Electricity and Magnetism
Gauss's Law
Ampère's Law

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Last time

- magnetic field of a moving charge
- magnetic field of a current
- the Biot-Savart law
- magnetic field around a straight wire

Overview

- Gauss's Law for magnetism
- Ampère's Law
- B-field outside and inside a wire
- Solenoids

B-Field around a wire revisited

Using the Biot-Savart law we found that the field around an infinitely long straight wire, carrying a current I was:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi a}$$

at a distance a from the wire.

Is the another way we could have solved this problem?

When we had an infinite line of charge, there was a law we could use to find the E-field...

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Is the another way we could have solved this problem?

When we had an infinite line of charge, there was a law we could use to find the E-field... Gauss's law.

Could we use something similar here?

Gauss's Law for Magnetic Fields

Gauss's Law for Electric fields:

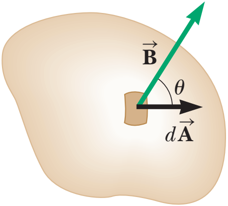
$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

The electric flux through a closed surface is equal to the charge enclosed by the surface, divided by ϵ_0.

There is a similar expression for magnetic flux!

First we must define magnetic flux, Φ_B.

Magnetic Flux



Magnetic flux

The magnetic flux of a magnetic field through a surface **A** is

$$\Phi_B = \int \mathbf{B} \cdot \mathrm{d}\mathbf{A}$$

Units: Tm^2

If the surface is a flat plane and **B** is uniform, that just reduces to:

$$\Phi_B = \mathbf{B} \cdot \mathbf{A}$$

Gauss's Law for Magnetic Fields

Gauss's Law for magnetic fields.:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

Where the integral is taken over a closed surface A.

We can interpret it as an assertion that magnetic monopoles do not exist.

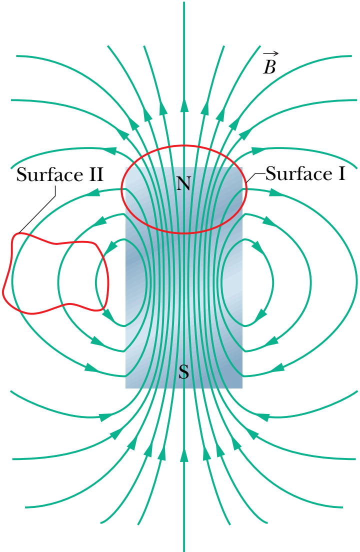
In differential form:

$$\nabla \cdot \mathbf{B} = 0$$

The magnetic field has no sources or sinks.
(It is "divergence-free"; we can write $\mathbf{B} = \nabla \times \mathbf{a}$, where $\mathbf{a}$ is the "vector potential".)

Gauss's Law for Magnetic Fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$



B-Field around a wire revisited

Gauss's law will not help us find the strength of the B-field around the wire: the flux through any closed surface will be zero.

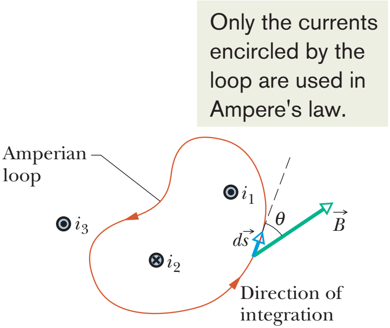
Another law can.

Ampère's Law

For constant currents (magnetostatics):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

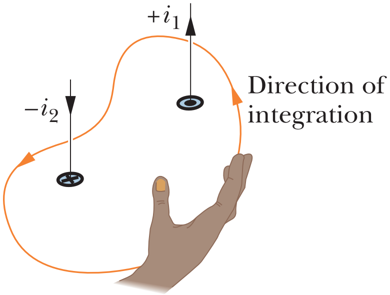
The line integral of the magnetic field around a closed loop is proportional to the current that flows *through* the loop.[1]



[1] That is, the current that flows through any surface bounded by the loop.

Ampère's Law

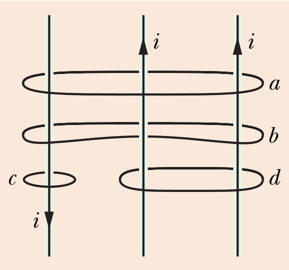
This is how to assign a sign to a current used in Ampere's law.



A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Question

The figure here shows three equal currents i (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \mathbf{B} \cdot d\mathbf{s}$ along each, greatest first.



A a, b, c, d

B d, b, c, a

C (a and b), d, c

D d, (a and c), b

[1] Halliday, Resnick, Walker, page 773.

