Electricity and Magnetism
Current and Resistance
Ohm's Law
Exotic Conductors
Power

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Last time

- resistance
- resistivity
- conductivity
- Ohm's Law

Overview

- Drude model
- semiconductors
- superconductors
- power

Resistance of Resistors with Non-Uniform Area

For a resistor with uniform cross-section A, made of material with resistivity ρ:

$$R = \frac{\rho L}{A}$$

What if the cross section isn't uniform?

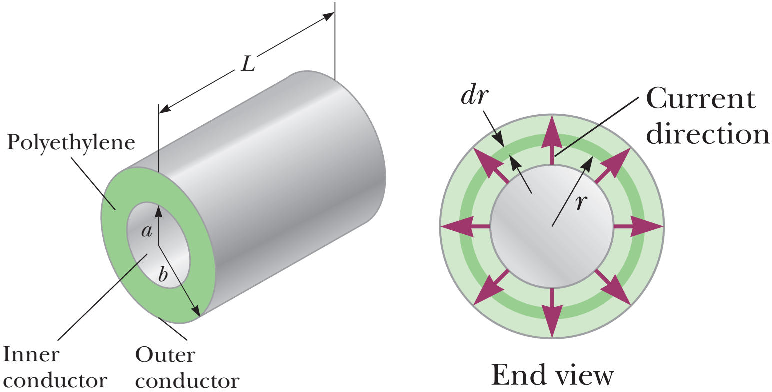
Use:

$$\mathrm{dR} = \frac{\rho}{A(\ell)}\,\mathrm{d}\ell$$

$A(\ell)$ means Area is a function of position, ℓ, along the length of the conductor. (Not A times ℓ.)

Example: Coaxial Cable

Find the resistance between the two conducting layers.



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At radius r the area a current can pass through is $A(r) = 2\pi r L$

$$R = \int_a^b \frac{\rho}{2\pi r L}\,\mathrm{dr}$$

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$$
\begin{aligned}
R &= \int_a^b \frac{\rho}{2\pi r L}\,\mathrm{dr} \\
&= \frac{\rho}{2\pi L}[\ln b - \ln a] \\
&= \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)
\end{aligned}
$$

Ohm's Law

Ohm's Law

The current through a device is directly proportional to the potential difference applied across the device.

$$\Delta V \propto I$$

Not all devices obey Ohm's Law!

In fact, for all materials, if ΔV is large enough, Ohm's law fails.

They only obey Ohm's law when the **resistance of the device is independent of the applied potential difference** and its polarity (that is, which side is the higher potential).

Ohm's Law Question

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). Which of the devices obeys Ohm's law?

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

(A) 1 only

(B) 2 only

(C) both

(D) neither

[1] Halliday, Resnick, Walker, page 692.

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A reason for Ohm's Law?

Electrons in an electric field accelerate.

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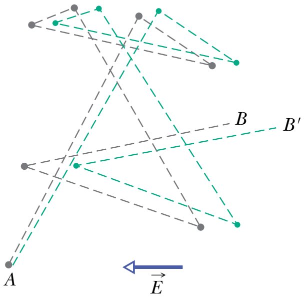
We supply a constant potential difference across a resistor. Why do the electrons not move faster and faster?

The mechanism for resistance is that the electrons collide with atoms in the resistive material.



The collisions slow the drift of the electrons.

A reason for Ohm's Law?

We will argue that we can model resistivity as being inversely proportional to the average time between collisions τ:

$$\rho \sim \frac{1}{\tau}$$

The model is the Drude model. We pretend that electrons are like little balls that collide from time to time with atoms, and are accelerated by the field.

$$\mathbf{a}_e = \frac{\mathbf{F}_{\text{net}}}{m_e} = \frac{(-e)\mathbf{E}}{m_e}$$

The acceleration stops when a collision occurs, after an average time interval, τ.

$$v_d = a_e \tau = \frac{eE\tau}{m_e}$$

A reason for Ohm's Law?

$$v_d = a_e \tau = \frac{eE\tau}{m_e}$$

Last lecture we found for a current of electrons:

$$J = n\, e\, v_d$$

And we can also write $J = \frac{E}{\rho}$.
Equating them, and using the expression for the drift velocity:

$$\frac{E}{\rho} = ne\,\frac{eE\tau}{m_e}$$

Rearranging:

$$\rho = \frac{m_e}{ne^2\tau}$$

m_e, n, and e are constants, so $\rho \propto \frac{1}{\tau}$.

