Electricity and Magnetism
DC Circuits
Load Matching
Kirchhoff's Laws

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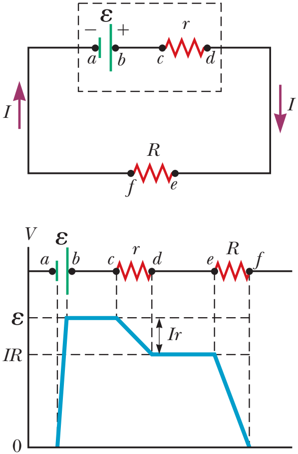
Last time

- power
- emf
- internal resistance of batteries

Overview

- load matching
- potential drops
- Kirchhoff's laws
- using KIrchhoff's laws

Internal resistance



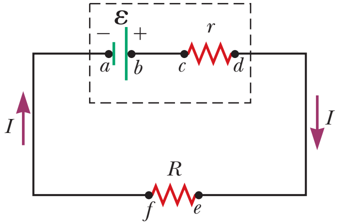
Let ΔV be the potential difference supplied by the battery to the rest of the circuit:

$$\Delta V = \mathcal{E} - Ir$$

ΔV is the potential difference between the terminals of the battery at points a and d in the diagram.

Internal resistance and current

The current that flows in the circuit, I, will in turn depend on the **load resistance** R, *ie.* the resistance in the rest of the circuit.



$\Delta V = IR$
and so, $IR = \mathcal{E} - Ir$ and:

$$I = \frac{\mathcal{E}}{r + R}$$

Internal resistance, potential difference, and power

$$I = \frac{\mathcal{E}}{r + R}$$

The potential difference supplied to the circuit ΔV:

$$\Delta V = IR = \frac{\mathcal{E} R}{r + R}$$

It depends on both the internal and external ("load") resistances.

Power:

$$\begin{aligned} \text{power supplied} &= \text{total power delivered} \\ I\mathcal{E} &= I^2 r + I^2 R \end{aligned}$$

Question

Quick Quiz 28.1: To maximize the **percentage** of the power from the emf of a battery that is delivered to a device external to the battery, what should the internal resistance of the battery be?

(A) It should be as low as possible.

(B) It should be as high as possible.

(C) The percentage does not depend on the internal resistance.

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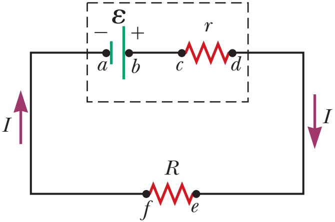
Load Matching

Assuming r is fixed, for what value of the load resistance R is the power delivered to R greatest?



Idea: maximize $P_R = I^2 R$.

Load Matching

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$$\begin{aligned}\frac{\mathrm{dP_R}}{\mathrm{dR}} &= \frac{\mathrm{d}}{\mathrm{dR}}\left(\frac{\mathcal{E}^2 R}{(r+R)^2}\right) \\ &= \frac{\mathcal{E}^2}{(r+R)^2} + (-2)\frac{\mathcal{E}^2 R}{(r+R)^3}\end{aligned}$$

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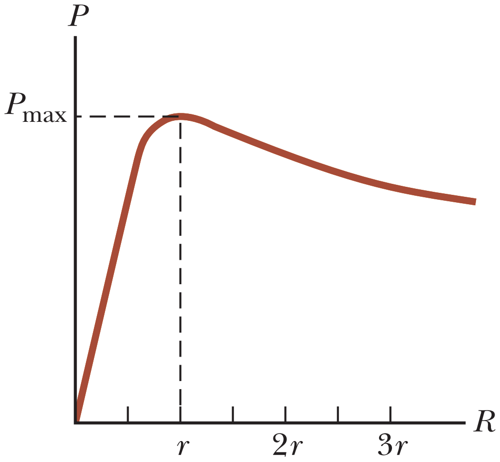
Set equal to zero to find stationary point:

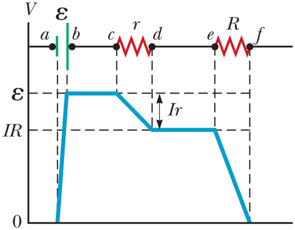
$$\begin{aligned}0 &= \frac{\mathcal{E}^2}{(r+R)^2} + (-2)\frac{\mathcal{E}^2 R}{(r+R)^3} \\ 1 &= \frac{2R}{r+R} \\ R &= r\end{aligned}$$

Load Matching



The y-axis is the power delivered to the load resistance.

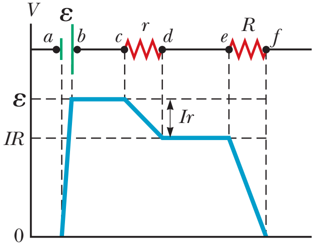
Potential difference between two points



For any circuit we can find the potential difference between points in the circuit by finding the potential drop or jump across the elements between those points.

