Electricity and Magnetism
Force on a Wire
Torque on a Wire Loop

Lana Sheridan

De Anza College

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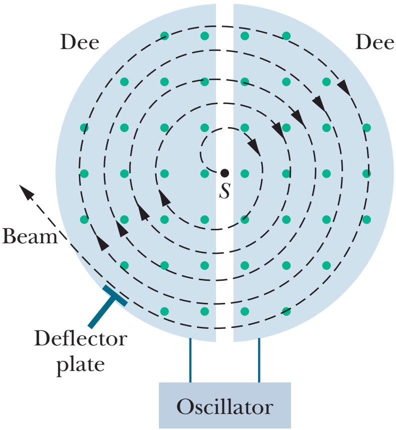
Last time

- accelerating charged particles
- the electron-Volt
- cyclotrons

Overview

- synchotrons
- magnetic force on a current carrying wire
- torque on a wire loop in a B-field

Cyclotrons



$$f = f_{\text{osc}} = \frac{|q|B}{2\pi m}$$

Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

$$r = \frac{mv}{|q|B}$$

So, $v = qBr/m$.
When the ion exits the cyclotron, it will have kinetic energy:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ K &= \frac{(qBR)^2}{2m} \end{aligned}$$

where R is the radius of the "dee".

Cyclotron

The first cyclotron was built in 1934.

The world's largest cyclotron has a maximum radius of 7.9 m.

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[1] Photo from Lawrence Berkeley National Laboratory.

Synchrotron

Once the charged particles reach $\sim 10\%$ of the speed of light this stops working.

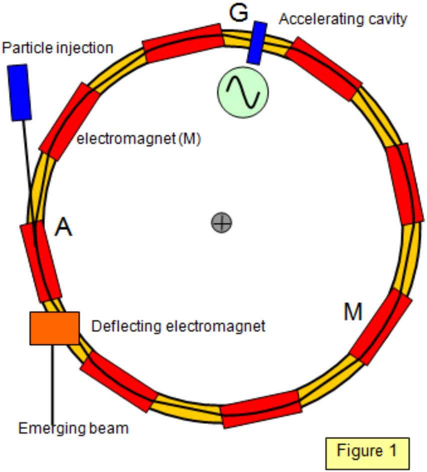
This is because the effective mass of the particles is increasing, so $f_{\text{osc}} = \frac{2\pi m}{|q|B}$ is no longer a constant.

Also, at these speeds the area of the magnetic field for a cyclotron must be quite big as the radius of the path becomes large.

A solution to this is the synchrotron.

Synchrotron

Synchrotrons operate similarly to cyclotrons, but the frequency of the potential switching can vary.



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[1] Figure from schoolphysics.co.uk.

Synchrotron

This also means that the particles can be kept on a single loop, even as their velocity increases.

The magnetic field only has to cover the ring itself.
(Not the area in the middle of the ring.)

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The LHC (Large Hadron Collider) at CERN is a type of synchrotron.

The Tevatron at Fermilab was also one, but it has been shutdown due to Congressional budget cuts.

Synchrotron

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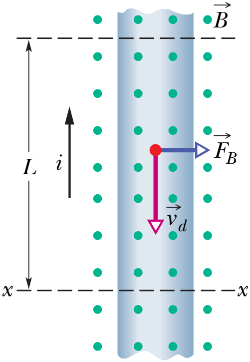
[1] Photo copyright © Synchrotron Soleil, used with permission

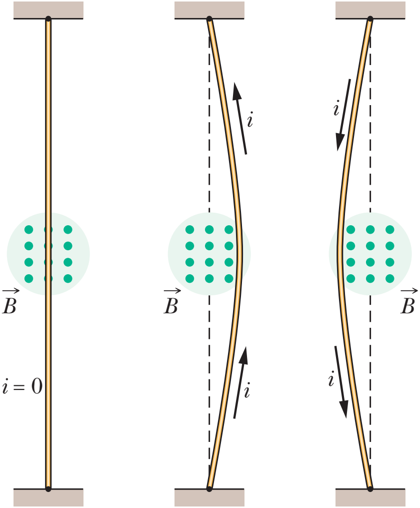
Magnetic Force on a Current Carrying Wire

Charged particles moving in a magnetic field experience a force.



A wire carrying a current also experiences a force, since there is a force on each moving charge confined to the wire.

Magnetic Force on a Current Carrying Wire



The direction of the force depends on the direction of the current.

Magnetic Force on a Current Carrying Wire

The force on the wire in a uniform magnetic field is given by:

$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

where **L** is a distance vector that points along the length of the wire in the direction of the conventional current I and is as long as the part of the wire inside the field is.

By considering the force on an individual charge, we can motivate this equation.

Magnetic Force on a Current Carrying Wire

The force on an individual conduction electron is
$\mathbf{F}_B = (-e)\,\mathbf{v}_d \times \mathbf{B}$.

The total force will be the sum of the force on all the moving charges together.

How much conduction charge is in the wire?

Magnetic Force on a Current Carrying Wire

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$\mathbf{F}_B = (-e)\,\mathbf{v}_d \times \mathbf{B}$.

