Electricity and Magnetism
Charges in Crossed E- and B-Fields

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Last time

- magnetic force on a charge
- circular trajectories
- helical trajectories

Overview

- charged particle in E and B fields
- applications of crossed fields
- discovery of the electron
- Hall effect

The Lorentz Force

A charged particle can be affected by both electric and magnetic fields.

This means that the total force on a charge is the sum of the electric and magnetic forces:

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

This total force is called the **Lorentz force**.

This can always be used to deduce the electromagnetic force on a charged particle in E- or B-fields.

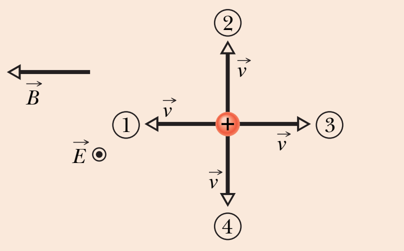
Crossed Fields

Both electric and magnetic fields interact with moving charges and produce forces on them.

This can be used to study charged particles.

Warm Up Question: Crossed Fields

The diagram shows four possible directions for the velocity **v** of a positively-charged particle: which direction could possibly result in a net force of zero on the particle?[1]



(A) 1 (left)

(B) 2 (up)

(C) 3 (right)

(D) 4 (down)

[1] Halliday, Resnick, Walker, 9th ed., page 741.

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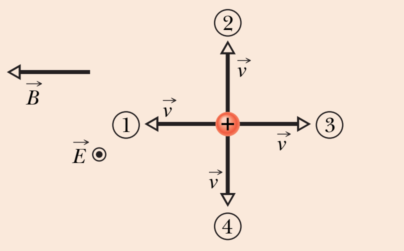

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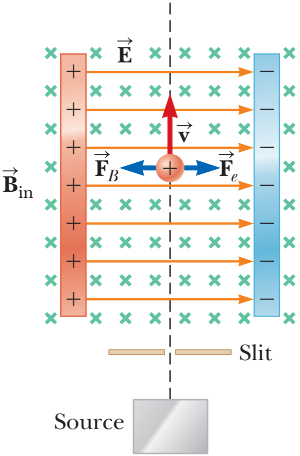
Velocity Selector: Using both electric and magnetic fields

Charges are accelerated with and electric field then travel down a channel with uniform electric and magnetic fields.



Velocity Selector: Using both electric and magnetic fields

The particles only reach the end of the channel if $\mathbf{F} = 0$.

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

so that means

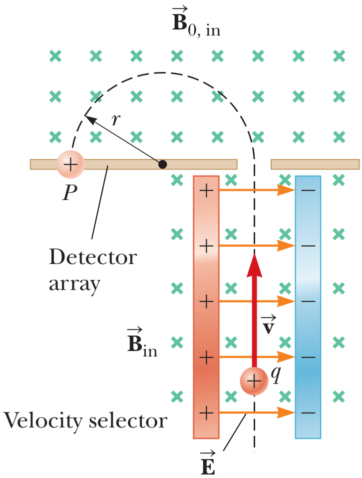
$$q\mathbf{E} = -q\mathbf{v} \times \mathbf{B}$$

supposing $\mathbf{v}$ and $\mathbf{B}$ are perpendicular:

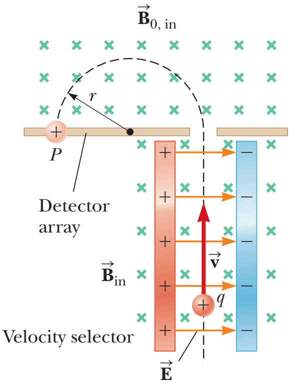
$$v = \frac{E}{B}$$

Mass Spectrometer

After selecting particles to have velocity $\mathbf{v} = E/B$ along the channel, they are fed into a magnetic field.



Mass Spectrometer



Where they collide with the detector allows us to find the radius of the path, r.

