Electricity and Magnetism
Circuits
EMF

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Last time

- the Drude model of conduction
- semiconductors
- superconductors
- power

Overview

- power and high voltage transmission
- resistors in series and parallel
- electromotive force
- internal resistance of a battery (?)

Power

Power is the rate of energy transfer or the rate at which work is done:

$$P = \frac{\mathrm{dW}}{\mathrm{dt}} = \frac{\mathrm{dQ}\,\Delta \mathrm{V}}{\mathrm{dt}} = \frac{\mathrm{dQ}}{\mathrm{dt}}\,(\Delta V)$$

where charge is moved through a potential difference ΔV at a rate $\frac{\mathrm{dQ}}{\mathrm{dt}} = I$.

For an electrical circuit we can ask about the rate at which a battery or other power supply transfers energy to a device.

This depends on the current and the potential difference:

$$P = I\,\Delta V$$

Power "Dissipated" in Resistors

$$P = I\,\Delta V$$

where, again, I is the current through the resistor and ΔV is the potential difference across the resistor.

Using $R = \frac{\Delta V}{I}$,

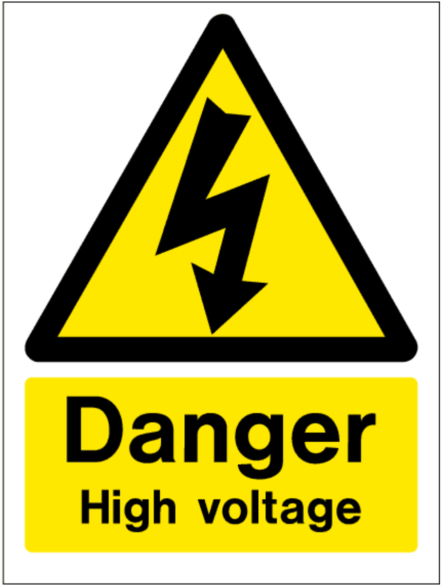
Power dissipated as heat in a resistor:

$$P = I^2 R$$

or equivalently,

$$P = \frac{(\Delta V)^2}{R}$$

Example: Why High Voltage?



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$$R = \frac{\rho L}{A} = 13.4\ \Omega$$

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$$P = I^2 R = (500 \text{ A})^2 (13.4 \ \Omega) = 3.36 \text{ MW}$$

Much more loss!

High Voltage Transmission

This is why power stations transmit power at a very high voltage.

The voltage is "stepped down" before being delivered to your house.

Mains electricity in the US is distributed throughout a house at 120 V. (The "line voltage".)

Circuits (Ch 28)

Circuits consist of a collection of electrical components connected by conducting wires through which charge is driven by an energy source.

Right now we focus on **direct-current (DC)** circuits.

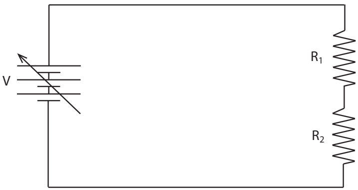
In a direct-current circuit current flows in one direction only.

This is the only type of situation we have been considering so far. However, in the coming labs you may look at some situations with **alternating-current (AC)**, in which the current flows forward, then backward, through the circuit.

Resistors Series and Parallel

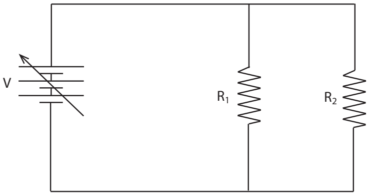
Series

When components are connected one after the other along a single path, they are connected in series.

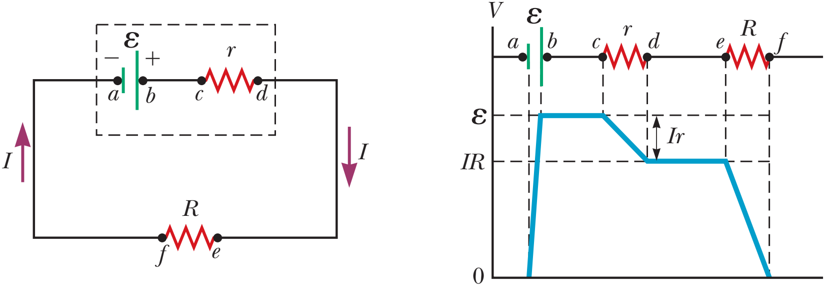


Parallel

When components are connected side-by-side on different paths, they are connected in parallel.



Potential in a Circuit



The potential drops across each resistor in the circuit as each transforms electrical power to heat.

Equivalently, the potential energy of a charge q decreases as it moves through a resistor.

We can assign a value of the electric potential to each point in a circuit.

Resistors in Series

The current though resistors in series in a loop is the same.