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How much conduction charge is in the wire?

$$q = -ne\mathcal{V}$$

where n is the volume density of charge carriers, and $\mathcal{V}$ is the volume of the wire.

Also, this charge is negative, since the flowing charges are electrons.

Magnetic Force on a Current Carrying Wire

$$\mathbf{F}_B = -ne\mathcal{V}\,\mathbf{v}_d \times \mathbf{B}$$

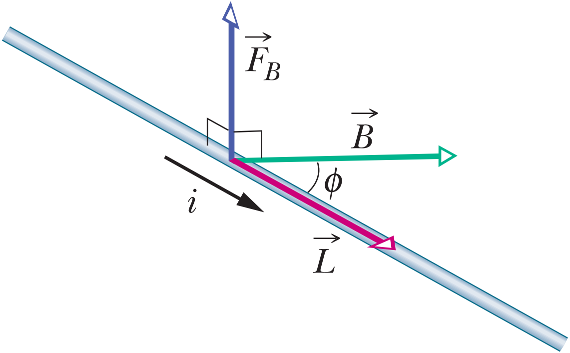
$$v_d = \frac{I}{neA} \;\rightarrow\; I\,L = ne\mathcal{V}v_d$$

since $\mathcal{V} = AL$ where L is the length of the wire.

If the wire is straight and in a uniform field and we define **L** to be a vector of length L pointed in the direction of the conventional current, then:

$$\mathbf{F}_B = I\,\mathbf{L} \times \mathbf{B}$$

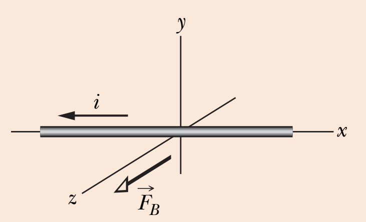
Magnetic Force on a Current Carrying Wire



$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

Question

A current i passes through a wire in a uniform magnetic field $\mathbf{B}$. The magnetic force $\mathbf{F}_B$ acts on the wire. The field is oriented so that the force is maximum. In what direction is the field?



(A) $+y$

(B) $-y$

(C) $+z$

(D) $-z$

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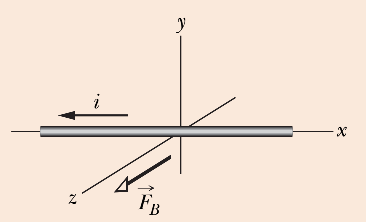

(A) $+y$

(B) $-y$ ←

(C) $+z$

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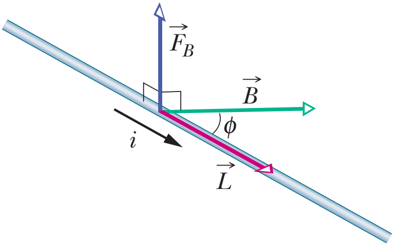
Problem

A straight, horizontal length of copper wire has a current $I = 28$ A through it. The linear density (mass per unit length) of the wire is 46.6 g/m.

What are the magnitude and direction of the **minimum** magnetic field **B** needed to suspend the wire – that is, to balance the gravitational force on it?



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[1] Halliday, Resnick, Walker, page 751.

Problem

At equilibrium

$$F_B = F_g$$

Problem

At equilibrium

$$F_B = F_g$$

$$ILB \sin\theta = mg$$

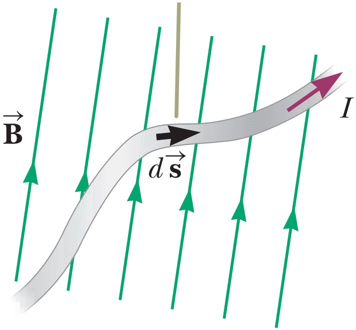
$$B = 1.6 \times 10^{-2}\ \mathrm{T}$$

Magnetic Force on a Current Carrying Wire

What about the case where the wire is curved, or the B-field is not uniform?

Magnetic Force on a Current Carrying Wire

What about the case where the wire is curved, or the B-field is not uniform?

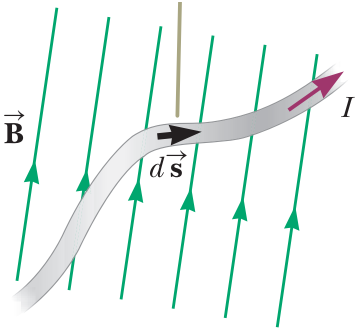


Now we must consider each infinitesimal segment of wire:

$$\mathrm{d}\mathbf{F} = I\ \mathrm{d}\mathbf{s} \times \mathbf{B}$$

Remember that **B** could depend on **s** if the field is not uniform.

Magnetic Force on a Current Carrying Wire



$$\mathbf{F} = I \int d\mathbf{s} \times \mathbf{B}$$

The integral is taken over the length of the wire.

Summary

- synchotrons
- force on a curved wire

Homework Serway & Jewett:

- PREVIOUS: Ch 29, Problems: 27, 33, 35
- NEW: Ch 29, Problems: 37, 41