Mass-to-charge ratio:

$$\frac{m}{|q|} = \frac{rB_0}{v}$$

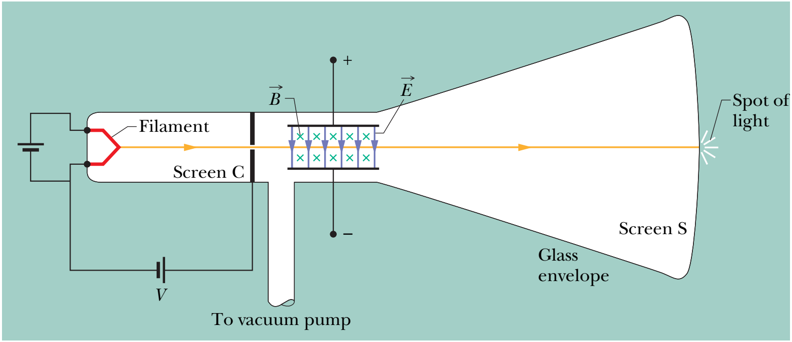
The Discovery of the Electron

Orienting a magnetic field at right angles to an electric field allowed J.J. Thompson in 1897 to determine the ratio of the electron's charge to its mass: $\frac{|q|}{m}$.

This was significant because it showed that the electron was much lighter than other known particles, establishing it as a new kind of particle.

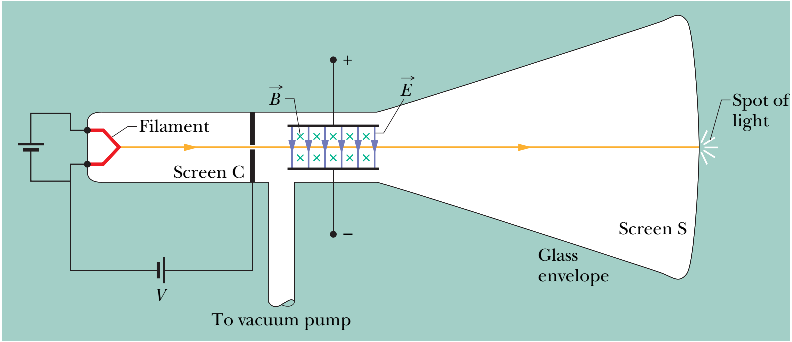
Discovery of the Electron: Main Idea

Electrons are accelerated along the yellow line.



Discovery of the Electron: Main Idea

Electrons are accelerated along the yellow line.



The electric field deflects them upward.

The magnetic field deflects them downward.

Adjust the magnetic field until the deflections cancel out and the spot returns to the center.

How to determine v, the speed of the electrons

The deflection of a charged particle moving through the fields is 0, only if $\mathbf{F}_{\text{net}} = 0$.

Assuming $\mathbf{v} \perp \mathbf{B}$:

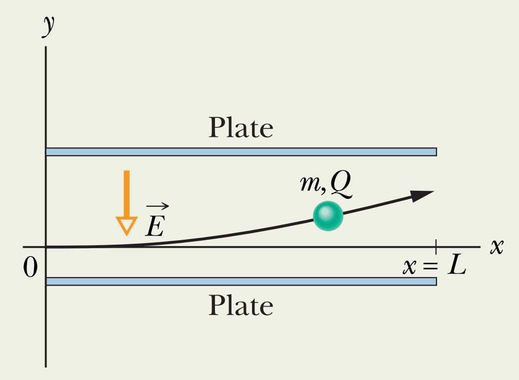
$$\begin{aligned} F_E &= F_B \\ qE &= qvB \\ v &= \frac{E}{B} \end{aligned}$$

(same as before)

Switch on both fields to get a measurement of v. Then switch off the magnetic field and, using the E-field only, measure the deflection distance y.

Why does that tell us about q/m?

Consider only the E-field from 2 parallel charged plates:



A charge particle follows a parabola, because the field is **uniform**.

This is exactly like projectile motion.

[1]Figure from Halliday, Resnick, Walker, 9th ed, page 593.

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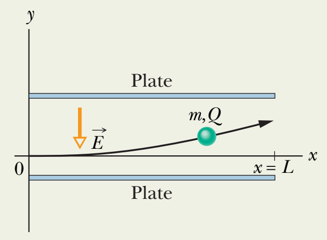

The displacement in the vertical (y) direction (same dir. as field lines)

$$y = v_{i,y}t + \frac{1}{2}at^2$$

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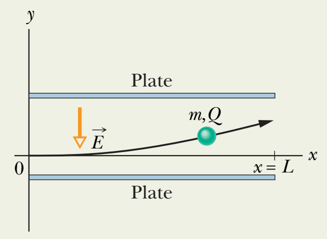

The displacement in the vertical (y) direction (same dir. as field lines)

$$y = v_{i,y} t + \frac{1}{2} a t^2$$

If the particle is moving horizontally only on entry into the field, $v_{i,y} = 0$.