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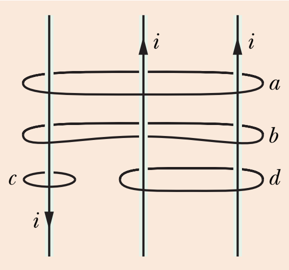

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D d, (a and c), b ←

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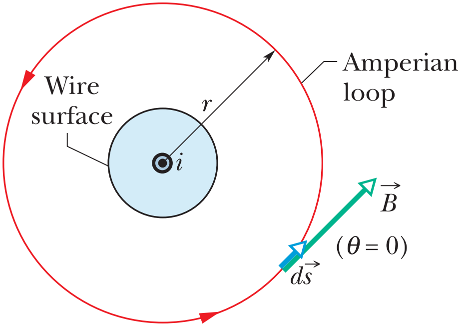
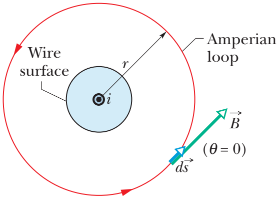
Ampère's Law and the Magnetic Field from a Current Outside a wire

Suppose we want to know the magnitude of the magnetic field at a distance r outside a wire. Using Ampère's Law?



Ampère's Law and the Magnetic Field from a Current Outside a wire



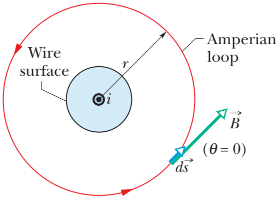
Ampère's Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

To find the B-field at a distance r from the wire's center choose a circular path of radius r.

By cylindrical symmetry, everywhere along the circle $\mathbf{B} \cdot d\mathbf{s}$ is constant.

Ampère's Law and the Magnetic Field from a Current Outside a wire



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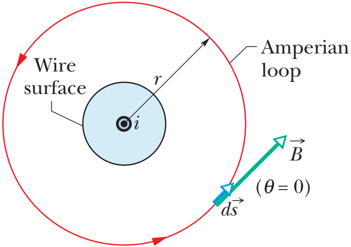
The magnetic field lines must form a closed loop $\Rightarrow$ $\mathbf{B} \cdot d\mathbf{s} = B\, ds$.

Ampère's Law and the Magnetic Field from a Current Outside a wire



$$B \oint ds = \mu_0 I_{enc}$$

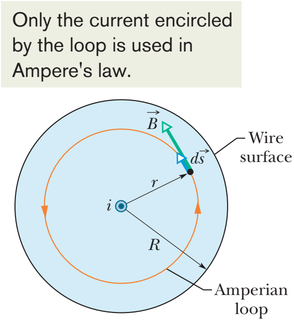
$$B(2\pi r) = \mu_0 I$$

And again we get

$$B = \frac{\mu_0 I}{2\pi r}$$

Ampère's Law and the Magnetic Field from a Current Inside a wire

We can also use Ampère's Law in another context, where using the Biot-Savart Law is harder.



Now we place the Amperian loop *inside* the wire.

We still have $\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B$, but now the current that flows through the loop is reduced.

Ampère's Law and the Magnetic Field from a Current Inside a wire

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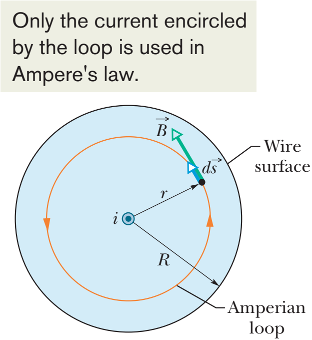
Assuming the wire has uniform resistivity, I_{enc}:



Ampère's Law and the Magnetic Field from a Current Inside a wire

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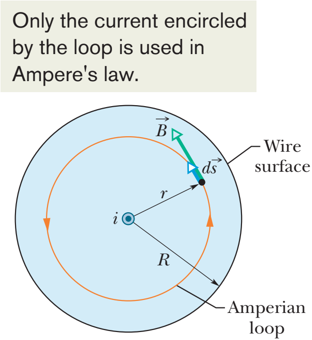


Assuming the wire has uniform resistivity, I_{enc}:

$$I_{\text{enc}} = \frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I$$

Ampére's Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B = \mu_0 \frac{r^2}{R^2} I$$

So,

$$B = \frac{\mu_0 I \, r}{2\pi R^2} = \frac{\mu_0 I}{2\pi R} \left(\frac{r}{R} \right)$$

Ampère's Law

For constant currents (magnetostatics):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

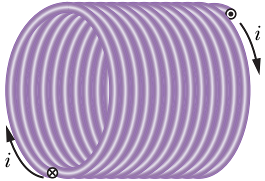
The line integral of the magnetic field around a closed loop is proportional to the current that flows *through* the loop.

