Electricity and Magnetism
Current

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Feb 2, 2018

Last time

- dielectrics and capacitors
- uses of capacitors

Overview

- current
- current density
- drift speed

Motion of Charge

Up until now, we have mostly considered charges in fixed positions.

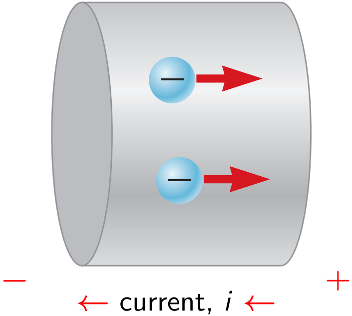
We will now look at steadily moving charges, particularly in circuits.

Flow of charge in a circuit

Current is the rate of flow of charge.

Conventional current is said to flow from the positive terminal to the negative terminal.

However, actually it is negatively charged electrons that flow through metal wires:



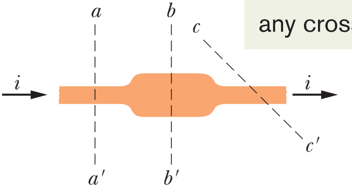
[1]Figure from Serway and Jewett, 9th ed.

Electric Current

Electric current, I, is the rate of flow of charge through some defined plane:

$$I = \frac{\mathrm{dQ}}{\mathrm{dt}}$$

The defined plane might be aa'. However, since charge is conserved if an amount of charge Q flows through aa', then the same amount of charge Q must flow through bb' and cc' in the same time interval.



The current is the same in any cross section.

Average Electric Current

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

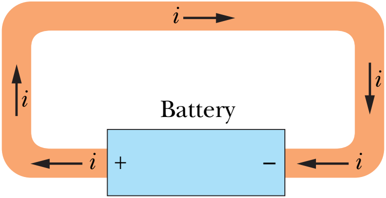
ΔQ is a net amount of charge and Δt is a time interval.

The flowing charge could be electrons in a conductor, positive or negative ions in a solution, electrons and ions in a plasma, etc.

Current

Charge will only move when there is a net force on it. A supplying a potential difference across two points on a wire will do this.

(a)

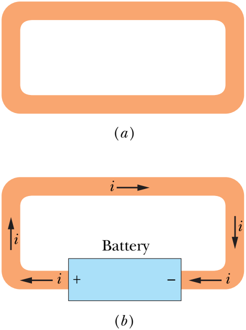


(b)

Current

Charge will only move when there is a net force on it. A supplying a potential difference across two points on a wire will do this.

However, notice that if there is a potential difference between two points in a wire, that must mean that there is a non-zero electric field between those points - even though the wire is a conductor!



Electric Current

The units of current are Amps, A. Formally, amperes. (After André-Marie Ampère.)

$1\ \text{A} = 1\ \text{C/s}$

Current is a scalar, however, a negative sign can be used to indicate a current flowing backwards through a loop.

Conventional Current

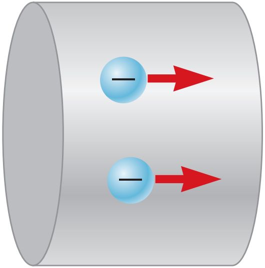
By convention, current is labeled indicating the direction in which **positive charge carriers would move**.

Of course, in very many circumstances, and particularly in conducting metals, electrons, which are negative charge carries, are the moving charges.

This means that a current arrow is drawn opposite to the direction of motion of electrons.

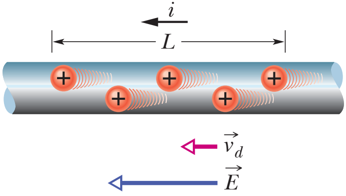
Conventional Current

A conducting wire:



− ← current, i ← +

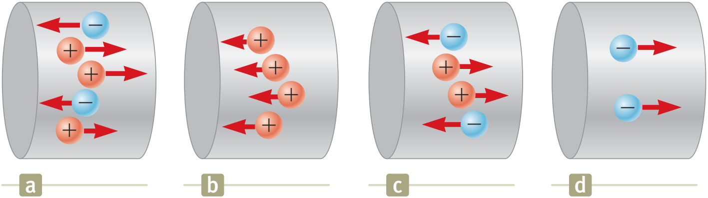
We imagine positive charges moving:



− +

Current Question

QuickQuiz 27.1: Consider positive and negative charges moving horizontally through the four regions. Rank the current in these four regions from highest to lowest.