Let the total potential difference across two resistors be ΔV, then

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Then the effective equivalent resistance of both together is just the sum

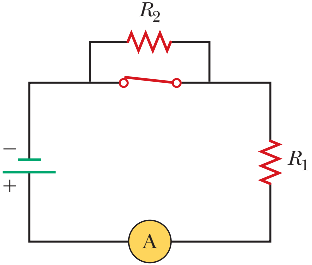
$$R_{\text{eq}} = R_1 + R_2$$

For n resistors in **series**:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n = \sum_{i=1}^{n} R_i$$

Resistors in Series Question

With the switch in the circuit closed, there is no current in R_2 because the current has an alternate zero-resistance path through the switch. There is current in R_1, and this current is measured with the ammeter at the bottom of the circuit. If the switch is opened, there is current in R_2.



What happens to the reading on the ammeter when the switch is opened?

(A) The reading goes up.
(B) The reading goes down.
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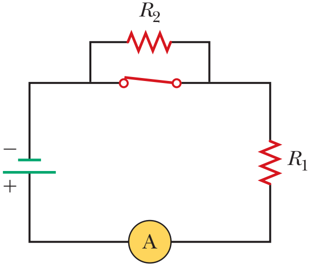

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Resistors in Parallel

The potential difference across two resistors in parallel is the same.

Let I be the total current that flows through both resistors:
$I = I_1 + I_2$. (Junction rule.)

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

Dividing the equation by ΔV:

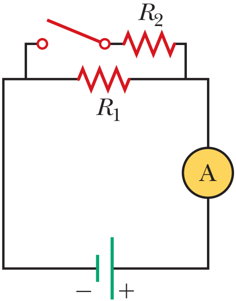
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For n of resistors in **parallel**:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Resistors in Parallel Question

Initially the switch in the circuit shown is open.



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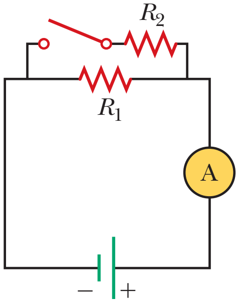

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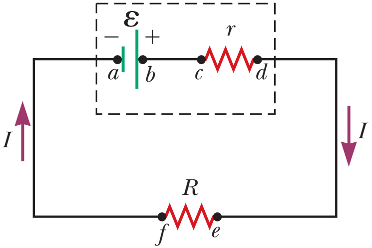
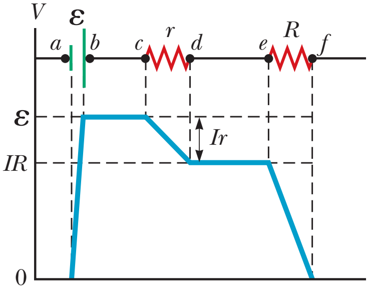
Resistors vs. Capacitors

Table of equivalent capacitances and resistances for series and parallel.

	resistors	capacitors
series	$R_{\text{eq}} = \sum R_i$	$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$
parallel	$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i}$	$C_{\text{eq}} = \sum C_i$

Potential in a Circuit





The potential drops across each resistor in the circuit.

Batteries increase the potential energy of a charge / raise the potential.

A closer look at batteries and power supplies

Batteries and power supplies fill a critical role in circuits.

They supply the energy to drive the charges around the circuit.

They do this by creating a charge imbalance and causing each charge to experience a force.

“Electomotive Force”

We say that a battery or power supply contributes an electromotive force (emf) and we can call batteries and power supplies emf devices.

These devices act as “charge pumps” in a circuit.

emf device

A device that maintains a potential difference between two points (terminals) in the circuit.

Electomotive “Force” (emf)

There is a force on each free charge in the system because there is an electric field.

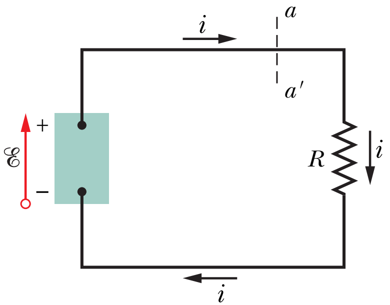
$$F = qE$$

The electric field exists because of the potential difference supplied to the circuit by the battery.

But this is not what we mean by emf! The emf is not actually a force.

Electomotive "Force"

We write an emf as $\mathcal{E}$, and label the battery with it:



Emf is actually a energy supplied per unit charge!
(Measured in volts.)

This makes calling it a "force" a bit misleading: the electromotive "force" is **not a force**.

(The name is an unfortunate choice that stuck.)

EMF

We can define emf by the following relation:

$$\mathcal{E} = \frac{dW}{dq}$$

meaning, an emf device does a work dW on an infinitesimal amount of charge dq:

$$dW = \mathcal{E}\, dq$$

while moving the infinitesimal charge dq from the negative terminal to the positive terminal. (Imagining dq to be positive.)

The amount of work that is done "lifting" this charge to the higher potential terminal depends only on the potential difference, so $\mathcal{E}$ is like a potential difference measured in volts.

Power Supplied

This definition for emf gives the power supplied by an emf device.

$$\mathsf{dW} = \mathcal{E}\ \mathsf{dq}$$

Power is the rate at which the work is done:

$$P = \frac{\mathsf{dW}}{\mathsf{dt}} = \mathcal{E}\ \frac{\mathsf{dq}}{\mathsf{dt}}$$

(assuming that the emf supplied by a source is constant.)
Then notice that $I = \frac{\mathsf{dq}}{\mathsf{dt}}$, so

$$P = I\mathcal{E}$$

This is the total power supplied by an emf device!

