Electricity and Magnetism
Motion of Charges in Magnetic Fields

Lana Sheridan

De Anza College

Feb 21, 2018

Last time

- introduced magnetism
- magnetic field
- Earth's magnetic field
- force on a moving charge

Overview

- charged particles' motion in magnetic fields

Force on a Moving Charge

The force on a moving electric charge in a magnetic field:

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

where **B is the magnetic field**, $\mathbf{v}$ is the velocity of the charge, and q is the electric charge.

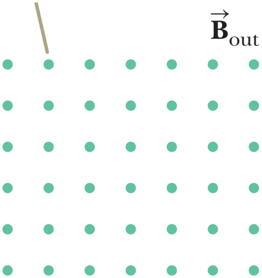
The magnitude of the force is given by:

$$F_B = q\,vB\sin\theta$$

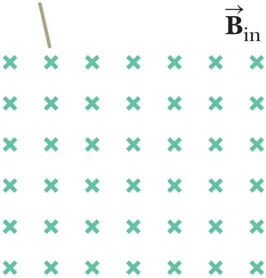
if θ is the angle between the $\mathbf{v}$ and $\mathbf{B}$ vectors.

Magnetic field direction reminder

B points **out of** the page.

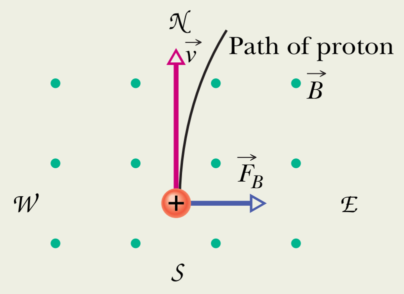


B points **into** the page.



Force on a Moving Charge

For example: here the dots indicate the field is directed upward out of the slide.



The force on the particle is $\perp$ to its velocity and the field.

[1] Figure from Halliday, Resnick, Walker, 9th ed.

Circular Motion of a Charge

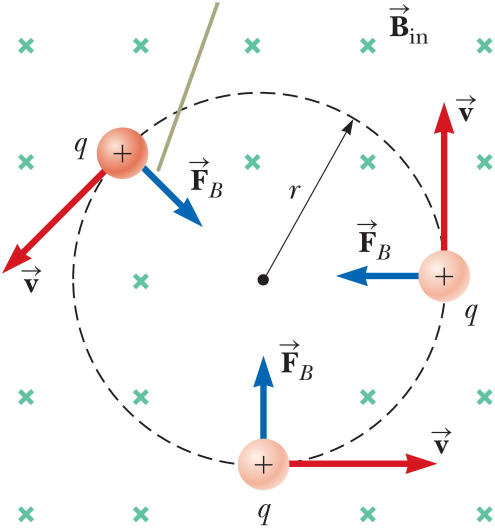
If a charge enters a magnetic field with a velocity at right angles to the field, it will feel a force perpendicular to its velocity.

This will change its trajectory, but not its speed.
$\Rightarrow$ Uniform Circular Motion!

The radius of the circle will depend on 4 things:

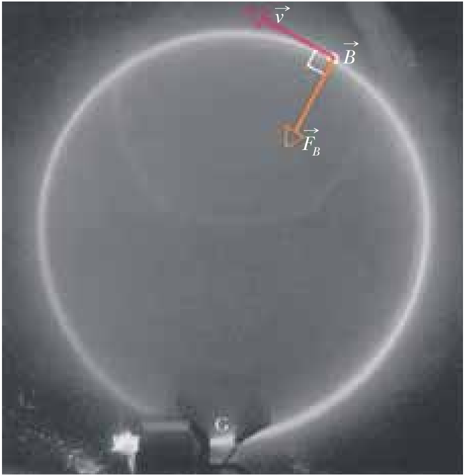
- mass of the particle
- charge of the particle
- initial velocity
- strength of the field

Circular Motion of a Charge



Circular Motion of a Charge

Electrons in a uniform magnetic field:



[1]Photo from Halliday, Resnick, Walker 9th ed, John Le P. Webb, Sussex University.

Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

Since **v** and **B** are perpendicular $F_B = qvB$:

$$\begin{aligned} \frac{mv^2}{r} &= |q|vB \\ r &= \frac{mv}{|q|B} \end{aligned}$$

The sign of q will determine whether the charge circulates clockwise or counter-clockwise.

Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

Since $\mathbf{v}$ and $\mathbf{B}$ are perpendicular $F_B = qvB$:

$$\begin{aligned} \frac{mv^2}{r} &= |q|vB \\ r &= \frac{mv}{|q|B} \end{aligned}$$

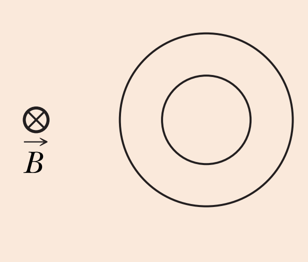
The sign of q will determine whether the charge circulates clockwise or counter-clockwise.

$$\frac{mv^2}{r} = |q|vB$$

Question

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field **B**, which is directed into the page. One particle is a proton; the other is an electron (which is less massive).

Which particle follows the smaller circle?



(A) proton

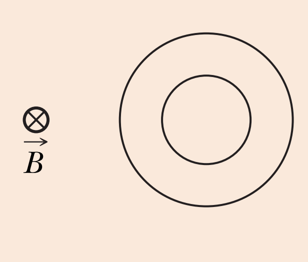
(B) electron

[1]Halliday, Resnick, Walker, 9th ed., page 746

Question

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field **B**, which is directed into the page. One particle is a proton; the other is an electron (which is less massive).

Which particle follows the smaller circle?



(A) proton

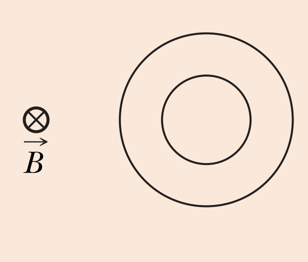
(B) electron ←

[1]Halliday, Resnick, Walker, 9th ed., page 746

Question

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field **B**, which is directed into the page. One particle is a proton; the other is an electron (which is less massive).

Which particle travels clockwise (as viewed in the diagram)?



(A) proton

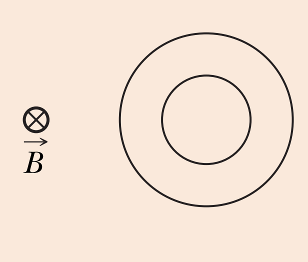
(B) electron

[1] Halliday, Resnick, Walker, 9th ed., page 746

Question

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field **B**, which is directed into the page. One particle is a proton; the other is an electron (which is less massive).

Which particle travels clockwise (as viewed in the diagram)?



(A) proton

(B) electron ←

[1] Halliday, Resnick, Walker, 9th ed., page 746

Circular Motion of a Charge

Can you find the period T for the orbit? (Time for the particle to make a full circle?)

Circular Motion of a Charge

Can you find the period T for the orbit? (Time for the particle to make a full circle?)

$$T = \frac{2\pi r}{v}$$

Circular Motion of a Charge

Can you find the period T for the orbit? (Time for the particle to make a full circle?)

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{|q|B}$$

Circular Motion of a Charge

Can you find the period T for the orbit? (Time for the particle to make a full circle?)

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{|q|B}$$

Also, angular frequency

$$\omega = \frac{|q|B}{m}$$

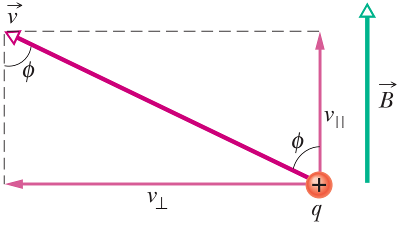
More general case

What if the velocity vector of a charge particle is not perpendicular to the magnetic field?

More general case

What if the velocity vector of a charge particle is not perpendicular to the magnetic field?

There will be some component of the velocity in the direction of the magnetic field.



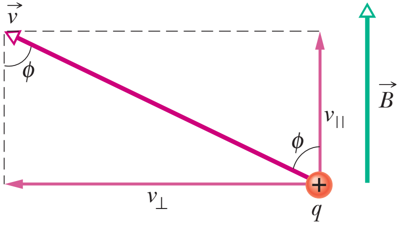
For the cross product:

$$|\mathbf{v} \times \mathbf{B}| = vB\sin\phi = (v\sin\phi)B = v_\perp B$$

More general case

What if the velocity vector of a charge particle is not perpendicular to the magnetic field?

