Electricity and Magnetism
Capacitors in Series
Energy Stored in a Capacitor

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Last time

- cylindrical and spherical capacitors
- Parallel plate capacitors
- Circuits and circuit diagrams
- Capacitors in parallel

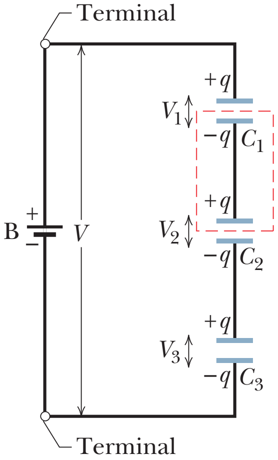
Overview

- capacitors in series
- practice with capacitors in circuits
- energy stored in a capacitor
- dielectrics
- molecular view of dielectrics

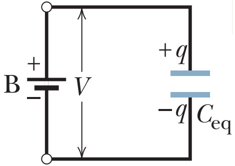
Capacitors in Series

Capacitors in series all store the **same charge**.

Three capacitors in series:



Equivalent circuit:



Capacitors in Series

Again, we could replace all three capacitors in the circuit with one equivalent capacitance and we can find the capacitance of this equivalent capacitor.

The **sum of the potential differences** across capacitors in series is V, the battery's supplied potential difference.

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

where $\Delta V_1 = q/C_1$, etc.
Then,

$$C_{\text{eq}} = \frac{q}{\Delta V}$$

Capacitors in Series

Equivalent capacitance:

$$
\begin{aligned}
C_{\text{eq}} &= \frac{q}{\Delta V} \\
&= \frac{q}{\Delta V_1 + \Delta V_2 + \Delta V_3} \\
&= \left[\frac{V_1 + V_2 + V_3}{q}\right]^{-1} \\
&= \left[\frac{\Delta V_1}{q} + \frac{\Delta V_2}{q} + \frac{\Delta V_3}{q}\right]^{-1} \\
&= \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}
\end{aligned}
$$

Capacitors in Series

In general, for any number n of capacitors in **series**, we can always relate the effective capacitance of them all together to the individual capacitances by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

The equivalent capacitance of capacitors in series is always less than the smallest capacitance in the series.

Practice

A 5.0 μF capacitor is connected in parallel with a 10 μF capacitor. What is the equivalent capacitance of this arrangement?

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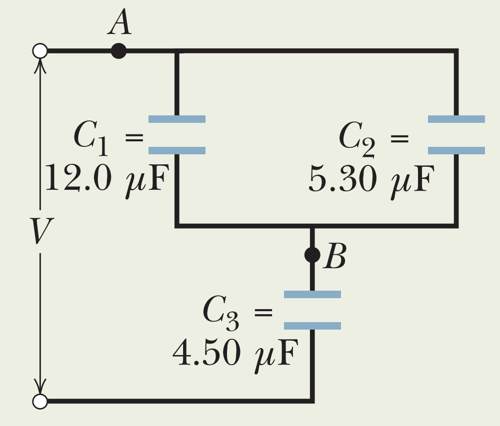
$$C_{\text{eq}} = 3.3\ \mu\text{F}$$

More Practice

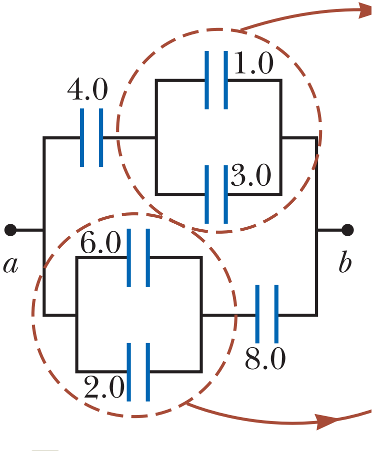
What is the equivalent capacitance of this arrangement?



More Practice

When solving this type of problem, take an iterative approach.

