Electricity and Magnetism
Current and Resistance
Resistance and Resistivity

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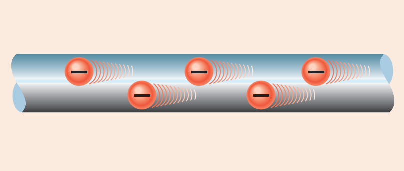
Last time

- current
- current density
- drift velocity

Warm Up Question

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward:
(a) the (conventional) current I,
(b) the current density $\mathbf{J}$,
(c) the electric field $\mathbf{E}$ in the wire?



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(B) all rightwards

(C) leftward, leftward, rightward

(D) rightward, rightward, leftward

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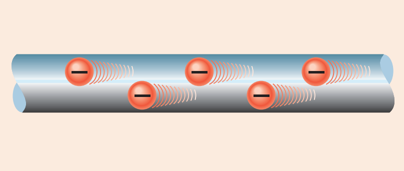

(A) all leftwards

(B) all rightwards $\leftarrow$

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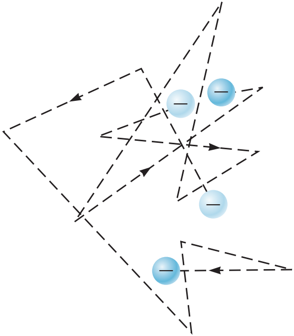
Overview

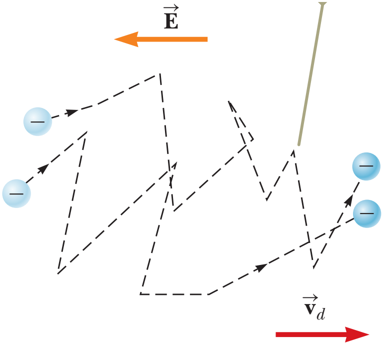
- resistance
- resistivity
- conductivity
- Ohm's Law

Drift Speed

Electrons with $E = 0$:



with an E-field:



$$v_d = \frac{I}{nAe} = \frac{J}{ne}$$

$(J = I/A)$

Drift Speed of an Electron in Copper

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900$ μm when it has a uniform current $I = 17$ mA?

Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

Drift Speed of an Electron in Copper

How many electrons per unit volume? Same as number of copper atoms:

$$n = \frac{N_A \, \rho}{M} = \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(8.96 \times 10^3 \text{ kg/m}^3)}{63.54 \times 10^{-3} \text{ kg/mol}}$$

N_A is Avagadro's number, M is the molar mass (kgs per mole of copper), and ρ is copper's density.

$$n = 8.49 \times 10^{28} \text{ m}^{-3}$$

This is the number of free conduction electrons in a cubic meter of copper. (A lot.)

Drift Speed of an Electron in Copper

$$
\begin{aligned}
v_d &= \frac{I}{nAe} \\
&= \frac{(17 \times 10^{-3}\ \text{A})}{(8.49 \times 10^{28}\ \text{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \text{C})}
\end{aligned}
$$

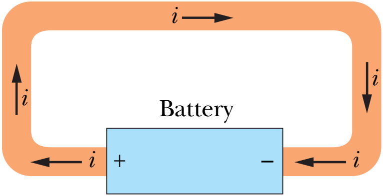
Drift Speed of an Electron in Copper

$$\begin{aligned} v_d &= \frac{I}{nAe} \\ &= \frac{(17 \times 10^{-3}\ \text{A})}{(8.49 \times 10^{28}\ \text{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \text{C})} \\ v_d &= 4.9 \times 10^{-7}\ \text{m/s} \end{aligned}$$

Very slow!

Resistance

When a potential difference is applied across a conductor, current begins to flow.



However, different amounts of current will flow in different conductors, even when the applied potential difference is the same.

What is the characteristic of the conductor which determines the **amount** of current that will flow?

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Resistance

Resistance

The resistance of a conductor is given by the ratio of the applied potential to the current that flows through the conductor at that potential:

$$R = \frac{\Delta V}{I}$$

The units of resistance are Ohms, Ω, symbol is the capital Greek letter "Omega". $1\ \Omega\ = 1$ V/A

We can think of a high resistance as *resisting*, or impeding, the flow of current.

Resistivity

An individual conductor or circuit component has a resistance.