There will be some component of the velocity in the direction of the magnetic field.

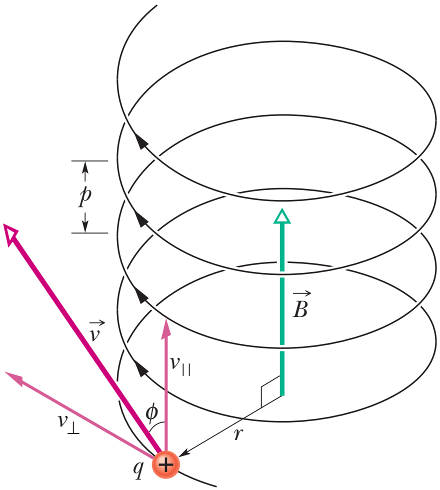


For the cross product:

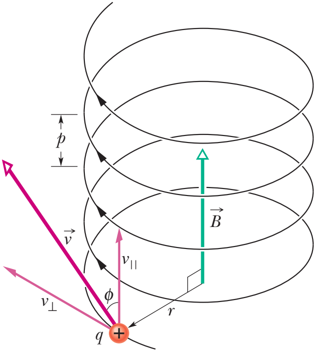
$$|\mathbf{v} \times \mathbf{B}| = vB \sin \phi = (v \sin \phi) B = v_\perp B$$

The force will not depend on the $\|$-component and the $\|$-component of velocity will not be changed.

Helical Trajectories



Helical Trajectories



The pitch, p, of the helix is

$$p = v_\parallel T = \frac{2\pi v_\parallel m}{|q|B}$$

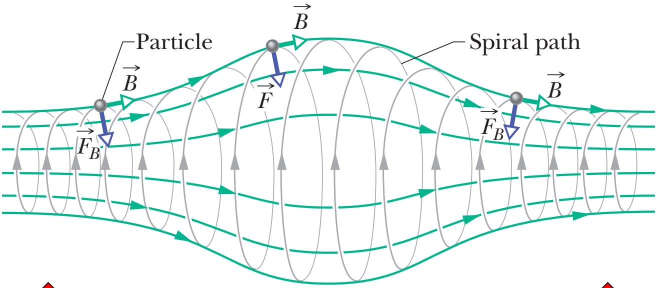
where T is the time period.

The radius is

$$r = \frac{m v_\perp}{|q|B}$$

using our equation from earlier.

