Electricity and Magnetism
DC Circuits
Resistance-Capacitance Circuits

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Feb 12, 2018

Last time

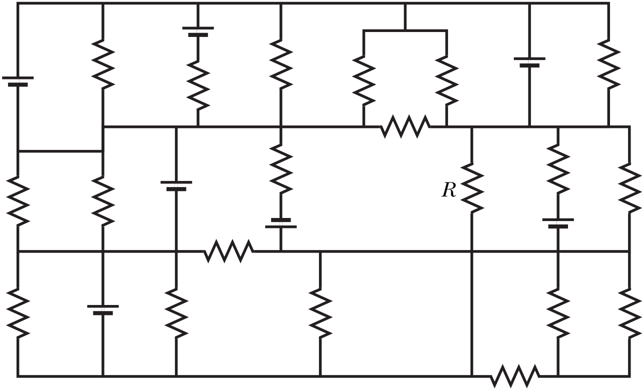
- using Kirchhoff's laws

Overview

- two Kirchhoff trick problems
- resistance-capacitance circuits

Using Kirchhoff's Laws examples

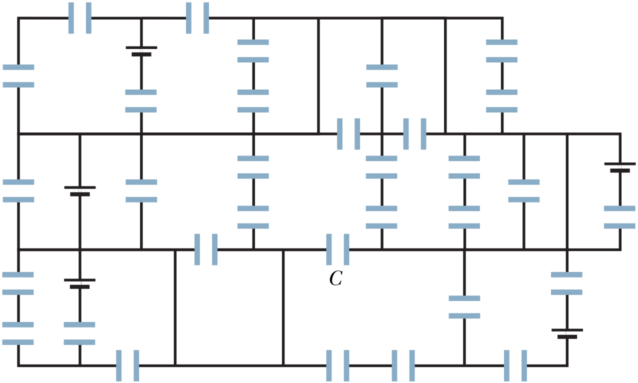
6 *Res-monster maze.* In Fig. 27-21, all the resistors have a resistance of 4.0 Ω and all the (ideal) batteries have an emf of 4.0 V. What is the current through resistor R? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



[1] Halliday, Resnick, Walker, 8th ed, page 725, question 6.

Using Kirchhoff's Laws examples

8 *Cap-monster maze.* In Fig. 27-22, all the capacitors have a capacitance of 6.0 μF, and all the batteries have an emf of 10 V. What is the charge on capacitor C? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



[1] Halliday, Resnick, Walker, 8th ed, page 726, question 8.

Time Varying Circuits

In circuits charge is not static, but moving.

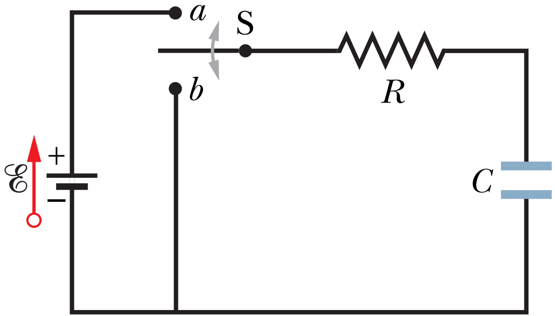
Current does not necessarily have to remain constant in time.

Capacitors take some time to charge and discharge due to resistances in the wires.

Other components also cause current to behave differently at different times, but for now, we will concentrate on circuits with resistors and capacitors.

RC Circuits

Circuits with resistors and capacitors are called "RC circuits."



Charging a Capacitor

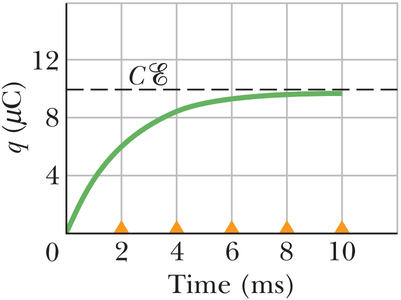
When an uncharged capacitor is first connected to an electrical potential difference, a current will flow.

Once the capacitor is fully charged however, $q = C\,(\Delta V)$, current has no where to flow and stops.

The capacitor gently “switches off” the current.

Charge varies with time

The charge on the capacitor changes with time.



It is possible to determine how if changes by considering the loop rule for a resistor in series with a capacitor:

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

Current is the rate of charge flow with time: $i = \frac{\mathrm{dq}}{\mathrm{dt}}$.

Charge varies with time

If we replace i in our equation with the derivative:

$$\mathcal{E} - R\,\frac{\mathrm{dq}}{\mathrm{dt}} - \frac{q}{C} = 0$$

This is a differential equation. There is a way to solve such equations to find solutions for how q depends on time.

Here, separation of the variables q and t is possible.

Charge varies with time

$$\mathcal{E} - R\,\frac{\mathrm{dq}}{\mathrm{dt}} - \frac{q}{C} = 0$$

Rearranging:

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &= \frac{C\mathcal{E}}{RC} - \frac{q}{RC} \\ \int \frac{1}{C\mathcal{E} - q}\,\mathrm{dq} &= \int \frac{1}{RC}\,\mathrm{dt} \end{aligned}$$

The limits of our integral will be determined by the initial conditions for the situation we are considering.

