Electricity and Magnetism
Dielectrics and Capacitors

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Last time

- capacitors in series
- practice with capacitors in circuits
- Energy stored in a capacitor
- Dielectrics
- molecular view of dielectrics
Warm Up Question

Two capacitors of values 4.0 nF and 6.0 nF are connected in a circuit as shown:

(A) 4.0 nF
(B) 6.0 nF
(C) 10 nF
(D) 2.4 nF
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Overview

- Dielectrics
- Gauss’s law with dielectrics
- Electric displacement
- Some uses of dielectrics
Dielectrics

dielectric
an insulating material that can affects the strength of an electric field passing through it

Different materials have different dielectric constants, $\kappa$.

For air $\kappa \approx 1$. (It is 1 for a perfect vacuum.)

$\kappa$ is never less than 1. It can be very large $> 100$.

The effect of sandwiching a dielectric in a capacitor is to change the capacitance:

$$C \rightarrow \kappa C$$
Why do dielectrics effect the strength of the electric field?

The external electric field from the aligns dipoles in the dielectric material.

The separation produces a nonpolar dielectric slab is zero electric field in the direction opposite that of the applied electric field. The resultant field inside the dielectric (the vector sum of fields and ) has the same direction as but a smaller magnitude. Both the field produced by the surface charges in Fig. 25-15 and the dielectric by one of the methods described in Section 25-7.

For the situation of Fig. 25-16, we can find the electric field between the plates as we did in Fig. 25-5: We enclose the charge on one face of the slab (due to the positive ends of dipoles there) and negative charges set up a field which opposes the applied field. Thus, the effect of both polar and nonpolar dielectrics opposes the applied field. Therefore, the effect of both polar and nonpolar dielectrics is to reduce the electric field strength.
Electric field inside the dielectric

The polarized dielectric contributes its own field, $E'$. 

The electric field from the charged plates alone $E_0$, is reduced.

The resulting reduced field is $E = \frac{E_0}{\kappa}$.
Dielectric in a Capacitor

\[ \epsilon_0 \rightarrow \kappa \epsilon_0 \]

For a parallel plate capacitor with a dielectric, the capacitance is now:

\[ C = \frac{\kappa \epsilon_0 A}{d} \]
Dielectric in a Capacitor

If we add a dielectric while the capacitor is connected to a battery:

\[ \frac{C}{\epsilon_0} \frac{1}{d} \]

where \( \epsilon_0 \) has the dimension of length. For example, \( \frac{C}{\epsilon_0} \frac{1}{d} = \frac{A}{d} \) for a parallel-plate capacitor.

Faraday's discovery was that, with a dielectric completely filling the space between the plates, Eq. 25-26 becomes

\[ \frac{C}{\epsilon_0} \frac{k}{\epsilon_0} \frac{1}{d} \]

where \( C_{air} \) is the value of the capacitance with only air between the plates. For example, if we fill a capacitor with strontium titanate, with a dielectric constant of 310, we multiply the capacitance by 310.

Both these observations are consistent (through the relation \( q = CV \)) with the increase in capacitance caused by the dielectric.

Comparison of Eqs. 25-26 and 25-27 suggests that the effect of a dielectric can be summed up in more general terms:

In a region completely filled by a dielectric material of dielectric constant \( k \), all electrostatic equations containing the permittivity constant \( \epsilon_0 \) are to be modified by replacing \( \epsilon_0 \) with \( k \epsilon_0 \).

Thus, the magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form of Eq. 23-15:

\[ E = \frac{1}{4 \pi \epsilon_0} \frac{q}{r^2} \]

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric (see Eq. 23-11) becomes

\[ E = \frac{1}{4 \pi \epsilon_0} \frac{q}{r^2} \]

Because \( k \) is always greater than unity, both these equations show that for a fixed distribution of charges, the effect of a dielectric is to weaken the electric field that would otherwise be present.
Dielectric in a Capacitor

If we add a dielectric while the capacitor is connected to a battery:

- $q$ will increase. ($q = CV$)
- $U$ will increase. ($U = \frac{1}{2} CV^2$)
Dielectric in a Capacitor

If we add a dielectric while the capacitor is isolated so charge cannot leave the plates:

\[ q = \text{a constant} \]
Dielectric in a Capacitor

If we add a dielectric while the capacitor is isolated so charge cannot leave the plates:

- $V$ will decrease. \((V = \frac{q}{C})\)
- $U$ will decrease. \((U = \frac{q^2}{2C})\)
Effect of a Dielectric on Field

Imagine again the isolated conductor: charge density $\sigma$ is constant.