Also $a = F_E/m$, giving:

$$y = \frac{1}{2} \frac{F_E}{m} t^2$$

Why does that tell us about q/m?

There is no acceleration in the x direction:

$$x = L = v_x t \qquad \Rightarrow \qquad t = \frac{L}{v}$$

Therefore the deflection in the y direction due to the electric field by the end of the plates (length L):

$$y = \frac{(qE)L^2}{2mv^2}$$

This gives an expression for q/m:

$$\frac{|q|}{m} = \frac{2\,y\,v^2}{E\,L^2}$$

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We already found the speed v: $v = E/B$

$$\frac{|q|}{m} = \frac{2\,y\,E}{B^2\,L^2}$$

Discovery of the Electron

For an electron, $|q| = e$:

$$\frac{e}{m_e} = 1.759 \times 10^{11} \text{ C/kg}$$

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$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

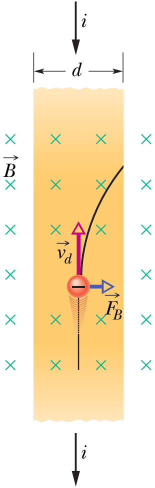
The Hall effect

Or, how to use a current and a field to create a potential difference.

The Hall effect

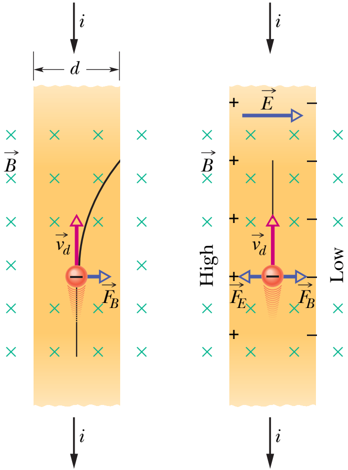
Or, how to use a current and a field to create a potential difference.

Electrons flowing in a conductor can also be deflected by a magnetic field!



The Hall effect

Electrons are pushed to the right until so much negative charge has built up on the right side that the electrostatic force balances the magnetic force.



At this point we have crossed fields and the potential difference between the left and the right side stabilizes.

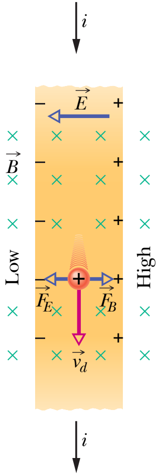
The Hall effect

The Hall effect allows us to learn many things about the charge carriers in a conductor:

- their charge
- their volume density
- their drift velocity (for a given current)

The Hall effect

Suppose the charge carriers in a conductor were positively charged:



We would get the opposite polarity for the potential difference!

The Hall effect

The constant potential difference that appears across the conductor once the current has stabilized is called the *Hall potential difference*.

$$\Delta V = Ed$$

where d is the width of the conductor.

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Since the electric force and magnetic force balance:

$$\begin{aligned} F_E &= F_B \\ eE &= ev_d B \\ v_d &= \frac{E}{B} \end{aligned}$$

We can use our knowledge to estimate v_d.

The Hall effect

Alternatively, we can estimate the density of charge carriers, n.

Remember:

$$v_d = \frac{I}{n\,e\,A}$$

Equating this with our expression for v_d on the previous slide:

$$\frac{E}{B} = \frac{I}{n\,e\,A}$$

Rearranging, and using $\Delta V = Ed$ and letting $t = A/d$ be the conductor **thickness**:

$$n = \frac{B\,I}{e(\Delta V)t}$$

The Hall effect

Remembering $\Delta V = Ed$ and $t = A/d$ is the conductor **thickness**:

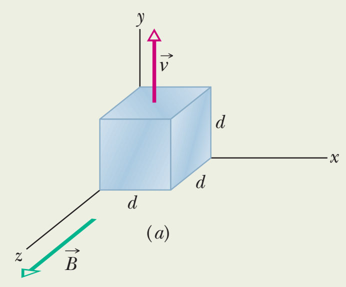
$$n = \frac{B\,I}{e(\Delta V)t}$$

ΔV is called the Hall Potential Difference:

$$\Delta V = \frac{B\,I}{nte}$$

The Hall effect - example question

A solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\mathbf{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\mathbf{B}$ of magnitude 0.050 T in the positive z direction.