(A) b, a, c, d

(B) a, (b and c), d

(C) d, (b and c), a

(D) b, d, a, c

Current Question

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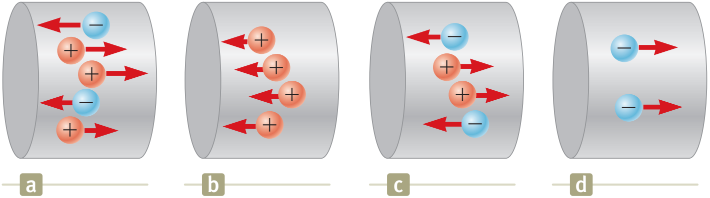

(A) b, a, c, d

(B) a, (b and c), d ←

(C) d, (b and c), a

(D) b, d, a, c

Current and Junctions

Since charge is conserved, all charge that flows into a point, must flow out of it as well.

We can apply this to a junction: a point at which wires join or split.

This gives Kirchhoff's junction rule:

Junction Rule

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Current and Junctions

Junction Rule

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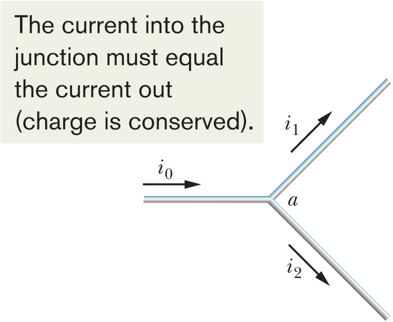
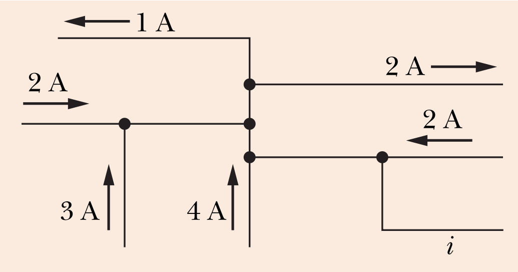


In the diagram, $i_0 = i_1 + i_2$

Question

What are the magnitude and direction of the current i in the lower right-hand wire?



[1]Halliday, Resnick, Walker, 9th ed, page 684.

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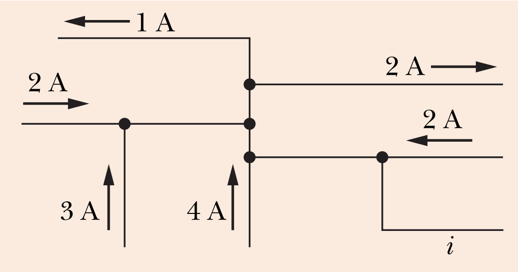


$i = 8$ A to the right.

[1]Halliday, Resnick, Walker, 9th ed, page 684.

Current Density

Current Density, J

The current per unit area through a conductor.

$$J = \frac{I}{A}$$

Strictly, this is the **average current density** through the area A, assuming the area A is perpendicular to the direction of the current.

This view of current density will be sufficient for most purposes in this course.

Current Density

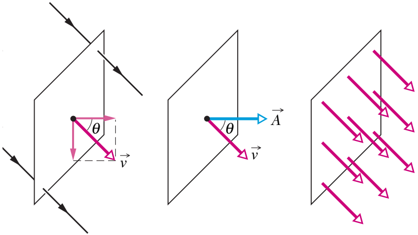
More formally, current density can be defined so that current is very similar to flux:

$$I = \int_A \mathbf{J} \cdot \mathrm{d}\mathbf{A}$$

Whereas flux:

$$\Phi_E = \int_A \mathbf{E} \cdot \mathrm{d}\mathbf{A}$$

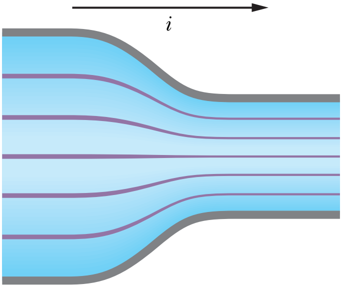
Current density J can be compared with the electric field, E.



Current Density

Current density can be represented with streamlines that are denser where the current density is higher.

