Electricity and Magnetism
Magnetic Force on a Curved Wire
Torque on a Wire Loop
Magnetic Dipole Moment

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De Anza College

Feb 27, 2018

Last time

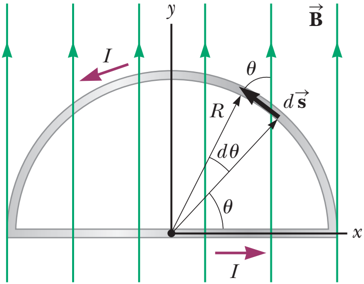
- particle accelerators: synchotrons
- force on a wire with a current in a B-field

Overview

- force on a curved wire with a current in a B-field
- torque on a wire loop in a magnetic field
- motors
- relating a current loop to a magnet
- magnetic dipole moment
- torque and potential energy of magnetic dipole

Example 29.4: Magnetic Force on a Wire

What is the net force on this semicircular wire loop in a uniform B-field, given that the current in the loop is I?



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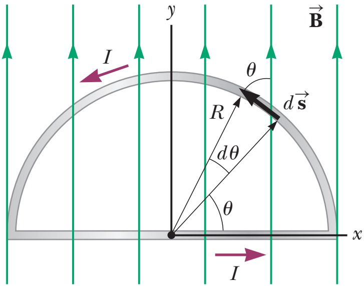

First: use symmetry. The wire is in the x, y-plane, $\mathbf{B} = B\mathbf{j}$, any magnetic force can only point in the $\mathbf{k}$-direction.

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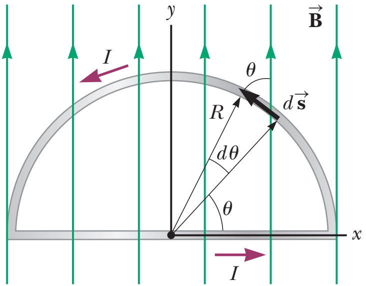

First: use symmetry. The wire is in the x, y-plane, $\mathbf{B} = B\mathbf{j}$, any magnetic force can only point in the $\mathbf{k}$-direction.

Mentally, break the wire into two pieces, the bottom, straight piece and the top curved piece.

Example 29.4: Magnetic Force on a Wire

Bottom segment:

$$\mathbf{F}_b = I\mathbf{L} \times \mathbf{B}$$

Since $\mathbf{L} = 2R\mathbf{i}$, and $\mathbf{B} = B\mathbf{j}$:

$$\mathbf{F}_b = 2RIB\,\mathbf{k}$$

Top segment:

$$\mathbf{F}_t = I \int \mathrm{d}\mathbf{s} \times \mathbf{B}$$

Example 29.4: Magnetic Force on a Wire

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Example 29.4: Magnetic Force on a Wire

Top segment:

$$\mathbf{F}_t = I \int d\mathbf{s} \times \mathbf{B}$$

The top segment is semi-circular. A path along it is a circular arc: $s = R\theta \ \rightarrow d\mathbf{s} = R\,d\theta(\hat{\boldsymbol{\theta}})$.

$$\begin{aligned}
\mathbf{F}_t &= I \int RB \sin\theta \, d\theta \, (-\mathbf{k}) \\
&= -IRB\,\mathbf{k} \int_0^{\pi} \sin\theta \, d\theta \\
&= -IRB\,\mathbf{k} \, [\cos\theta]_0^{\pi} \\
&= -2RIB\,\mathbf{k}
\end{aligned}$$

Example 29.4: Magnetic Force on a Wire

Bottom segment:

$$\mathbf{F}_b = 2RIB\,\mathbf{k}$$

Top segment:

$$\mathbf{F}_t = -2RIB\,\mathbf{k}$$

Total:

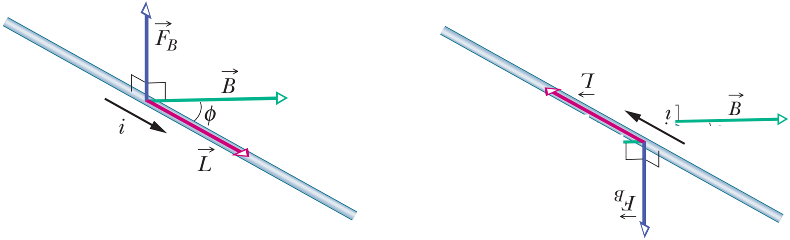
$$\mathbf{F}_{\text{net}} = 0$$

This is a general result. The force on any loop of wire in a **uniform** magnetic field is zero!

Torque on a Loop of Wire with a Current

Or, how to turn electricity into motion.

