Electricity and Magnetism
Dielectrics and Capacitors

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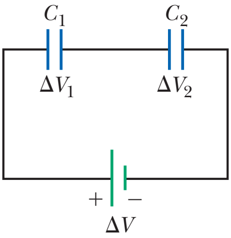
Last time

- capacitors in series
- practice with capacitors in circuits
- Energy stored in a capacitor
- Dielectrics
- molecular view of dielectrics

Warm Up Question

Two capacitors of values 4.0 nF and 6.0 nF are connected in a circuit as shown:



(A) 4.0 nF

(B) 6.0 nF

(C) 10 nF

(D) 2.4 nF

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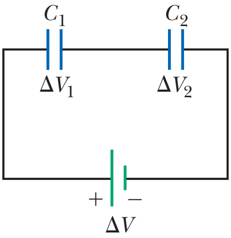

(A) 4.0 nF

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Overview

- Dielectrics
- Gauss's law with dielectrics
- electric displacement
- some uses of dielectrics

Dielectrics

dielectric

an insulating material that can affects the strength of an electric field passing through it

Different materials have different **dielectric constants**, κ.

For air $\kappa \approx 1$. (It is 1 for a perfect vacuum.)

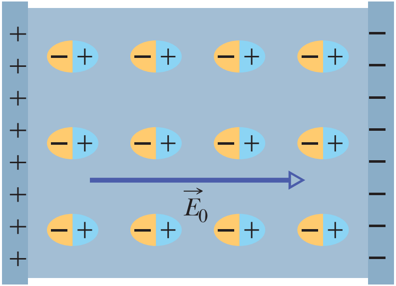
κ is never less than 1. It can be very large > 100.

The effect of sandwiching a dielectric in a capacitor is to change the capacitance:

$$C \to \kappa C$$

Dielectrics and Electric Field

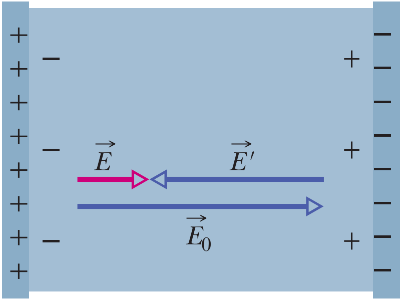
Why do dielectrics effect the strength of the electric field?



The external electric field from the aligns dipoles in the dielectric material.

Electric field inside the dielectric

The polarized dielectric contributes its own field, E'.



The electric field from the charged plates alone E_0, is reduced.

The resulting reduced field is $E = \frac{E_0}{\kappa}$

Dielectric in a Capacitor

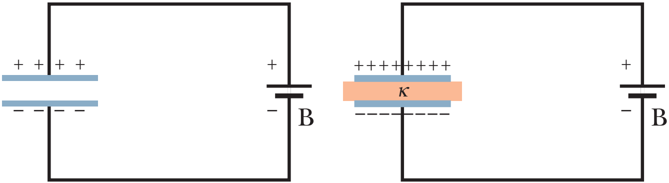
$$\epsilon_0 \;\rightarrow\; \kappa\epsilon_0$$

For a parallel plate capacitor with a dielectric, the capacitance is now:

$$C = \frac{\kappa\epsilon_0 A}{d}$$

Dielectric in a Capacitor

If we add a dielectric while the capacitor is connected to a battery:



V = a constant

Dielectric in a Capacitor

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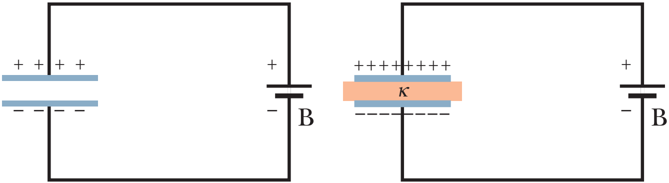
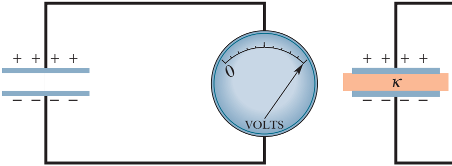


V = a constant

- q will increase. $(q = CV)$
- U will increase. $(U = \frac{1}{2}CV^2)$

Dielectric in a Capacitor

If we add a dielectric while the capacitor is isolated so charge cannot leave the plates:



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Dielectric in a Capacitor

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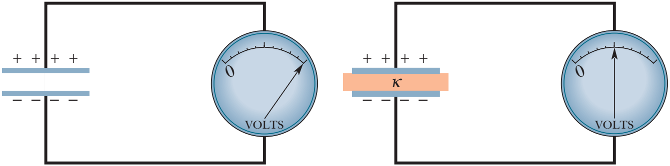
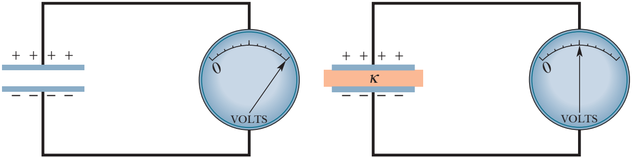


q = a constant

- V will decrease. $\left(V = \frac{q}{C}\right)$
- U will decrease. $\left(U = \frac{q^2}{2C}\right)$

Effect of a Dielectric on Field

Imagine again the isolated conductor: charge density σ is constant.



q = a constant

The electric field between the plates is $E = \frac{\sigma}{\epsilon_0}$ originally.

With dielectric added: $E \rightarrow \frac{\sigma}{\kappa \epsilon_0}$.

The field strength decreases: $E \rightarrow \frac{E}{\kappa}$ (as we know it should)

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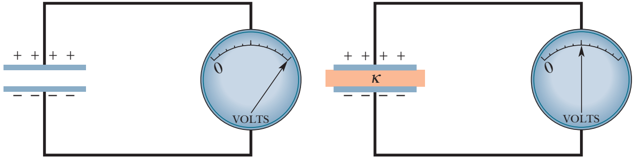

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What happens to the energy density u?

Effect of a Dielectric on Field

What happens to the energy density? Was: $u_0 = \frac{1}{2}\epsilon_0 E_0^2$.

$$u = \frac{1}{2}\left(\kappa\epsilon_0\right)\left(E\right)^2$$

Effect of a Dielectric on Field

What happens to the energy density? Was: $u_0 = \frac{1}{2}\epsilon_0 E_0^2$.

$$\begin{aligned}
u &= \frac{1}{2}(\kappa\epsilon_0)(E)^2 \\
&= \frac{1}{2}(\kappa\epsilon_0)\left(\frac{\sigma}{\kappa\epsilon_0}\right)^2 \\
&= \frac{1}{2}\epsilon_0\kappa\left(\frac{1}{\kappa^2}\right)E_0^2 \\
&= \frac{1}{\kappa}\left(\frac{1}{2}\epsilon_0 E_0^2\right) \\
u &= \frac{u_0}{\kappa}
\end{aligned}$$

Energy density decreases.

Dielectrics and Electric Field

Dielectrics effect the field around a charge

$$E \rightarrow \frac{E}{\kappa}$$

For example, for a point charge q in free space:

$$E_0 = \frac{k\,q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

But in a dielectric, constant κ:

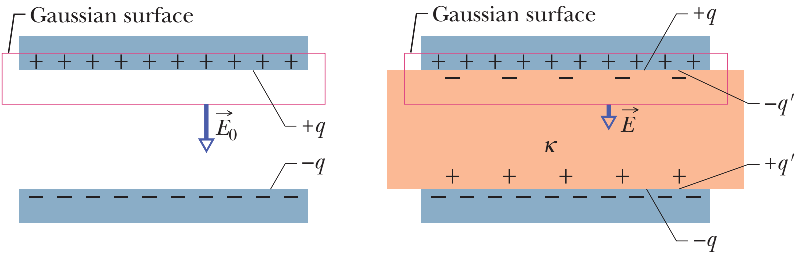
$$E = \frac{1}{4\pi(\kappa\epsilon_0)}\frac{q}{r^2} = \frac{E_0}{\kappa}$$

Guass's Law with dielectrics

$$\kappa\epsilon_0\Phi_E = q_{\text{free}}$$

or:

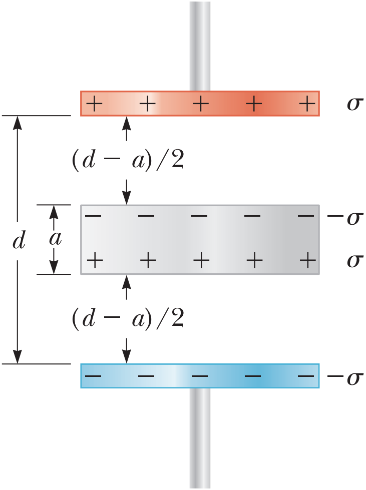
$$\oint_A \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{free}}}{\kappa\epsilon_0}$$



The charge $q_{\text{free}} = q$ in the diagram. It is just the charge on the plates, the charge that is free to move.