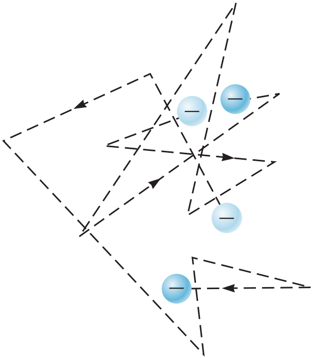
(*cf.* electric field and electric field lines)



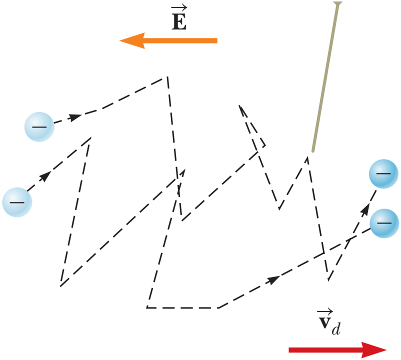
Microscopic Model of Current

Conduction electrons can be though of as moving in a random way, colliding with atoms.

Electrons with $E = 0$:



with an E-field:



With an external field, they tend to drift in the opposite direction to the field lines.

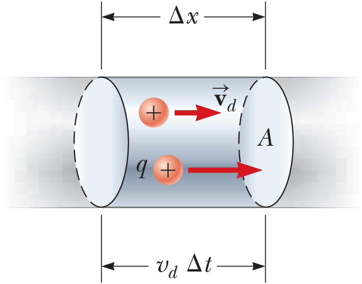
Drift Speed

The drift speed v_d of charge carriers in a conductor is the average speed at which a charge carrier is expected to move through a conductor.

The average speed of a charge carrier through a circuit, by definition is:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

How far (Δx) do we expect a charge carrier to move in time Δt?



Drift Speed

Need an expression relating v_{avg} to current.

Suppose there are n free conduction electrons per unit volume.

Then $n\,A\,\Delta x$ electrons move through a cross section A in time Δt.
(Vol $= A\,\Delta x$)

$$I = \frac{Q}{\Delta t} = \frac{(nA\,\Delta x)e}{\Delta t}$$

Drift Speed

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Then we can rearrange:

$$\frac{\Delta x}{\Delta t} = \frac{I}{nAe}$$

Drift Speed

Putting this back into the expression for v_d:

$$v_d = \frac{\Delta x}{\Delta t} = \frac{I}{nAe}$$

$$v_d = \frac{I}{nAe} = \frac{J}{ne}$$

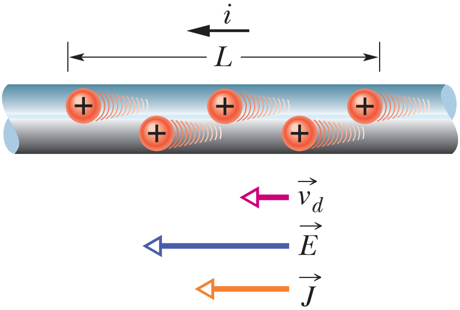
$(J = I/A)$

Drift velocity

We can also express this as a vector relation:

$$\mathbf{J} = n\, q\, \mathbf{v}_d$$

where q is the charge of the charge carrier.



Drift Speed of an Electron in Copper

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900$ μm when it has a uniform current $I = 17$ mA?

Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

Drift Speed of an Electron in Copper

How many electrons per unit volume? Same as number of copper atoms:

$$n = \frac{N_A \, \rho}{M} = \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(8.96 \times 10^{3} \text{ kg/m}^{3})}{63.54 \times 10^{-3} \text{ kg/mol}}$$

N_A is Avagadro's number, M is the molar mass (kgs per mole of copper), and ρ is copper's density.

$$n = 8.49 \times 10^{28} \text{ m}^{-3}$$

This is the number of free conduction electrons in a cubic meter of copper. (A lot.)

Drift Speed of an Electron in Copper

$$
\begin{aligned}
v_d &= \frac{I}{nAe} \\
&= \frac{(17 \times 10^{-3}\ \mathrm{A})}{(8.49 \times 10^{28}\ \mathrm{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \mathrm{C})}
\end{aligned}
$$

Drift Speed of an Electron in Copper

$$\begin{aligned} v_d &= \frac{I}{nAe} \\ &= \frac{(17 \times 10^{-3}\ \text{A})}{(8.49 \times 10^{28}\ \text{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \text{C})} \\ v_d &= 4.9 \times 10^{-7}\ \text{m/s} \end{aligned}$$

Very slow!

Summary

- current
- current density
- drift speed

2nd Collected Homework due Monday, Feb 12.

Homework

Serway & Jewett:

- PREVIOUS: Ch 26, onward from page 799. Problems: 43, 47, 49, 53, 63
- NEW: Ch 27, onward from page 824. Problems: 1, 5, 7