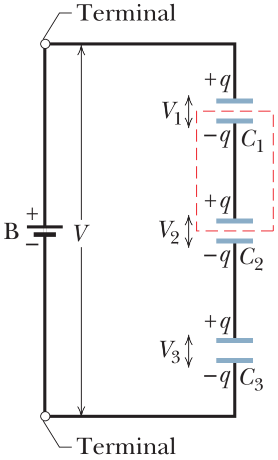
So in general, for any number n of capacitors in **parallel**, the effective capacitance of them all together is:

$$C_{\text{eq}} = C_1 + C_2 + ... + C_n = \sum_{i=1}^{n} C_i$$

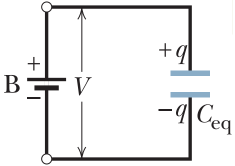
Capacitors in Series

Capacitors in series all store the **same charge**.

Three capacitors in series:



Equivalent circuit:



Summary

- parallel plate capacitors
- capacitors of different shapes
- circuits, circuit diagrams
- capacitors in parallel

Homework

Serway & Jewett:

- PREVIOUS: Ch 26, onward from page 799. Problems: 1, 5, 7, 11, 51
- NEW: Ch 26. Problems: 13