Electricity and Magnetism
B-Fields from Moving Charges

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Last time

- force on a curved current carrying wire
- torque on a wire loop
- magnetic dipole moment

Overview

- B-field from a moving charge
- fields from currents

Magnetic fields from moving charges and currents

We are now moving into chapter 30.

Anything with a magnet moment creates a magnetic field.

This is a logical consequence of Newton's third law.

Magnetic Permeability

A constant we will need is:

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$

μ_0 is called the **magnetic permeability of free space**.

It arises when we look at magnetic fields because of our choice of SI units.

Whenever we use μ_0 we assume we are considering the magnetic field to be in a vacuum or air.

μ_0 is not the magnetic dipole moment $\boldsymbol{\mu}$!
Another notation coincidence.

Magnetic fields from moving charges

A moving charge will interact with other magnetic poles.

This is because it has a magnetic field of its own.

The field for a moving charge is given by the Biot-Savart Law:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\,\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

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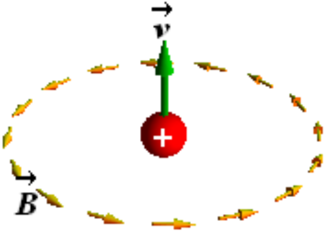
Quite similar to the E-field from a charge:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\,\hat{\mathbf{r}}$$

Magnetic fields from moving charges

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\,\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$



[1]Figure from rakeshkapoor.us.

Magnetic fields from currents

Consider the field from an infinitesimal amount of charge, dq:

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathrm{dq}\ \mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

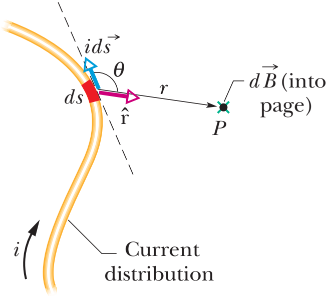
We can deduce from this what the magnetic field due to the current in a small piece of wire is.

Current is made up of moving charges!

$$\mathrm{dq}\ \mathbf{v} = \mathrm{dq}\, \frac{\mathrm{d}\mathbf{s}}{\mathrm{dt}} = \frac{\mathrm{dq}}{\mathrm{dt}}\, \mathrm{d}\mathbf{s} = I\, \mathrm{d}\mathbf{s}$$

We can replace dq $\mathbf{v}$ in the equation above.

Magnetic fields from currents



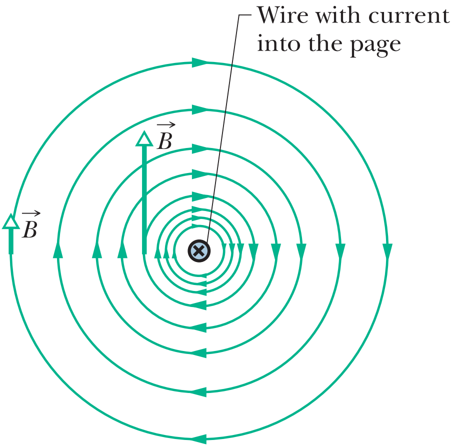
This is another version of the Biot-Savart Law:

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I \ \mathrm{d}\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

where d**B** is the magnetic field from a small segment of wire, of length ds.

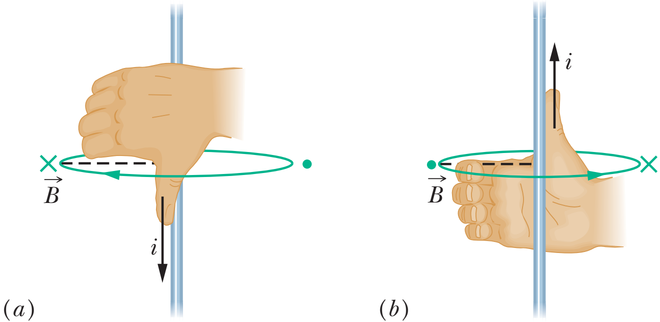
Magnetic fields from currents

Magnetic field around a wire segment, viewed end-on:



Magnetic fields from currents

How to determine the direction of the field lines (right-hand rule):



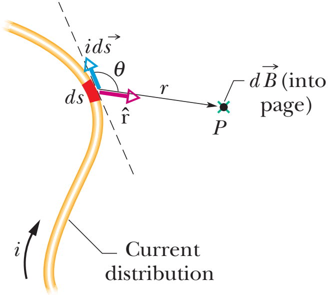
Using the Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\ d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

To calculate a B-field vector, we will need to integrate over **s**.