Non-Uniform Fields: Magnetic Bottle



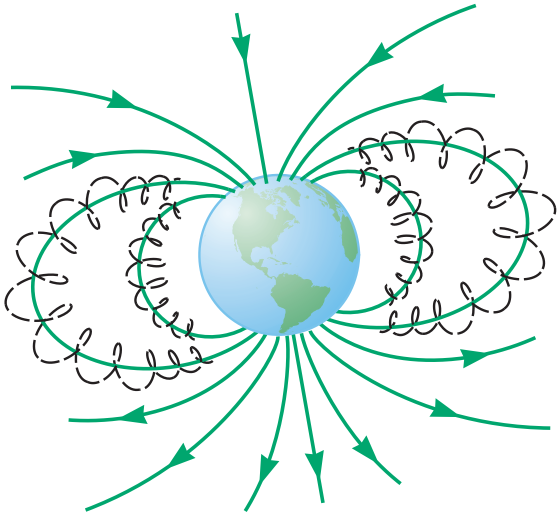
Strong field

Weaker field

Strong field

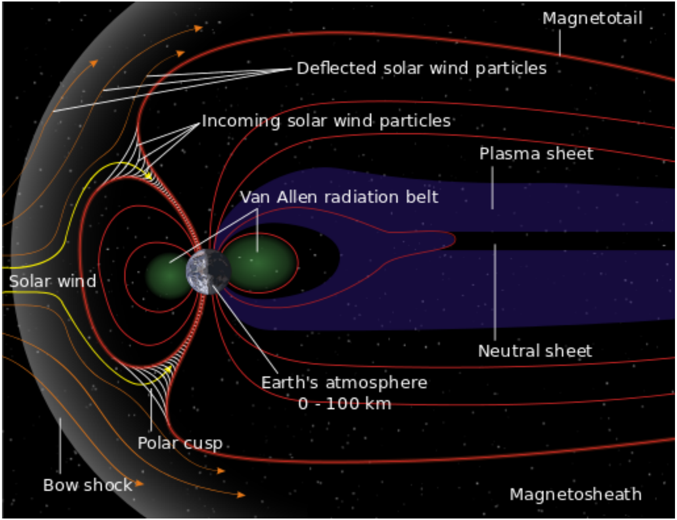
Non-Uniform Fields: Van Allen Belts

Earth's magnetic field acts as a magnetic bottle for cosmic rays.



Non-Uniform Fields: Van Allen Belts

When these charges particles in the belts are disturbed by the solar wind they can drop down into the atmosphere.



[1]Figure by NASA.

Non-Uniform Fields: Van Allen Belts

When these charges particles in the belts are disturbed by the solar wind they can drop down into the atmosphere. The resulting glow is the aurora borealis.

[1]Photo by Donald R. Pettit, Expedition Six NASA ISS science officer, 2013.

The Lorentz Force

A charged particle can be affected by both electric and magnetic fields.

This means that the total force on a charge is the sum of the electric and magnetic forces:

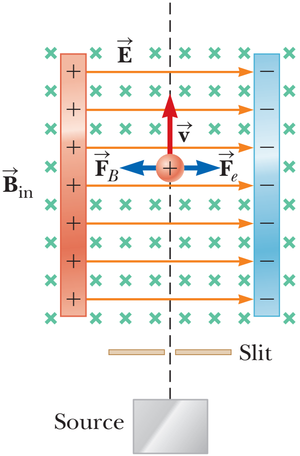
$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

This total force is called the **Lorentz force**.

This can always be used to deduce the electromagnetic force on a charged particle in E- or B-fields.

Velocity Selector: Using both electric and magnetic fields

Charges are accelerated with and electric field then travel down a channel with uniform electric and magnetic fields.



Velocity Selector: Using both electric and magnetic fields

The particles only reach the end of the channel if $\mathbf{F} = 0$.

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

so that means

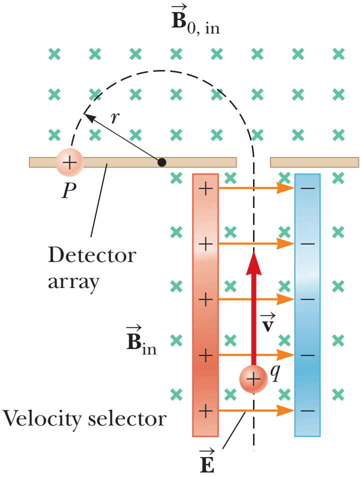
$$q\mathbf{E} = -q\mathbf{v} \times \mathbf{B}$$

supposing $\mathbf{v}$ and $\mathbf{B}$ are perpendicular:

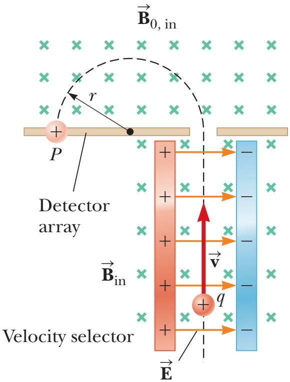
$$v = \frac{E}{B}$$

Mass Spectrometer

After selecting particles to have velocity $\mathbf{v} = E/B$ along the channel, they are fed into a magnetic field.



Mass Spectrometer



Where they collide with the detector allows us to find the radius of the path, r.

Mass-to-charge ratio:

$$\frac{m}{|q|} = \frac{rB_0}{v}$$

Summary

- magnetic field lines
- charged particles in crossed-fields
- properties of the electron

Homework Serway & Jewett:

- PREVIOUS: Ch 29, Obj Qs: 1, 3, 5; Conc. Qs: 1, 7; Problems: 1, 8, 9
- Ch 29, Obj Qs: 7; Problems: 13, 15, 23, 73, 80