Capacitor with a Metal slab, Ex 26.7

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. Find the capacitance of the device.



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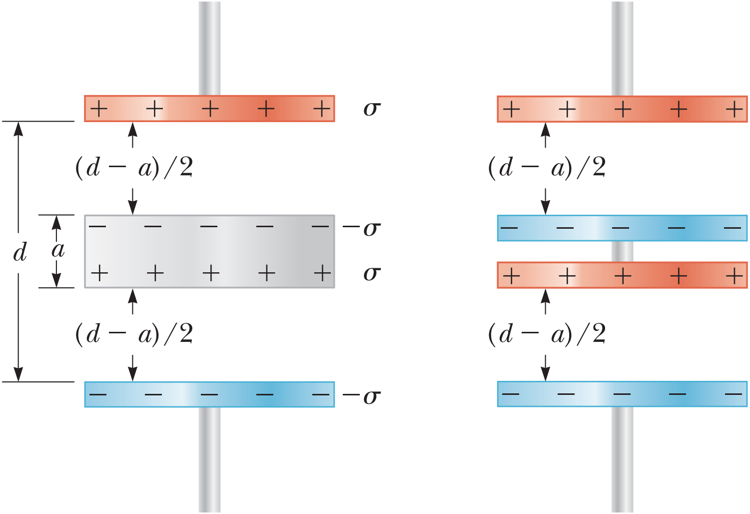

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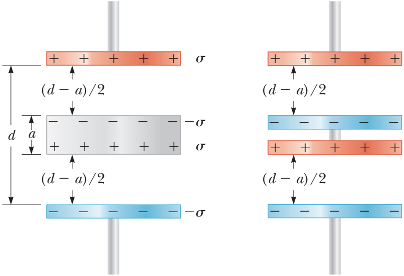

This is just 2 capacitors in series!

$$C_{eq} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1}$$

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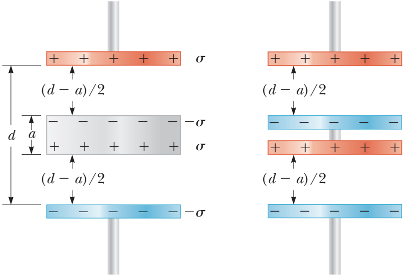

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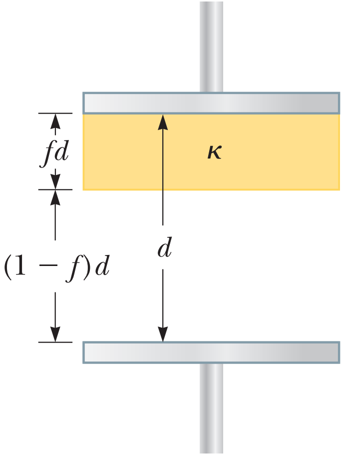
$$\begin{aligned} C_{\text{eq}} &= \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} \\ &= \left[\frac{(d-a)/2}{\epsilon_0 A} + \frac{(d-a)/2}{\epsilon_0 A}\right]^{-1} \\ &= \frac{\epsilon_0 A}{(d-a)} \end{aligned}$$

Partially-Filled Capacitor, Ex 26.8

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness fd is inserted between the plates, where f is a fraction between 0 and 1?



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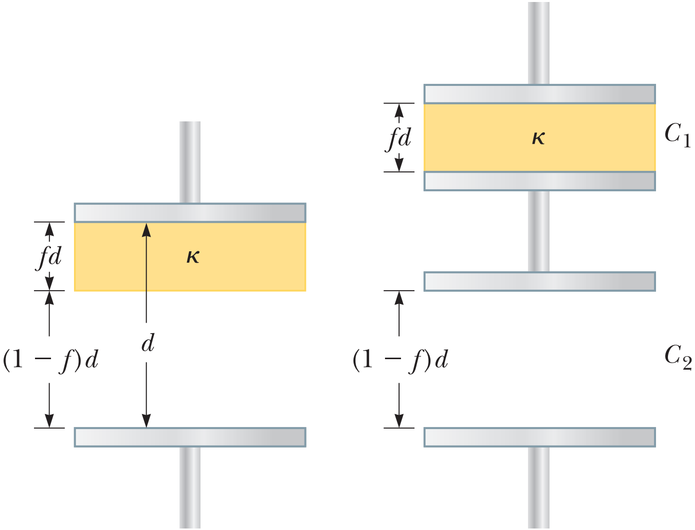

