Electricity and Magnetism
Particle Accelerators

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Last time

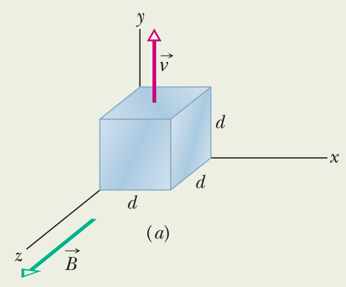
- charged particle in E and B fields
- applications of crossed fields
- discovery of the electron
- Hall effect

Overview

- Hall effect example
- cyclotrons
- synchotrons

The Hall effect - example question

A solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\mathbf{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\mathbf{B}$ of magnitude 0.050 T in the positive z direction.



Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

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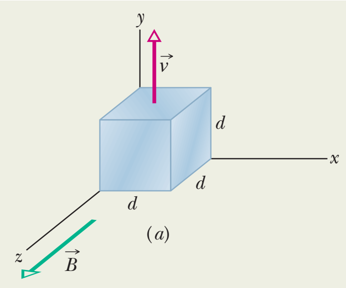
[1] Halliday, Resnick, Walker, 9th ed, page 743.

The Hall effect - example question

Free charges in the conductor will feel a force as they move along with the entire conductor through the field.

The free charges are electrons. We have to find the direction of the force on them.



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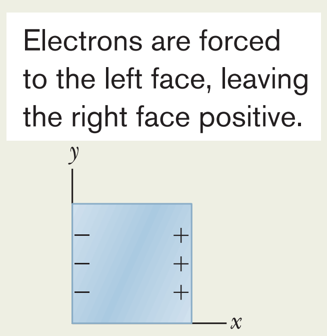

Electrons are forced to the left face, leaving the right face positive.

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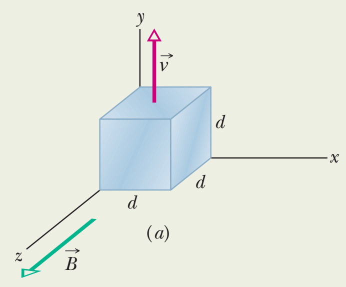

What is the potential difference between the faces of higher and lower electric potential?

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The Hall effect - example question

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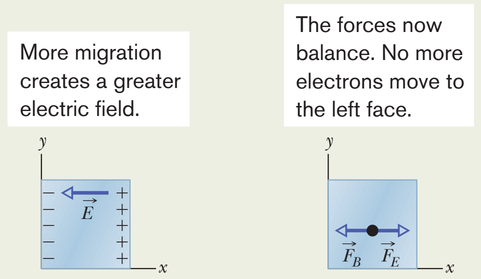

More migration creates a greater electric field.

The forces now balance. No more electrons move to the left face.

$$F_E = F_B$$

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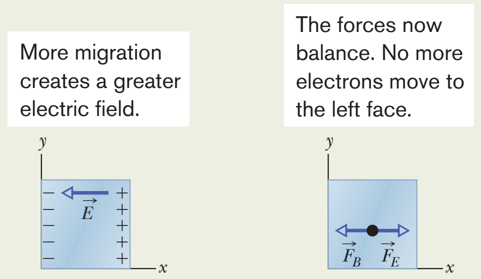

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$$\begin{aligned} F_E &= F_B \\ eE &= evB \\ \left(\frac{\Delta V}{d}\right) &= vB \\ \Delta V &= vBd \end{aligned}$$

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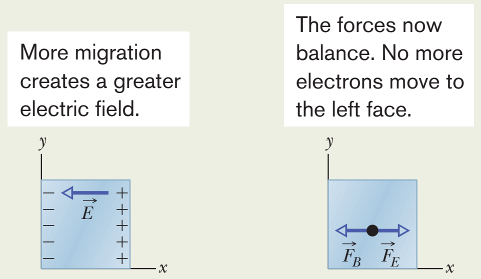

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$$\begin{aligned}
F_E &= F_B \\
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\left(\frac{\Delta V}{d}\right) &= vB \\
\Delta V &= vBd \\
\Delta V &= 3.0\ \text{mV}
\end{aligned}$$

Related Effects

- the Hall effect in semiconductors - can be more complex! Depends on the material.
- the quantum Hall effect - can observe quantization of the Hall potential difference. Can be used to measure the charge of the electron.

The Lorentz Force

A charged particle can be affected by both electric and magnetic fields.

This means that the total force on a charge is the sum of the electric and magnetic forces:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

This total force is called the **Lorentz force**.

This can always be used to deduce the electromagnetic force on a charged particle in E or B fields.

Accelerating Charged Particles

High speed beams of particles are useful for studying nuclear and particle physics.

They can be tricky to create, however.

Charged particles can be accelerated with a potential difference.

The electron-Volt (again)

One convenient unit of energy for particles is the electron-Volt, written eV.

This is the amount of energy that an electron accelerated through a potential difference of 1 Volt has.

$$U = qV$$

$$1\text{ eV} = (1.6 \times 10^{-19}\text{ C})(1\text{ V}) = 1.6 \times 10^{-19}\text{ J}.$$

Accelerating Charged Particles

The acceleration of a particle from a potential difference depends on its mass.

Suppose there is a practical limit on how strong an electric field can be created, E.

The force on the particle is:

$$F = qE$$

The acceleration can be deduced from Newton's second law:

$$a = \frac{F}{m} = \frac{qE}{m}$$

The final velocity of a particle with this acceleration will be:

$$v_f^2 = 2ad = \frac{qEd}{m}$$

If m is large than the accelerating distance d must be also. For protons the value of d necessary becomes impractical.