Later we will extend this law to deal with the situation where the fields / currents are changing.

Solenoids

solenoid

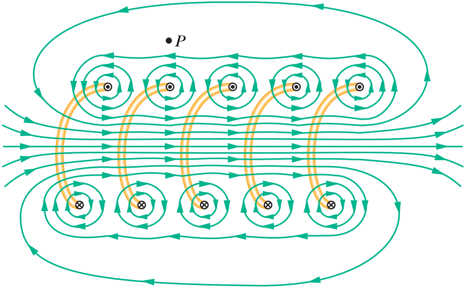
A helical coil of tightly wound wire that can carry a current.



turn

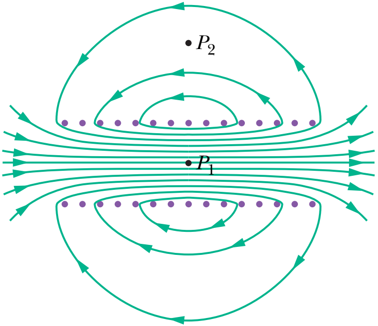
A single complete loop of wire in a solenoid. “This solenoid has 10 turns,” means it has 10 complete loops.

Magnetic Field inside and around a solenoid



Each turn of wire locally has a circular magnetic field around it. The fields from all the wires add together to create very dense field lines inside the solenoid.

Magnetic Field of a solenoid

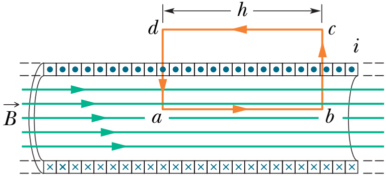


The wires on opposite sides (top and bottom in the picture) have currents in opposite directions. The fields add up between them, but cancel out outside of them.

Magnetic Field of an ideal solenoid

In an ideal solenoid (with infinite length) the field outside is small (and perpendicular to the Amp. loop) and inside is uniform. (Similar to a capacitor!)

Can use an Amperian loop to find the B-field inside:



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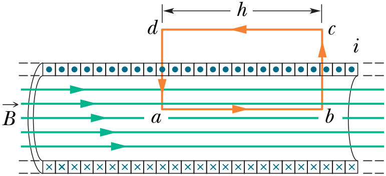


$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

Here, suppose there are n turns per unit length in the solenoid, then $I_{enc} = Inh$

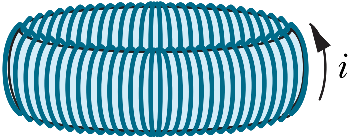
$$Bh = \mu_0 Inh$$

Inside an ideal solenoid:

$$B = \mu_0 In$$

Toroids

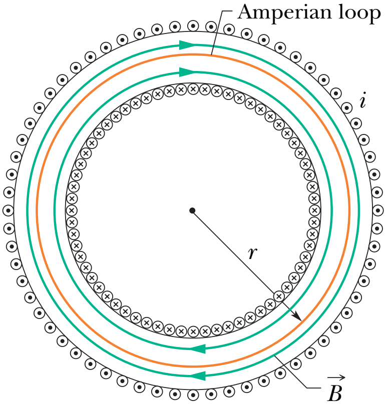
A toroid is a solenoid wrapped into a torus (donut) shape.



The two ends of the solenoid are wrapped around an attached to each other.

Magnetic Field in a Toroid

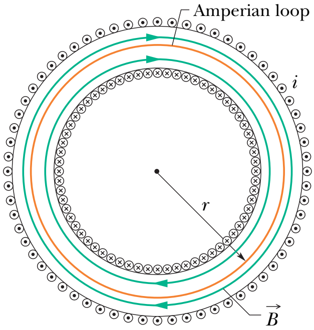
Cross section through a toroid:



We can use the Amperian loop shown to find the field inside the toroid's loop.

Magnetic Field in a Toroid

Suppose the toroid has N turns.



$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 N I$$

Inside a toroid: s

$$B = \frac{\mu_0 I N}{2\pi r}$$

This is not independent of the radius! The field is stronger closer to the inside: it is not uniform.

Summary

- Forces on parallel wires
- Gauss's Law for magnetism
- Ampère's Law
- Solenoids

3rd Test Friday, Mar 9.

Homework

- Collected homework 3, posted online, due on Monday, Mar 5.

Serway & Jewett:

- PREVIOUS: Ch 30, Problems: 3, 5, 9, 13, 19
- NEW: Ch 30, Problems: 21, 25, 31, 33, 34, 47