The resistance is based on

- the material it is made of,
- its geometry, and
- the temperature

The material that a component is made from affects the resistance, because different materials have different *resistivities*.

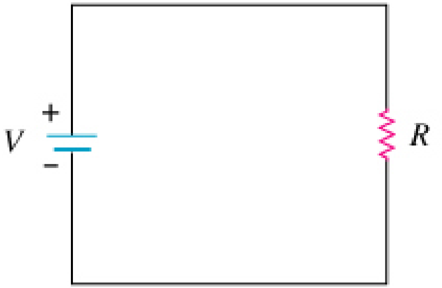
Resistor

A resistor is a component that can be incorporated into a circuit.

It has a particular resistance at a given voltage.

[1] Image from thomasnet.com

Resistor in a Circuit diagram



Resistivity

resistivity, ρ

the ratio of the electric field strength in a material to the current density this field causes in the material:

$$\rho = \frac{E}{J}$$

Resistivity is a property of a material. Its symbol is the Greek letter ρ, pronounced "rho".

The units of resistivity are Ω m.

$$1\ \Omega\ \text{m} = 1\ \frac{\text{V}}{\text{A}}\ \text{m} = 1\ \frac{\text{V/m}}{\text{A/m}^2}$$

which agrees with the definition of $\rho = E/J$.

Resistivity

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

Resistivity

Together with the geometry of the component made of that material, we can predict the resistance of the component.

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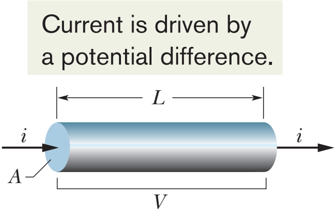
For a wire, cylinder, or anything with **uniform cross-section** A, made of material with resistivity ρ:

$$R = \frac{\rho L}{A}$$

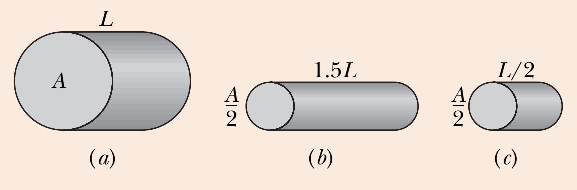
and L is the length of the wire.



(This follows from the definition of ρ.)

Question

Rank the three cylindrical copper conductors according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



(A) a, b, c

(B) c, b, a

(C) b, (a and c)

(D) (a and c), b

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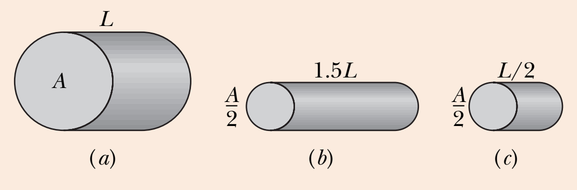

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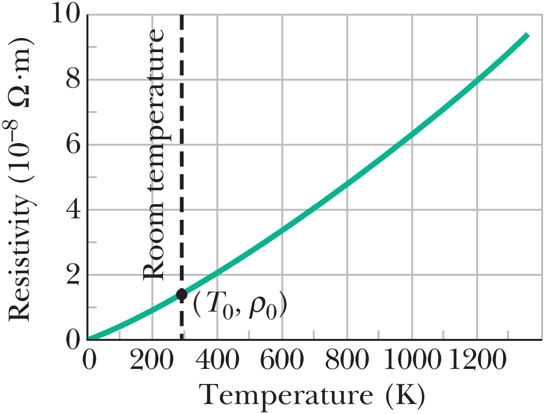
Resistivity can depend on Temperature



Resistivity can depend on Temperature

The relationship between resistivity and temperature is close to linear.

For most engineering purposes, a linear model is good enough.

The model:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

The resistivity varies linearly with the difference in temperature from some reference value T_0.

ρ_0 is the resistivity at T_0.

Resistivity can depend on Temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

α is just a constant, however it takes different values for different materials.

α is called the **temperature coefficient of resistivity**. It has units K^{-1}.

For example for copper[1]:

$\rho_0 = 1.62 \times 10^{-8}\ \Omega$ m   at 20° C

$\alpha = 4.3 \times 10^{-3}\ K^{-1}$

[1]Value from Halliday, Resnick, Walker, 8th ed.