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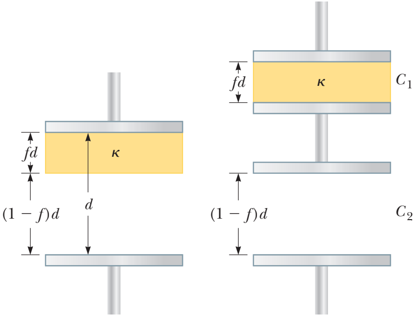

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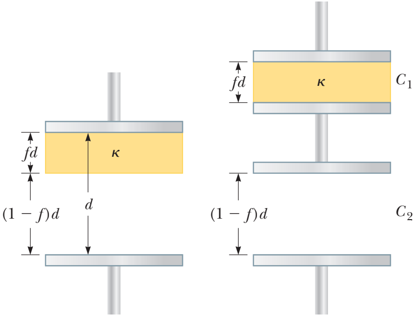
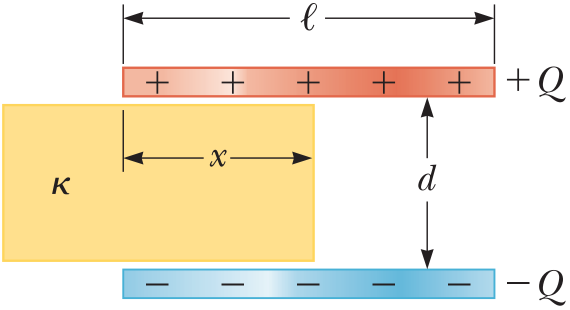


Again, 2 capacitors in series!

$$
\begin{aligned}
C_{\text{eq}} &= \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} \\
&= \left[\frac{df}{\kappa\epsilon_0 A} + \frac{(1-f)d}{\epsilon_0 A}\right]^{-1} \\
&= \frac{\kappa}{f + \kappa(1-f)}\, C_0
\end{aligned}
$$

Partially-Filled Capacitor

What about this case?



Electric Displacement Field

It is sometimes convenient to package the effect of the electric field together with the effect of the dielectric.

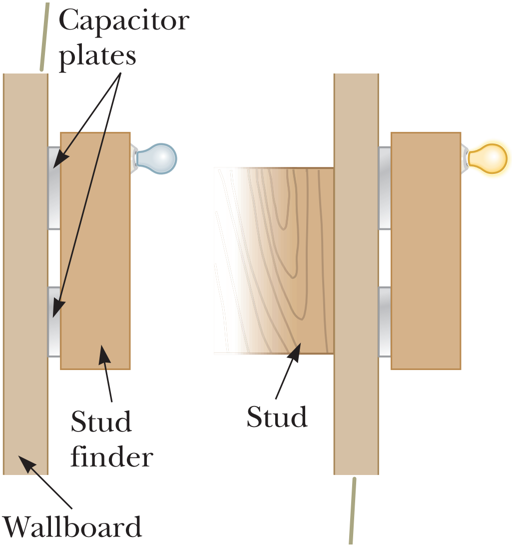
For this, people use a quantity, **Electric Displacement field**, which can be expressed[1]

$$\mathbf{D} = \kappa \epsilon_0 \mathbf{E}$$

Gauss's law is very often written in terms of the electric displacement, rather than the electric field, if the field being studied is in a polarizable material.

[1] In a linear, homogeneous, isotropic dielectric with instantaneous response.

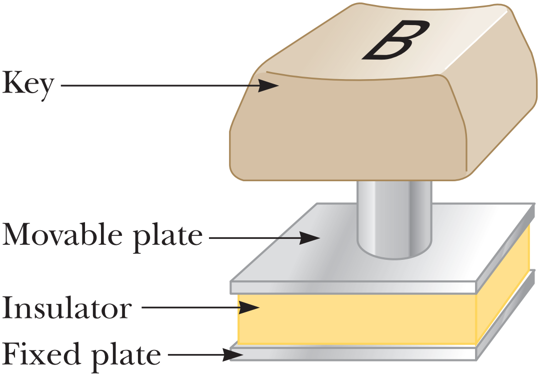
Uses of Dielectric Effects



[1]Figures from Serway & Jewett, 9th ed.

Uses of Dielectric Effects

Computer keyboard:



Summary

- dielectrics
- Gauss's law with dielectrics
- electric displacement
- some uses of dielectrics

Quiz tomorrow.

Homework

Serway & Jewett:

- PREVIOUS: Ch 26, onward from page 799. Problems: 13, 17, 21, 25, 31, 33, 35
- NEW: Ch 26. Problems: 43, 47, 49, 53, 63