Two rules can help us track this.

Potential difference between two points



"Voltage Drops":

resistance rule

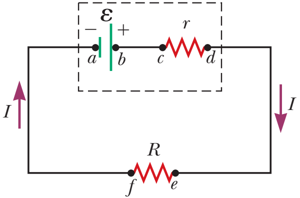
Going through a resistance R in the direction of the current, the change in potential is $-IR$; in the opposite direction it is $+IR$.

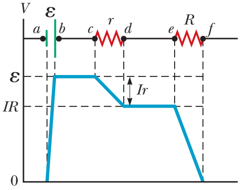
"Voltage jumps":

emf rule

Going through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.

The loop rule

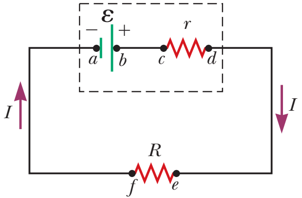


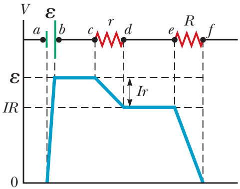


Notice in the lower diagram that we we come back at the right end to the same potential that we started at on the left end.

In fact, it doesn't matter what point we start at: if we go around a closed loop, when we return to the starting point, we must return to the starting potential also.

The loop rule





Kirchhoff's loop rule:

Loop Rule

The sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Kirchhoff's Laws

The loop rule for potential difference and the junction rule for current together are called Kirchhoff's laws.

Loop Rule

The sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

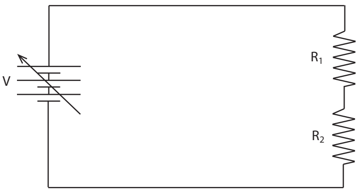
Junction Rule

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

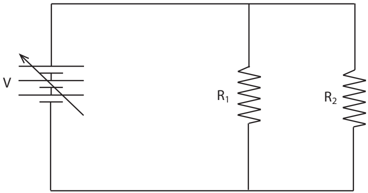
Using both it is possible to discover many things about how a circuit operates, for example how much power will be dissipated in a particular component.

Multiloop Circuits

Single loop

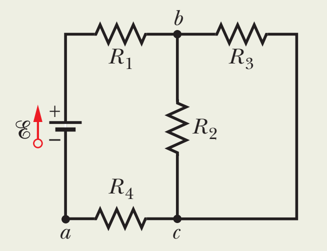


Multiloop



Example

Consider the circuit pictured with $\mathcal{E} = 12$ V, and the following resistor values: $R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$, and $R_4 = 8.0\ \Omega$.



What is the current through the battery?

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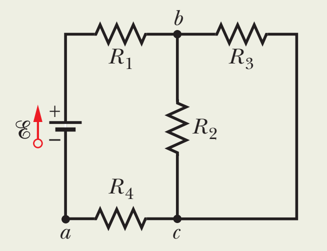

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answer: $I = 0.30$ A

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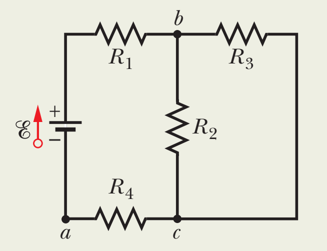

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What is the current through resistor R_2?

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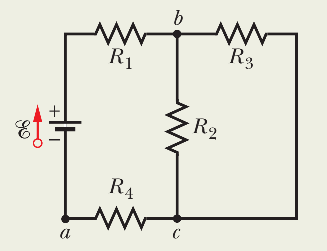
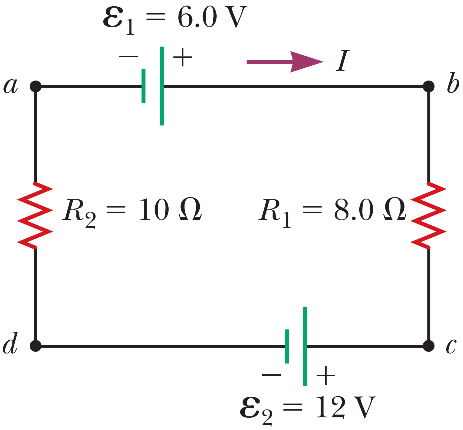


What is the current through the battery?
answer: $I = 0.30$ A

What is the current through resistor R_2?
answer: $I_2 = 0.18$ A

Example with Two Batteries

Find the current in the circuit.



Suppose the current flows in the direction shown.

Summary

- load matching
- Kirchhoff's laws

Next Test on Thursday, Feb 15.

Homework

- Collected homework 2, posted online, due on Monday, Feb 12.

Serway & Jewett:

- PREVIOUS: Ch 28, onward from page 857. Problems: 1, 3, 5, 7, 9, 15
- NEW: Ch 28, Problems: 23, 27, 31, 33, 35, 61