Identify sets of capacitors that are in parallel, then series, then parallel, etc. and at each step replace with the equivalent capacitance:



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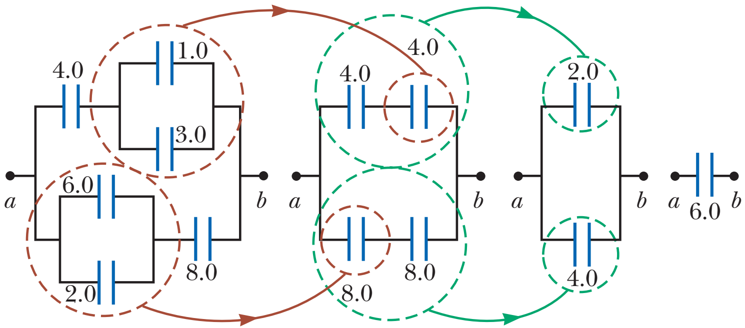
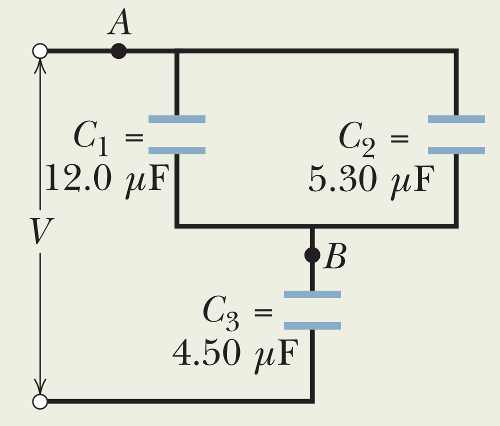


More Practice

What is the equivalent capacitance of this arrangement:



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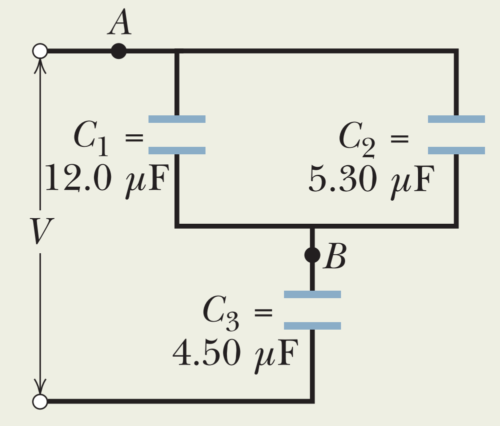

$C_{eq} = 3.57\,\mu F$.

Energy Stored in a Capacitor

A charged capacitor has an electric field between the plates. This field can be thought of as storing potential energy.

As you might expect, the energy stored is equal to the work done charging the capacitor. (Energy Conservation!)

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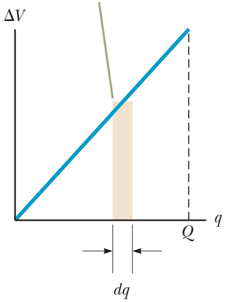
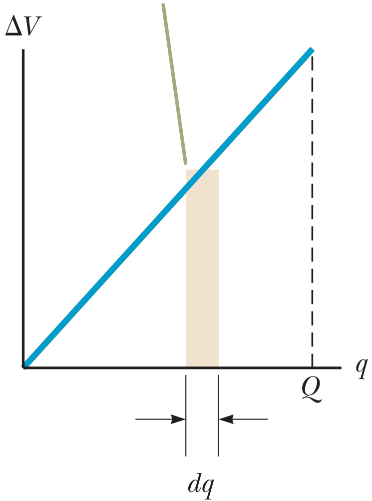
But how much work is done? $W_{\text{app}} = q\,\Delta V$, yet the potential difference across the plates changes as more charge is placed on the capacitor plates.



Energy Stored in a Capacitor

How much work is done? $dW_{app} = (\Delta V)\, dq$



$\rightarrow$ Need to integrate!