Compare to $P = I\,(\Delta V)$ as the power delivered to any component.

EMF

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Usually, we introduce emf when we want to make the battery more realistic: **batteries have some internal resistance**, so the potential the supplied is not the same in all circumstances.

The emf gives the maximum potential a battery can supply.

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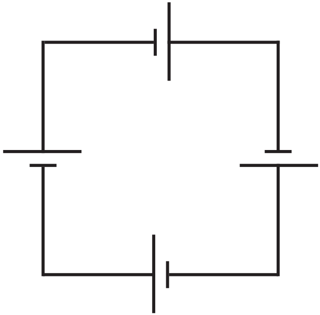
Usually, we introduce emf when we want to make the battery more realistic: **batteries have some internal resistance**, so the potential the supplied is not the same in all circumstances.

The emf gives the maximum potential a battery can supply.

There is one other important reason, however: we can now start to encounter circumstances where we cannot define electric potential.

EMF

We can still talk about emf, $\mathcal{E}$, even when we cannot define and electrical potential.



[1]Lithograph in the mathematically-inspired impossible reality style, by M.C. Escher.

Internal resistance

How does internal resistance affect the supplied potential difference?

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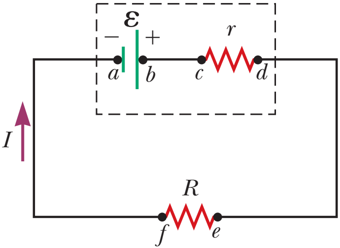
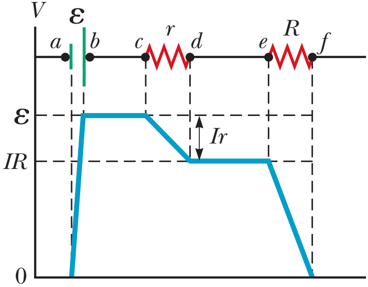
It is another resistance that is in series!



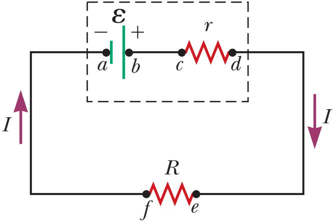


Let r be the internal resistance

$$\Delta V_r = Ir$$

ΔV_r is the potential drop across the internal resistance.

Internal resistance

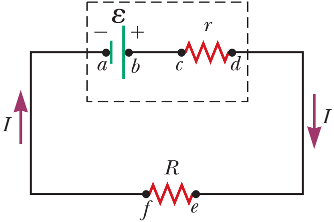


Let ΔV be the potential difference supplied by the battery to the rest of the circuit:

$$\Delta V = \mathcal{E} - Ir$$

V is the potential difference between the terminals of the battery at points a and d in the diagram.

Internal resistance



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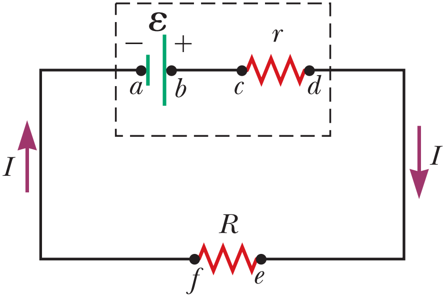
ΔV depends on the current that flows in the circuit!

Internal resistance



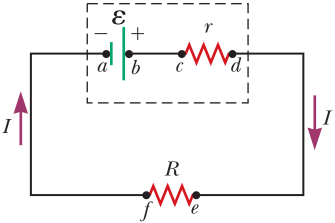
Ideal battery

An ideal battery has no internal resistance. ($r = 0$)

Real batteries do have internal resistance.

Internal resistance and current

The current that flows in the circuit, I, will in turn depend on the **load resistance** R, *ie.* the resistance in the rest of the circuit.



$\Delta V = IR$
and so, $IR = \mathcal{E} - Ir$ and:

$$I = \frac{\mathcal{E}}{r + R}$$

Internal resistance, potential difference, and power

$$I = \frac{\mathcal{E}}{r + R}$$

The potential difference supplied to the circuit ΔV:

$$\Delta V = IR = \frac{\mathcal{E} R}{r + R}$$

It depends on both the internal and external ("load") resistances.

Power:

$$\begin{aligned} \text{power supplied} &= \text{total power delivered} \\ I\mathcal{E} &= I^2 r + I^2 R \end{aligned}$$

Summary

- power and high voltage transmission
- resistors in series and parallel
- emf
- internal resistance of a battery

Quiz Friday.

Homework

- Collected homework 2, posted online, due on Monday, Feb 12.

Serway & Jewett:

- PREVIOUS: Ch 27, onward from page 824. Problems: 39, 43, 45, 57, 73, (85)
- NEW: Ch 28, Problems: 1, 3, 5, 7, 9, 15