RC Circuits: Charging Capacitor

When charging an initially uncharged capacitor: $q = 0$ at $t = 0$

$$
\begin{aligned}
\int_0^q \frac{1}{C\mathcal{E} - q}\,\mathrm{dq} &= \int_0^t \frac{1}{RC}\,\mathrm{dt} \\
-\ln(C\mathcal{E} - q) + \ln(C\mathcal{E} - 0) &= \frac{t}{RC} \\
\ln\left(\frac{C\mathcal{E}}{C\mathcal{E} - q}\right) &= \frac{t}{RC} \\
\frac{C\mathcal{E}}{C\mathcal{E} - q} &= e^{t/RC}
\end{aligned}
$$

The solution is:

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

RC Circuits: Charging Capacitor

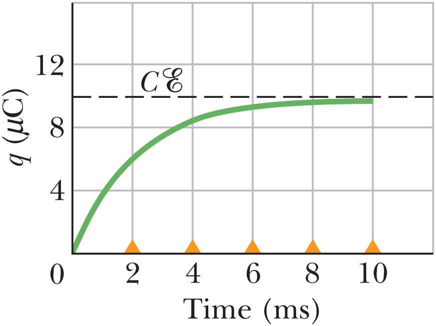
$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

This solution could also be written in a different way.

Notice $Q_{\text{max}} = C\mathcal{E}$.

($\mathcal{E}$ takes the place of the potential difference.)

$$q(t) = Q_{\text{max}}\left(1 - e^{-t/RC}\right)$$



RC Circuits: Charging Capacitor

Using the equation for q, an equation for current can also be found, since $i = \frac{\mathrm{dq}}{\mathrm{dt}}$:

$$i(t) = \left(\frac{\mathcal{E}}{R}\right) e^{-t/RC}$$

Dividing the charge by the capacitance C, we can also find the potential difference across the capacitor:

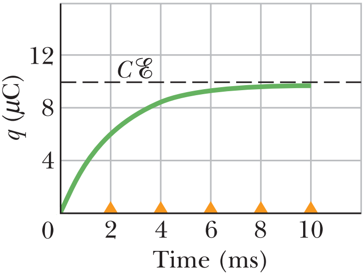
$$|\Delta V_C(t)| = \mathcal{E}(1 - e^{-t/RC})$$

RC Circuits: Charging Capacitor

How the solutions appear with time:

Charge:

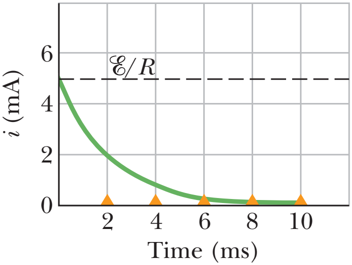
$q = Q_{\max}\left(1 - e^{-t/RC}\right)$



where for this circuit
$Q_{\max} = C\mathcal{E}$

Current:

$i = I_i\, e^{-t/RC}$



where for this circuit $I_i = \frac{\mathcal{E}}{R}$

RC Circuits: Time Constant

$$\tau = RC$$

τ is called the **time constant** of the circuit.

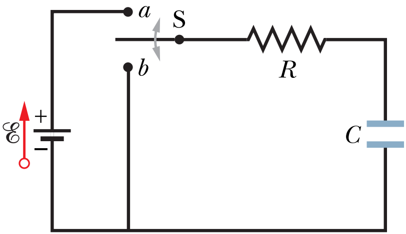
This gives the time for the current in the circuit to fall to $1/e$ of its initial value.

It is useful for comparing the “relaxation time” of different RC-circuits.

RC Circuits: Discharging Capacitor

Imagine that we have charged up the capacitor, so that the charge on it is Q_i.

Now we flip the switch to b, the battery is disconnected, but charge flows off the capacitor, creating a current:



$$-R\,\frac{\mathrm{dq}}{\mathrm{dt}} - \frac{q}{C} = 0$$

RC Circuits: Discharging Capacitor

What happens to the charge on the capacitor?

RC Circuits: Discharging Capacitor

What happens to the charge on the capacitor?

$$q(t) = Q_i \, e^{-t/RC}$$

It decreases exponentially with time.

RC Circuits: Discharging Capacitor

$$-R\,\frac{\mathrm{dq}}{\mathrm{dt}} - \frac{q}{C} = 0$$

When discharging an initially charged capacitor: $q = Q_i$ at $t = 0$

$$\begin{aligned}
\int_{Q_i}^{q} \frac{1}{q}\,\mathrm{dq} &= -\int_0^t \frac{1}{RC}\,\mathrm{dt} \\
\ln(q) - \ln(Q_i) &= -\frac{t}{RC} \\
\ln\left(\frac{q}{Q_i}\right) &= -\frac{t}{RC}
\end{aligned}$$

The solution is:

$$q(t) = Q_i\, e^{-t/RC}$$

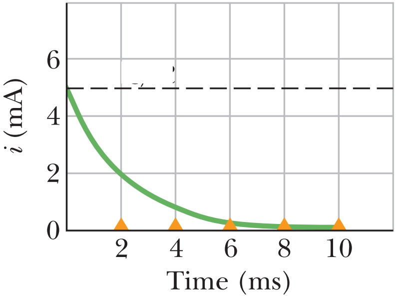
RC Circuits: Discharging Capacitor

What happens to the current?

$$i(t) = -I_i \, e^{-t/RC}$$

where $I_i = \frac{Q_i}{RC}$

The negative sign means the current flows in the opposite direction through the resistor when discharging as compared with charging.



RC Circuits: Discharging Capacitor

Multiplying the current by the resistance R gives the potential difference across the resistor:

$$|\Delta V_R(t)| = (\Delta V)_i \, e^{-t/RC}$$

The same expression describes the potential difference across the capacitor!

$$|\Delta V_C(t)| = (\Delta V)_i \, e^{-t/RC}$$

where $(\Delta V)_i = I_i R = \frac{Q_i}{C}$.

Summary

- resistance-capacitance circuits

Next Test on Feb 15.

Homework

Serway & Jewett:

- NEW: Ch 28, onward from page 857. CQs: 7; Problems: 37, 41, 43, 45, 65, 71
- NEW: Ch 26, prob: 78.