Energy Stored in a Capacitor

$$\Delta V = \frac{q}{C}$$

For a fixed capacitor (plates are not changing configuration or shape), C is a constant.

$$\begin{aligned} U_E = W_{\text{app}} &= \int_0^Q \frac{q}{C}\,\mathrm{dq} \\ &= \frac{1}{2}\frac{Q^2}{C} \end{aligned}$$

The energy stored in a capacitor with charge Q and capacitance C:

$$U = \frac{1}{2}\left(\frac{Q^2}{C}\right)$$

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Since $Q = C\left(\Delta V\right)$ we can also write this as:

$$U = \frac{1}{2}C\left(\Delta V\right)^2$$

And:

$$U = \frac{1}{2}Q\,\Delta V$$

Stored Energy Example

Suppose a capacitor with a capacitance 12 pF is connected to a 9.0 V battery.

What is the energy stored in the capacitor's electric field once the capacitor is fully charged?

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$$U_E = 4.9 \times 10^{-10} \text{ J}$$

Energy Density

It is sometimes useful to be able to compare the energy stored in different charged capacitors by their stored energy per unit volume.

We can link energy density to electric field strength.

This will make concrete the assertion that energy is stored in the field.

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For a parallel plate capacitor, energy density u is:

$$u_E = \frac{U_E}{Ad}$$

(Ad is the volume between the capacitor plates.)

Energy Density and Electric Field

$$
\begin{aligned}
u_E &= \frac{U_E}{Ad} \\
&= \frac{C(\Delta V)^2}{2Ad}
\end{aligned}
$$

Energy Density and Electric Field

$$u_E = \frac{U_E}{Ad} = \frac{C(\Delta V)^2}{2Ad}$$

Replace $C = \frac{\epsilon_0 A}{d}$:

$$u_E = \frac{\epsilon_0 A}{d} \frac{\Delta V^2}{2Ad} = \frac{\epsilon_0}{2} \left(\frac{\Delta V}{d} \right)^2$$

Energy Density and Electric Field

$$\begin{aligned} u_E &= \frac{U_E}{Ad} \\ &= \frac{C(\Delta V)^2}{2Ad} \end{aligned}$$

Replace $C = \frac{\epsilon_0 A}{d}$:

$$\begin{aligned} u_E &= \frac{\epsilon_0 A}{d} \frac{\Delta V^2}{2Ad} \\ &= \frac{\epsilon_0}{2} \left(\frac{\Delta V}{d} \right)^2 \end{aligned}$$

Lastly, remember $\Delta V = Ed$ in a parallel plate capacitor, so:

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

Energy Density and Electric Field

Energy density in a capacitor:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

The derivation of this expression assumed a parallel plate capacitor. However, it is **true more generally**. (General proof requires vector calculus.)

It is also true for varying electric fields, in which case the energy density varies.

Energy density of an electric field $\propto E^2$

Dielectrics

dielectric

an insulating material that can affects the strength of an electric field passing through it

Different materials have different **dielectric constants**, κ.

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Different materials have different **dielectric constants**, κ.

κ tells us how the capacitance of a capacitor changes if the material between the plates is changed.

For air $\kappa \approx 1$. (It is 1 for a perfect vacuum.)

κ is never less than 1. It can be very large > 100.

Dielectrics and Capacitance

dielectric

an insulating material that can affects the strength of an electric field passing through it

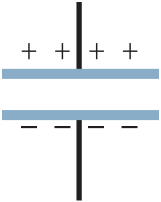
The effect of sandwiching a dielectric in a capacitor is to change the capacitance:

$$C \to \kappa C$$

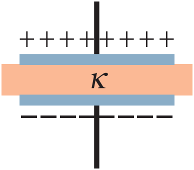
κ is the **dielectric constant**.

Dielectric in a Capacitor

Capacitance C



Capacitance κC



Adding a dielectric increases the capacitance.