This is a path integral, similar to what we needed to evaluate:

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$$



Now, however, we are finding the cross product: the perpendicular component of **s** relative to the vector **r**.

Line Integrals

There is one other special piece of notation used with some line integrals:

$$\oint$$

This symbol means that the integral starts and ends at the same point.

The path is a loop.

(We also used the $\oint$ notation before in Gauss's law to indicate a closed surface.)

Magnetic field from a long straight wire

The Biot-Savart Law,

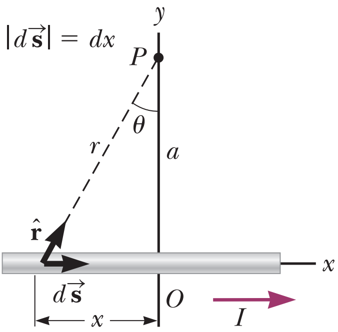
$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I \;\mathrm{d}\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

implies what the magnetic field is at a perpendicular distance a from an **infinitely long straight wire**:

$$B = \frac{\mu_0 I}{2\pi a}$$

Magnetic field from a long straight wire

Add up the B-field contributed by each infinitesimal length of wire carrying current I.



$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\ d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

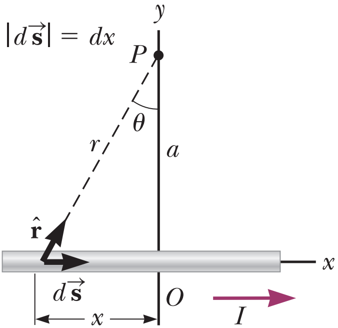
Following from the diagram:

$$d\mathbf{s} \times \hat{\mathbf{r}} = \cos\theta\, dx\, \mathbf{k} = \frac{a}{r}\, dx\, \mathbf{k}$$

$$\mathbf{B} \quad = \quad \frac{\mu_0}{4\pi} I \int \frac{a}{(x^2 + a^2)^{3/2}}\, dx\ \mathbf{k}$$

Magnetic field from a long straight wire

Add up the B-field contributed by each infinitesimal length of wire carrying current I.



$$\mathbf{B} = \frac{\mu_0}{4\pi} I \int \frac{a}{(x^2 + a^2)^{3/2}}\, dx\ \mathbf{k}$$

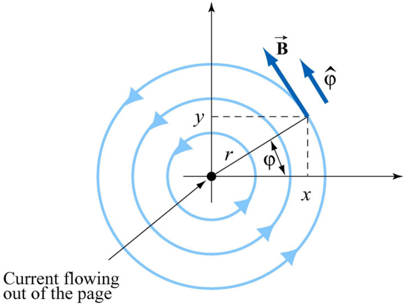
$$= \frac{\mu_0}{4\pi} I \left[\frac{x}{a\sqrt{x^2 + a^2}}\right]_{-\ell/2}^{\ell/2} \mathbf{k}$$

$$= \frac{\mu_0}{4\pi} I \left[\frac{\ell}{a\sqrt{(\ell/2)^2 + a^2}}\right] \mathbf{k}$$

Magnetic field from a long straight wire

Now consider a very long wire, $\ell \to \infty$.

$$
\begin{aligned}
\mathbf{B} &= \lim_{\ell \to \infty} \frac{\mu_0}{4\pi} I \left[\frac{\ell}{a\sqrt{(\ell/2)^2 + a^2}} \right] \mathbf{k} \\
&= \frac{\mu_0 I}{4\pi} \frac{2}{a} \mathbf{k}
\end{aligned}
$$



In general, the field will be a tangential vector:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\boldsymbol{\varphi}}$$

at a distance r from the center of the wire.

[1]Figure from MIT's Visualizing EM, Liao, Dourmashkin, and Belcher.

Summary

- field from a moving charge
- field from a current

3rd Test Friday, March 9.

Homework

- Collected homework 3, posted online, due on Monday, Mar 5.

Serway & Jewett:

- PREVIOUS: Ch 29, Problems: 47, 51, 57
- PREVIOUS: Ch 30, Problems: 49
- NEW: Ch 30, Problems: 3, 5, 9, 13, 19