A reason for Ohm's Law?

Resistivity is roughly inversely proportional to the average time between collisions τ:

$$\rho \sim \frac{1}{\tau}$$

So, does the time between collisions depend on the potential difference across the conductor?

It would, if the electric field caused a large change in the average electron's velocity. We would expect faster moving electrons to collide more frequently (τ would decrease).

A reason for Ohm's Law?

$$\rho \sim \frac{1}{\tau}$$

However, the average velocity of an electron in room temperature copper is $v \sim 1.6 \times 10^6$ m/s.

The drift velocity is perhaps $v_d \sim 10^{-7}$ m/s : $\frac{v_d}{v} \approx 10^{-13}$!

This means that varying the potential difference will have a negligible effect on τ and therefore also on the resisitivity ρ.

$\Rightarrow$ R is independent of ΔV in many cases.

$\Rightarrow$ ρ can depend on temperature!

More exotic conducting materials

So far, we have talked about conductors and insulators.

However, there are materials that behave in ways quite different from the conductors and insulators we have investigated so far. They are:

- semiconductors
- superconductors

Semiconductors

Semiconductors have resistivities between those of conductors and insulators.

However, their resistivities can be controlled by several different means (depending on the type of semiconductor):

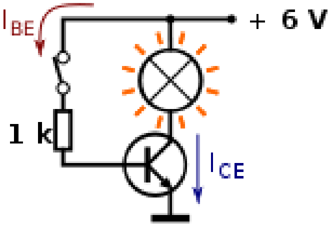
- by adding impurities during manufacture
- by electric fields
- by light

This allows for many new kinds of components in circuits: ones that amplify currents, emit light, are light sensitive, implement switching, *etc.*

Semiconductors

LED (light emitting diodes) are one application of semiconductors.

Transistors are another. Transistors can act as an amplifier or a switch in a circuit.



[1]Figure by FDominec, on Wikipedia.

Semiconductors

Silicon is perhaps the most famous semiconductor.

Recall that we had a model relating resistivity to temperature:

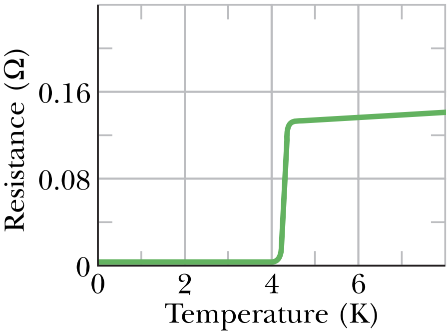
$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

For silicon α is negative! The resistivity decreases as temperature increases.

This is because at higher temperatures more electrons have enough energy to become freely-moving conducting electrons.

Superconductors

Superconducting materials are elements, alloys, or compounds that exhibit a remarkable property: below some *characteristic temperature* the resistivity of the material is effectively zero.



Examples of these materials are mercury and lead. Not all materials do this! Copper does not.

Mercury is superconducting below 4 K. (-269° C)

Superconductors

Before 1986, it seemed we had a good idea about how this happened and why.

[1] Drozdov, *et al.* (2015). Nature 525 (7567): 73-6. arXiv:1506.08190

Superconductors

Before 1986, it seemed we had a good idea about how this happened and why.

Then "high temperature" superconductors were found.

These are ceramics. One is yttrium barium copper oxide (YBCO).

The highest critical temperature, T_c, at atmospheric pressure found so far is ~ 138 K.

We do not really understand why these ceramics are superconductors.

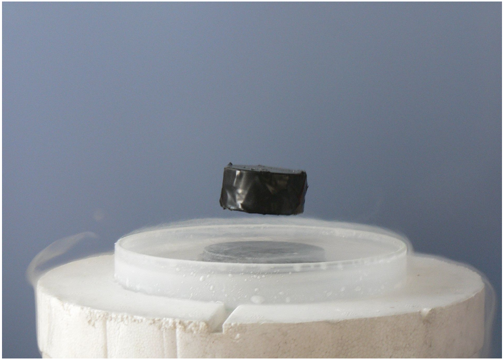
Hydrogen sulfide becomes a solid metal at extremely high pressures. It has $T_c = 203$ K at around 150 gigapascals pressure.[1]

[1] Drozdov, *et al.* (2015). Nature 525 (7567): 73-6. arXiv:1506.08190

Superconductors

Superconductors must be cooled to their critical temperature to reveal their superconducting properties.