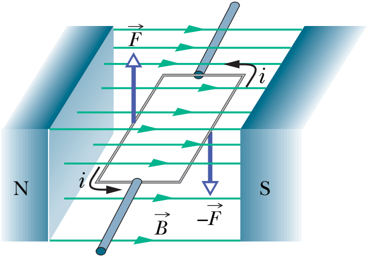
Consider two wires in a magnetic field with currents flowing in opposite directions.



They will experience forces in opposite directions.

Torque on a Loop of Wire with a Current

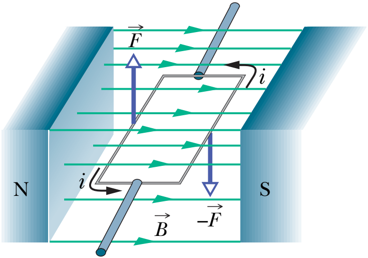
This is the situation that occurs when a loop of wire is placed in a B-field.



These opposing forces on opposite sides of the loop creates a torque on the loop.

Torque on a Loop of Wire with a Current

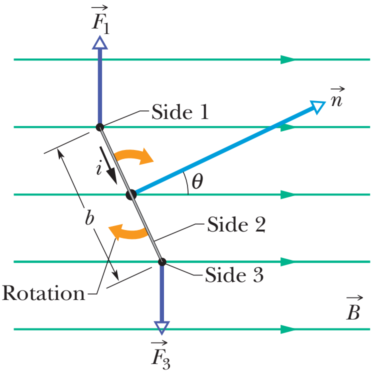
The current on the two sides away from the axle gives an upward force on the left and downward on the right.



On the two ends that connect to the axle, the force is zero when the loop lays flat parallel to the B-field.

When the loop rotates, the forces on those two ends are equal and opposite.

Torque on a Loop of Wire with a Current

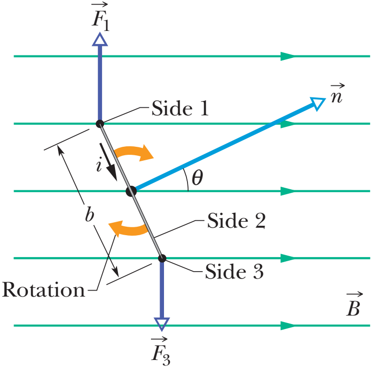


$$\boldsymbol{\tau}_F = \mathbf{r} \times \mathbf{F} \quad ; \qquad \boldsymbol{\tau}_{\text{net}} = \sum_i \boldsymbol{\tau}_i$$

$$\boldsymbol{\tau}_{\text{net}} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

Torque on a Loop of Wire with a Current

$$\mathbf{F}_1 = I\mathbf{a} \times \mathbf{B} = iaB\ \mathbf{j} = -\mathbf{F}_3$$



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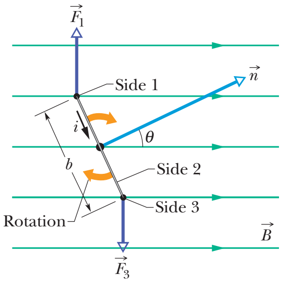
Torque on a Loop of Wire with a Current

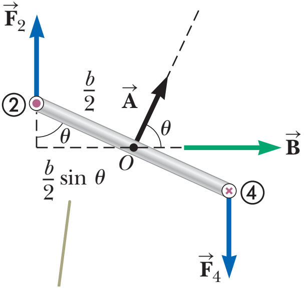


$$\begin{aligned}\boldsymbol{\tau}_{\text{net}} &= \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \\ &= \left(\frac{b}{2}\right)(IaB)\sin\theta + \left(\frac{b}{2}\right)(IaB)\sin\theta \quad \text{[cw in diag.]}\end{aligned}$$

Noting that the area of the loop $A = ab$:

$$\boldsymbol{\tau} = IAB\sin\theta$$

Torque on a Loop of Wire with a Current



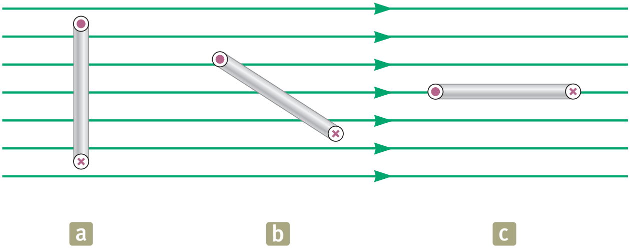
$$\boldsymbol{\tau} = IAB\sin\theta$$

We can make this expression more compact by defining $\mathbf{A} = A\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is normal to the loop plane.

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

Torque on a Loop of Wire Question

Which of the rectangular loops has the largest magnitude of the net force acting on it?



(A) a

(B) b

(C) c

(D) all the same

[1] Serway & Jewett, 9th ed.