Conductivity

Sometimes it is useful to represent how conductive a material is: how readily it permits current to flow, as opposed to how much it resists the flow of current.

conductivity, σ

a measure of what the current density is in a material for a particular electric field; the inverse of resistivity:

$$\sigma = \frac{1}{\rho} = \frac{J}{E}$$

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This is different than surface charge density (also written σ). This is just an unfortunate coincidence of notation.

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$$\sigma = \frac{1}{\rho} = \frac{J}{E}$$

The units of conductivity are $(\Omega \text{ m})^{-1}$.

We can use conductivity to relate the current density to the electric field in a material:

$$J = \sigma E$$

Resistance of Resistors with Non-Uniform Area

For a resistor with uniform cross-section A, made of material with resistivity ρ:

$$R = \frac{\rho L}{A}$$

What if the cross section isn't uniform?

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What if the cross section isn't uniform?

Integrate.

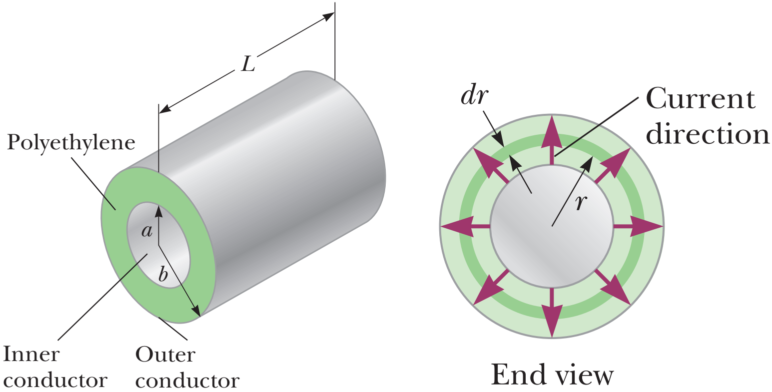
Use:

$$\mathrm{dR} = \frac{\rho}{A(\ell)}\,\mathrm{d}\ell$$

$A(\ell)$ means Area is a function of position, ℓ, along the length of the conductor. (Not A times ℓ.)

Example: Coaxial Cable

Find the resistance between the two conducting layers.



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At radius r the area a current can pass through is $A(r) = 2\pi r L$

$$R \;=\; \int_a^b \frac{\rho}{2\pi r L}\,\mathrm{dr}$$

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$$
\begin{aligned}
R &= \int_a^b \frac{\rho}{2\pi rL}\,\mathrm{dr} \\
&= \frac{\rho}{2\pi L}[\ln b - \ln a] \\
&= \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)
\end{aligned}
$$

Ohm's Law

Ohm's Law

The current through a device is directly proportional to the potential difference applied across the device.

$$\Delta V \propto I$$

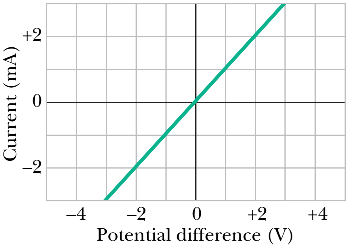
Not all devices obey Ohm's Law!

In fact, for all materials, if ΔV is large enough, Ohm's law fails.

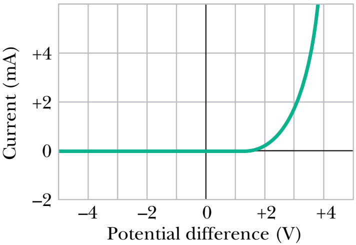
They only obey Ohm's law when the **resistance of the device is independent of the applied potential difference** and its polarity (that is, which side is the higher potential).

Ohm's Law

Obeys Ohm's law:



Does not obey Ohm's law:



We can write this linear relationship as $\Delta V = IR$ if and only if R is constant and independent of ΔV.

However, notice that we can always define $R(\Delta V) = \frac{\Delta V}{I}$ even when resistance does depend on ΔV.

Ohm's Law Question

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). Which of the devices obeys Ohm's law?

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

(A) 1 only

(B) 2 only

(C) both

(D) neither

[1] Halliday, Resnick, Walker, page 692.

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Summary

- Resistance
- resistivity
- conductance
- Ohm's Law

2nd Test Thursday, Feb 15.

Homework

- Collected homework 2, posted online, due on Monday, Feb 12.

Serway & Jewett:

- PREVIOUS: Ch 27, onward from page 824. Problems: 1, 5, 7
- NEW: Ch 27, Problems: 15, 23, 25, 29, 33, 71