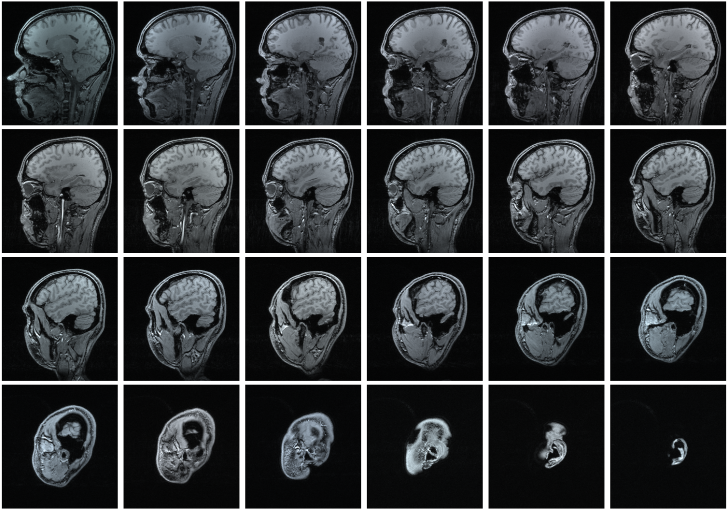
They expel magnetic field lines when cooled below their critical temperature as surface currents cancel out external magnetic fields.



[1]Magnet photo by Mai-Linh Doan, Wikipedia.

Superconductors

Superconductors are used as electromagents in MRI scanners, mass spectrometers, and particle accelerators.



[1]Taken at MPI fuer Biophysikalische Chemie Goettingen, by Daniel Schwen.

Superconductors

Superconductors can also be used very, very sensitive light detectors and for quantum logic circuits. ("Transition Edge Sensors")

If a material was found to have a critical temperature above or close to room temperature there would be a huge number of applications for it.

Power

Power is the rate of energy transfer or the rate at which work is done:

$$P = \frac{\mathrm{dW}}{\mathrm{dt}} = \frac{\mathrm{dQ}\,\Delta\mathrm{V}}{\mathrm{dt}} = \frac{\mathrm{dQ}}{\mathrm{dt}}\,(\Delta V)$$

where charge is moved through a potential difference ΔV at a rate $\frac{\mathrm{dQ}}{\mathrm{dt}} = I$.

For an electrical circuit we can ask about the rate at which a battery or other power supply transfers energy to a device.

This depends on the current and the potential difference:

$$P = I\,\Delta V$$

Power

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The units for power are Watts, W.

$1\ \mathrm{W} = 1\ \mathrm{J/s}$.

Does this unit agree with the new equation?

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Does this unit agree with the new equation?

1 A V $=$ (1 C/s) (1 J/C) $=$ 1 J/s . Yes.

Power Delivered

The power delivered to a device is

$$P = I\,\Delta V$$

where I is the current **through the device** and
ΔV is the potential difference **across the device**.

Power "Dissipated"

$$P = I\,\Delta V$$

We can use this expression along with $R = \frac{\Delta V}{I}$ to find the power dissipated as heat in a resistor.

Power dissipated as heat in a resistor:

$$P = I^2 R$$

or equivalently,

$$P = \frac{(\Delta V)^2}{R}$$

where I in the first equation is the current **in the resistor** and ΔV in the second equation is the potential difference **across the resistor**.

Summary

- a reason for Ohm's Law
- semiconductors and superconductors
- power
- high voltage transmission

2nd Test Thursday, Feb 15.

Homework

- Collected homework 2, posted online, due on Monday, Feb 12.

Serway & Jewett:

- PREVIOUS: Ch 27, onward from page 824. Problems: 15, 23, 25, 29, 33, 71
- NEW: Ch 27, Problems: 39, 43, 45, 57, 73, (85)