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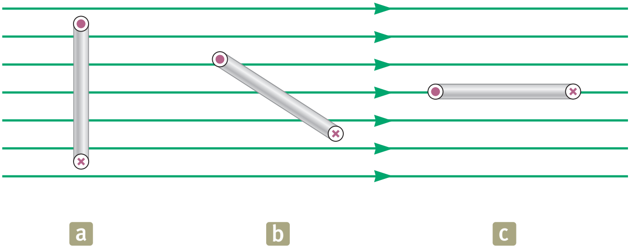

(A) a

(B) b

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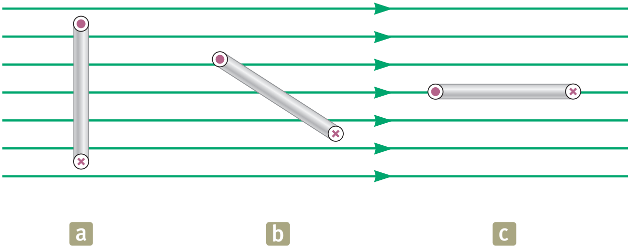
Torque on a Loop of Wire Question

Rank the magnitudes of the torques acting on the rectangular loops from highest to lowest.



(A) a, b, c

(B) b, a, c

(C) c, b, a

(D) c, a, b

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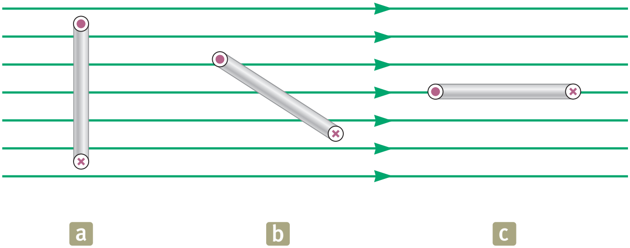
Torque on a Loop of Wire Question

Rank the magnitudes of the torques acting on the rectangular loops from highest to lowest.



(A) a, b, c

(B) b, a, c

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(D) c, a, b

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Torque on a Coil of Wire with a Current

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

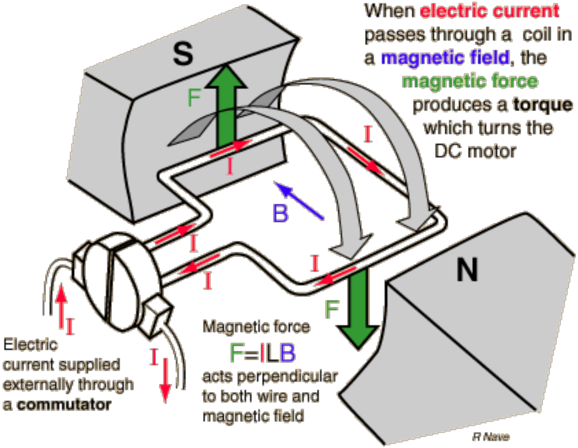
Remarkably, that equation also holds for other shapes of loop as long as they are flat (in one plane). A is the area of the loop.

For a *coil* of N loops stacked together, the effect of each loop adds up:

$$\boldsymbol{\tau} = NI\mathbf{A} \times \mathbf{B}$$

Electric Motors

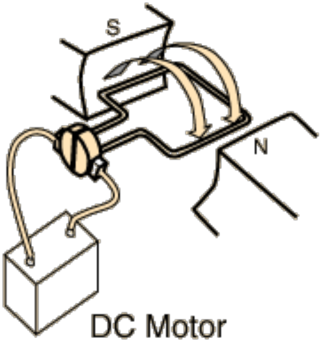
This effect can be used to turn electricity into mechanical work.

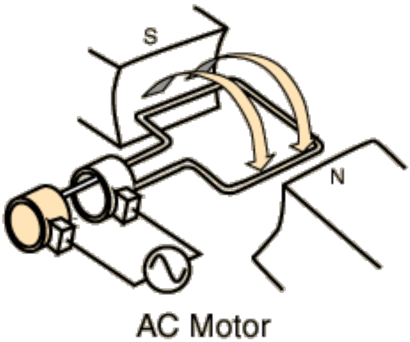


[1]Figure from hyperphysics.phys-arstr.gsu.edu

Electric Motors

Either direct current (DC) or alternating current (AC) can be used for a motor.





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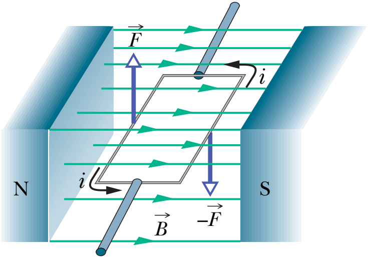
Torque on a Loop of Wire with a Current



$$\boldsymbol{\tau} = IAB\sin\theta$$

We can make this expression more compact by defining $\mathbf{A} = A\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is normal to the loop plane.