The electric field between the plates is $E = \frac{\sigma}{\epsilon_0}$ originally.

With dielectric added: $E \rightarrow \frac{\sigma}{\kappa \epsilon_0}$.

The field strength decreases: $E \rightarrow \frac{E}{\kappa}$ (as we know it should)
Imagine again the isolated conductor: charge density $\sigma$ is constant.

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With dielectric added: $E \to \frac{\sigma}{\kappa \varepsilon_0}$.

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What happens to the energy density $u$?
Effect of a Dielectric on Field

What happens to the energy density? Was: \( u_0 = \frac{1}{2} \varepsilon_0 E_0^2 \).

\[
u = \frac{1}{2} (\kappa \varepsilon_0) (E)^2\]
Effect of a Dielectric on Field

What happens to the energy density? Was: \( u_0 = \frac{1}{2} \epsilon_0 E_0^2 \).

\[
\begin{align*}
  u &= \frac{1}{2} (\kappa \epsilon_0) (E)^2 \\
  &= \frac{1}{2} (\kappa \epsilon_0) \left( \frac{\sigma}{\kappa \epsilon_0} \right)^2 \\
  &= \frac{1}{2} \epsilon_0 \kappa \left( \frac{1}{\kappa^2} \right) E_0^2 \\
  &= \frac{1}{\kappa} \left( \frac{1}{2} \epsilon_0 E_0^2 \right) \\
  u &= \frac{u_0}{\kappa}
\end{align*}
\]

Energy density decreases.
Dielectrics and Electric Field

Dielectrics effect the field around a charge

\[ E \rightarrow \frac{E}{\kappa} \]

For example, for a point charge \( q \) in free space:

\[ E_0 = \frac{k \ q}{r^2} = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \]

But in a dielectric, constant \( \kappa \):

\[ E = \frac{1}{4\pi (\kappa \epsilon_0)} \frac{q}{r^2} = \frac{E_0}{\kappa} \]
Guass’s Law with dielectrics

\[ \kappa \epsilon_0 \Phi_E = q_{\text{free}} \]

or:

\[ \oint_A \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{free}}}{\kappa \epsilon_0} \]

The charge \( q_{\text{free}} = q \) in the diagram. It is just the charge on the plates, the charge that is free to move.
A parallel-plate capacitor has a plate separation \( d \) and plate area \( A \). An uncharged metallic slab of thickness \( a \) is inserted midway between the plates. Find the capacitance of the device.
An Atomic Description of Dielectrics

Example 26.7   Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation \( d \) and plate area \( A \). An uncharged metallic slab of thickness \( a \) is inserted midway between the plates. Find the capacitance of the device.

![Capacitor with a Metal slab, Ex 26.7](image-url)
Capacitor with a Metal slab, Ex 26.7

This is just 2 capacitors in series!

\[
C_{\text{eq}} = \left[ \frac{1}{C_1} + \frac{1}{C_2} \right]^{-1}
\]
Capacitor with a Metal slab, Ex 26.7

This is just 2 capacitors in series!

\[
C_{eq} = \left[ \frac{1}{C_1} + \frac{1}{C_2} \right]^{-1}
\]
\[
= \left[ \frac{(d - a)/2}{\epsilon_0 A} + \frac{(d - a)/2}{\epsilon_0 A} \right]^{-1}
\]
\[
= \frac{\epsilon_0 A}{(d - a)}
\]
Partially-Filled Capacitor, Ex 26.8

A parallel-plate capacitor with a plate separation $d$ has a capacitance $C_0$ in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant $\kappa$ and thickness $fd$ is inserted between the plates, where $f$ is a fraction between 0 and 1?
Partially-Filled Capacitor, Ex 26.8