Effect of a Dielectric

The most straightforward way of tracking quantities that will change when a dielectric is added is by replacing ϵ_0 in all equations with ϵ using this relation:

$$\epsilon = \kappa\epsilon_0$$

(Or just think of the effect of the dielectric being $\epsilon_0 \rightarrow \kappa\epsilon_0$.)

The electrical permittivity increases.

Dielectrics and Electric Field

Why do dielectrics effect the strength of the electric field?

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Why do dielectrics effect the strength of the electric field?

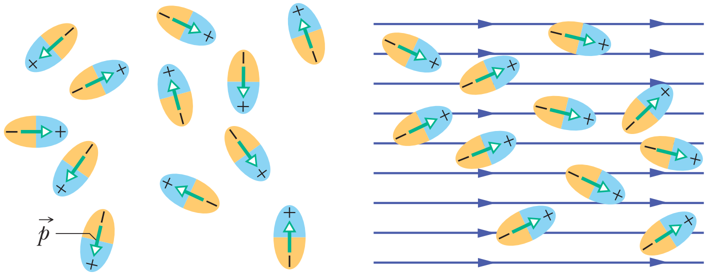
Dielectrics become polarized by the presence of an electric field.

There are two types of dielectrics, the process is a little different in each:

- polar dielectrics
- nonpolar dielectrics

Polar Dielectrics

The external electric field partially aligns the molecules of the dielectric with the field.



Since the dielectric is an insulator, there are no free charges to move through the substance, but molecules can align.

eg. distilled water

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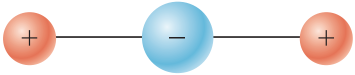
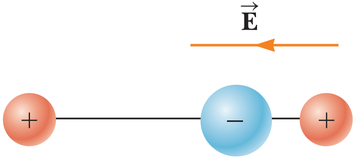
[1]Figures from Halliday, Resnick, Walker, 9th ed.

Nonpolar Dielectrics

Nonpolar dielectrics are composed of molecules which are not polar.

However, under the influence of a field, the distribution of the electrons in the molecules, or the shape of the molecule, is altered. Each molecule becomes slightly polarized.





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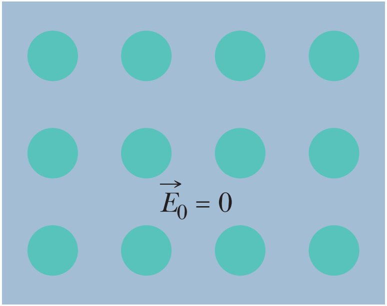
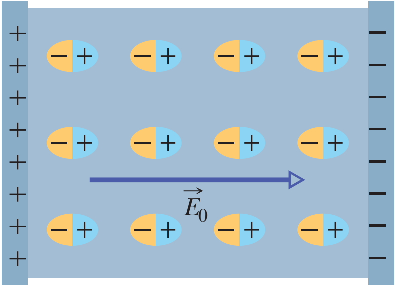
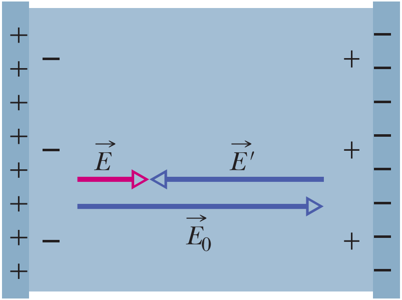




eg. nitrogen, benzene

Electric field inside the dielectric

The polarized dielectric contributes its own field, E'.



The electric field from the charged plates alone E_0, is reduced.

The resulting reduced field is $E = \frac{E_0}{\kappa}$

Summary

- capacitors in series
- practice with capacitors in circuits
- energy stored in a capacitor
- dielectrics
- molecular view of dielectrics

Quiz Friday.

Homework

Serway & Jewett:

- PREVIOUS: Ch 26, onward from page 799. Problems: 13
- NEW: Ch 26. Problems: 17, 21, 25, 31, 33, 35