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

Magnetic Moment for a Current Loop

For a current loop, we can define the **magnetic moment** of the loop as

$$\boldsymbol{\mu} = I\mathbf{A}$$

And for a coil N turns (loops) of wire carrying a current:

$$\boldsymbol{\mu} = NI\mathbf{A}$$

Then the expression for the torque can be written

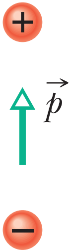
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Reminder: Electric Dipole Moment

Recall our definition for the Electric dipole moment:
dipole moment:

$$\mathbf{p} = q\,\mathbf{d}$$

where $\mathbf{d}$ is a vector pointing from the negative charge to the positive charge, and its magnitude d is the separation of the charges and each charge in the dipole has magnitude q.



Torque on a electric dipole in an electric field:

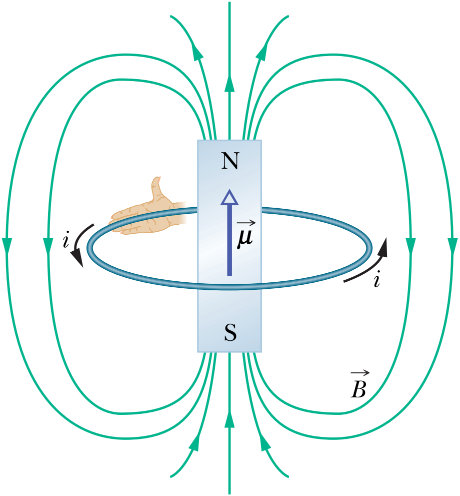
$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

Potential energy:

$$U = -\mathbf{p} \cdot \mathbf{E}$$

Current Loop vs Bar Magnet

A loop of wire with a current in it produces a magnetic field similar to a bar magnet.



Magnetic Dipole Moment

magnetic dipole moment, μ

The quantity relating an external magnetic field that a magnet or coil of wire is in to the torque on the magnet or coil due to that field.

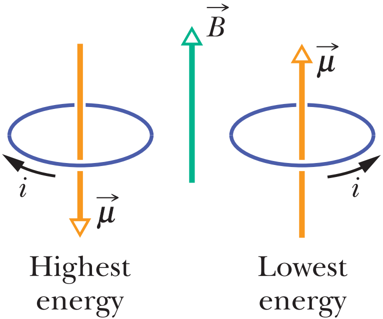
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

For a magnet, it is a vector pointing from the south pole of a magnet to the north pole, that is proportional to the strength of the B-field produced by the magnet itself.

For a coil, it is defined according the the right hand rule for current in a wire loop and is proportional to the coil area and current.

Potential Energy of a Dipole in a B-Field

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

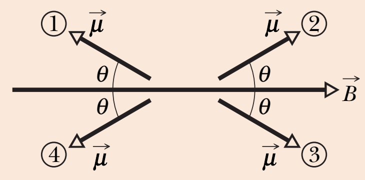


The energy can be found by integrating the torque over the angle of rotation.

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Question

The figure shows four orientations, at angle θ, of a magnetic dipole moment **μ** in a magnetic field. Rank the orientations according to the magnitude of the torque on the dipole, greatest first.



(A) 1 and 2, 3 and 4

(B) 1 and 4, 2 and 3

(C) 3, 2, 1, 4

(D) all the same

[1]Halliday, Resnick, Walker, 9th ed, page 745.

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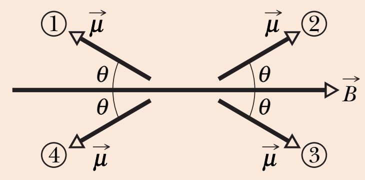

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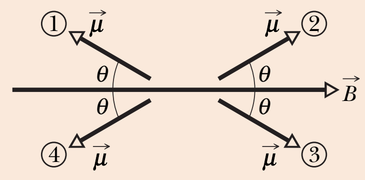

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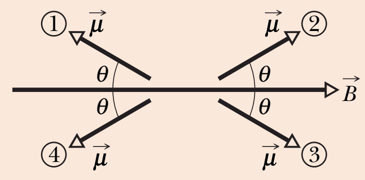

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Electric Dipole and Magnetic Dipole

	electric dipole	magnetic dipole
torque $\boldsymbol{\tau}$	$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
potential energy U	$U = -\mathbf{p} \cdot \mathbf{E}$	$U = -\boldsymbol{\mu} \cdot \mathbf{B}$

Summary

- force on a curved wire in a magnetic field
- torque on a current-carrying wire loop
- relating a current loop to a magnet
- magnetic dipole moment
- torque and potential energy of magnetic dipole

3rd Test Friday, Mar 9.

Homework

- Collected homework 3, due on Monday, Mar 5.

Serway & Jewett:

- PREVIOUS: Ch 29, Problems: 37, 41
- NEW: Ch 29, Problems: 42, 47, 48, 49, 51, 53, 57