What is the capacitance when a slab of dielectric material of dielectric constant \( \kappa \) and thickness \( fd \) is inserted between the plates, where \( f \) is a fraction between 0 and 1?

\[
C = \frac{\kappa fd}{d} \left( 1 - \frac{1}{1 + f} \right)
\]
Partially-Filled Capacitor, Ex 26.8

Again, 2 capacitors in series!

\[ C_{\text{eq}} = \left[ \frac{1}{C_1} + \frac{1}{C_2} \right]^{-1} \]
Again, 2 capacitors in series!

\[
C_{eq} = \left[ \frac{1}{C_1} + \frac{1}{C_2} \right]^{-1} = \left[ \frac{df}{\kappa \epsilon_0 A} + \frac{(1 - f)d}{\epsilon_0 A} \right]^{-1} = \frac{\kappa}{f + \kappa(1 - f)} C_0
\]
What about this case?
Electric Displacement Field

It is sometimes convenient to package the effect of the electric field together with the effect of the dielectric.

For this, people use a quantity, Electric Displacement field, which can be expressed\(^1\)

\[
D = \kappa \varepsilon_0 E
\]

Gauss’s law is very often written in terms of the electric displacement, rather than the electric field, if the field being studied is in a polarizable material.

\(^1\)In a linear, homogeneous, isotropic dielectric with instantaneous response.
Chapter 26 Capacitance and Dielectrics

Types of Capacitors
Many capacitors are built into integrated circuit chips, but some electrical devices have a polarity, which is indicated by positive and negative signs marked on the plates of the capacitor. When the capacitor moves across a dielectric, the capacitor and the dielectric.

Find the energy stored in the system before and after the dielectric is inserted.

A parallel-plate capacitor is charged with a battery to a charge $Q$. The plates are separated by a wallboard and the wood stud.

The change in the dielectric constant is due to the presence of the capacitor and the dielectric. If the polarity of the applied voltage is the opposite of what is intended, the capacitor cannot function. When the capacitor moves across a dielectric, the capacitor and the dielectric.

Figure 26.16 Three commercial capacitor designs.

- A high-voltage capacitor consisting of metallic plates, one fixed and the other movable, and contain air as the dielectric. These types of capacitors are often used in radio tuning circuits.
- An electrolytic capacitor because the dielectric layer is very thin and therefore the capacitance is low. This capacitor is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.14c, consists of a metallic foil in contact with a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this oxide layer serves as the dielectric. Very large values of capacitance can be obtained in electrolytic capacitors.
- Oil capacitors are used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.14b, consists of metallic foil interlaced with thin sheets of either paraffin-impregnated paper or Mylar, whose plates are separated by paper.

Often, an electrolytic capacitor is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.14c, consists of a metallic foil in contact with a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this oxide layer serves as the dielectric. Very large values of capacitance can be obtained in electrolytic capacitors.

Quick Quiz
If you have ever tried to hang a picture or a mirror, you know it can be a challenge. Often, an electrolytic capacitor is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.14c, consists of a metallic foil in contact with a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this oxide layer serves as the dielectric. Very large values of capacitance can be obtained in electrolytic capacitors.

A variable capacitor (typically 10 to 500 pF) usually consist of two interwoven sets of metallic plates separated by paper. The capacitance of a variable capacitor can be adjusted by moving the plates closer or farther apart. If the plates are moved closer together, the capacitance increases, and if they are moved farther apart, the capacitance decreases.

Uses of Dielectric Effects

1. Figures from Serway & Jewett, 9th ed.
Uses of Dielectric Effects

Computer keyboard:

Key
Movable plate
Insulator
Fixed plate
Summary

- dielectrics
- Gauss’s law with dielectrics
- electric displacement
- some uses of dielectrics

Quiz tomorrow.

Homework

Serway & Jewett:

- PREVIOUS: Ch 26, onward from page 799. Problems: 13, 17, 21, 25, 31, 33, 35
- NEW: Ch 26. Problems: 43, 47, 49, 53, 63