Cyclotrons

One way around this is to have the particles move in a circle.

The acceleration can take place in a limited space.

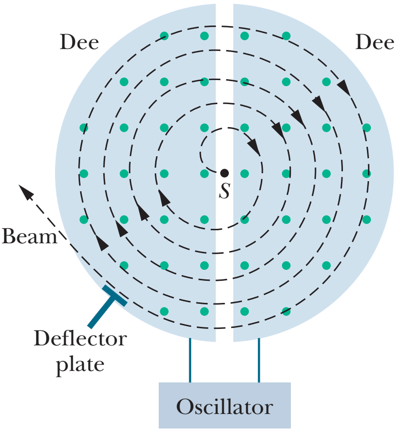
Magnetic fields cause the protons to follow circular arcs.

The protons are directed repeatedly through a potential difference that accelerates them.

The time period of the orbit does not depend on the velocity of the particles!

$$T = \frac{2\pi m}{|q|B}$$

Cyclotrons



$$f = f_{\text{osc}} = \frac{|q|B}{2\pi m}$$

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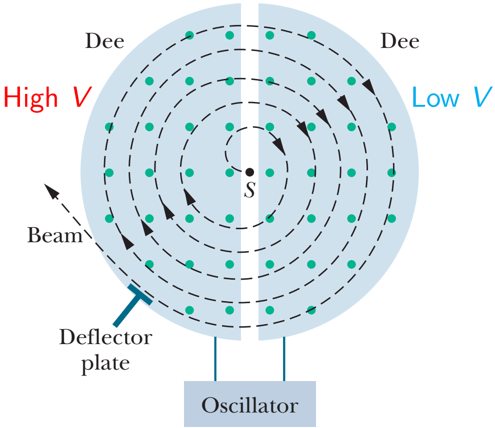

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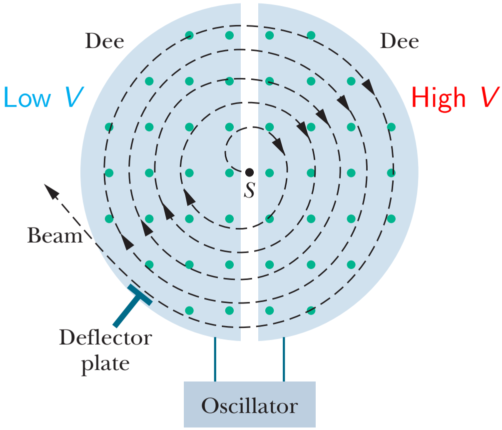

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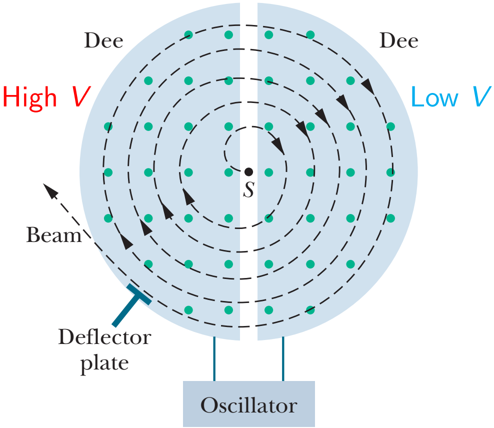

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Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

$$r = \frac{mv}{|q|B}$$

So, $v = qBr/m$.
When the ion exits the cyclotron, it will have kinetic energy:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ K &= \frac{(qBr)^2}{2m} \end{aligned}$$

Cyclotron

The first cyclotron was built in 1934.

The world's largest cyclotron has a maximum beam radius of 7.9 m.

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[1] Photo from Lawrence Berkeley National Laboratory.

Synchrotron

Once the charged particles reach $\sim 10\%$ of the speed of light this stops working.

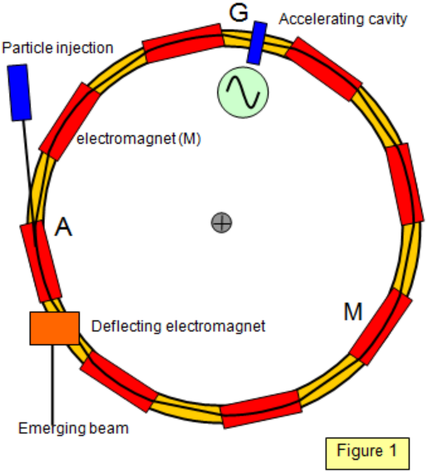
This is because the effective mass of the particles is increasing, so $f_{\text{osc}} = \frac{2\pi m}{|q|B}$ is no longer a constant.

Also, at these speeds the area of the magnetic field for a cyclotron must be quite big as the radius of the path becomes large.

A solution to this is the synchrotron.

Synchrotron

Synchrotrons operate similarly to cyclotrons, but the frequency of the potential switching can vary.



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[1] Figure from schoolphysics.co.uk.

Synchrotron

This also means that the particles can be kept on a single loop, even as their velocity increases.

The magnetic field only has to cover the ring itself.
(Not the area in the middle of the ring.)

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The LHC (Large Hadron Collider) at CERN is a type of synchrotron.

The Tevatron at Fermilab was also one, but it has been shutdown due to Congressional budget cuts.

Synchrotron

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[1] Photo copyright © Synchrotron Soleil, used with permission

Summary

- Hall effect type question
- cyclotrons
- synchotrons

Homework Serway & Jewett:

- Ch 29, Problems: 27 (in part (a), the textbook answer corresponds to ω, not f), 33, 35