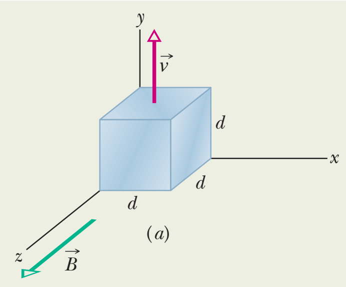
Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

[1] Halliday, Resnick, Walker, 9th ed, page 743.

The Hall effect - example question

Free charges in the conductor will feel a force as they move along with the entire conductor through the field.

The free charges are electrons. We have to find the direction of the force on them.



(a)

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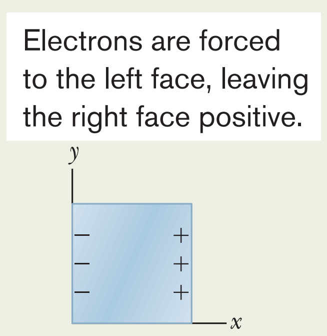

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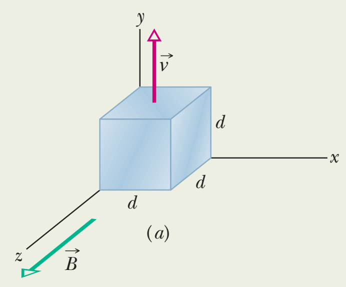

What is the potential difference between the faces of higher and lower electric potential?

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The Hall effect - example question

When does the potential difference between the faces stabilize?

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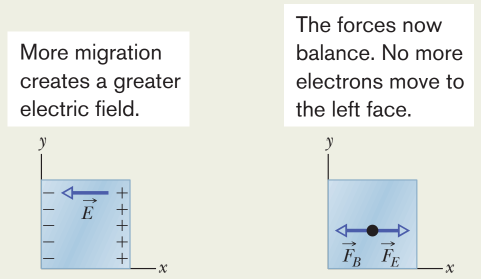
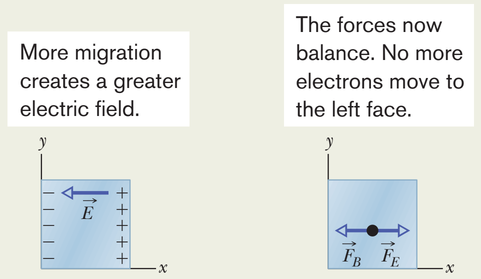


$$F_E = F_B$$

The Hall effect - example question

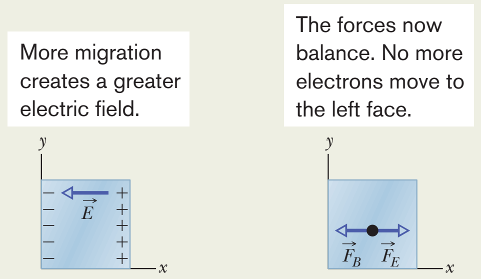
When does the potential difference between the faces stabilize?



$$
\begin{aligned}
F_E &= F_B \\
eE &= evB \\
\left(\frac{\Delta V}{d}\right) &= vB \\
\Delta V &= vBd
\end{aligned}
$$

The Hall effect - example question

When does the potential difference between the faces stabilize?



$$
\begin{aligned}
F_E &= F_B \\
eE &= evB \\
\left(\frac{\Delta V}{d}\right) &= vB \\
\Delta V &= vBd \\
\Delta V &= 3.0 \text{ mV}
\end{aligned}
$$

Related Effects

- the Hall effect in semiconductors - can be more complex! Depends on the material.

- the quantum Hall effect - can observe quantization of the Hall potential difference. Can be used to measure the charge of the electron.

Summary

- charged particles in crossed-fields
- charge and mass of the electron
- Hall effect

Homework Serway & Jewett:

- PREVIOUS: Ch 29, Obj Qs: 7; Problems: 13, 15, 23, 73, 80
- Ch 29, Problems: 25, 29, 55, 59