Electricity and Magnetism
Current Loops and Magnetic Dipoles
Magnetism in Matter

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Last time

- magnetic field inside a solenoid
- forces between current-carrying wires

Overview

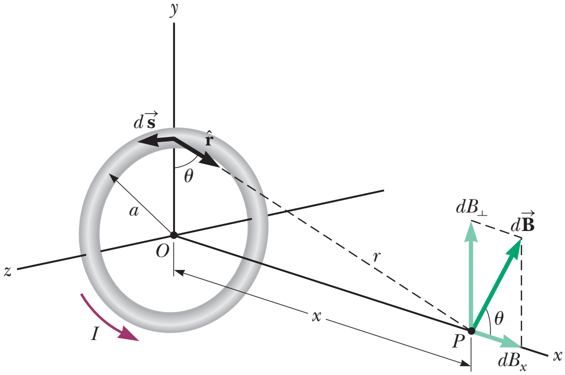
- magnetic field around a current loop
- more about magnetic dipoles
- magnetism of matter

Current loops and Magnetic dipoles

We are now going to return to current loops and see why they have associated magnetic dipole moments, and how these behave.

Magnetic field from a circular loop of wire

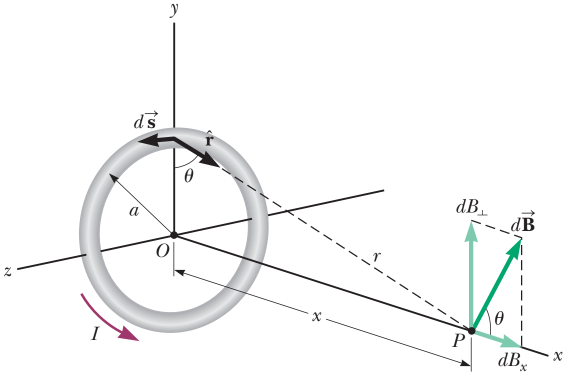
In the last lecture, we considered the magnetic moment of a loop of wire, now we look at the magnetic field along a line through the center of the loop.



Each little segment of wire d**s** with current I contributes a field d**B**.

By symmetry, we can see the the components that are parallel to the plane of the ring will cancel.

Magnetic field from a circular loop of wire



$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I \ \mathrm{d}\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

This time $\mathbf{r} \perp \mathrm{d}\mathbf{s}$

$$\mathrm{d}\mathbf{s} \times \hat{\mathbf{r}} = \mathrm{d}s \left(\cos\theta \mathbf{i} + \sin\theta \mathbf{j} \right)$$

Magnetic field from a circular loop of wire

(y-comp. cancels)

$$\mathbf{B} = \frac{\mu_0}{4\pi} I \oint \frac{\cos\theta}{r^2}\,\mathsf{ds}\;\mathbf{i}$$

Notice $\cos\theta = \frac{a}{r}$, and $r = \sqrt{a^2 + x^2}$.

They are independent of the integration variable!

$$\mathbf{B} \;=\; \frac{\mu_0}{4\pi}\frac{I a}{(a^2 + x^2)^{3/2}} \oint \mathsf{ds}\;\mathbf{i}$$

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They are independent of the integration variable!

$$\begin{aligned} \mathbf{B} &= \frac{\mu_0}{4\pi} \frac{I a}{(a^2 + x^2)^{3/2}} \oint \mathsf{ds}\ \mathbf{i} \\ &= \frac{\mu_0}{4\pi} \frac{I a}{(a^2 + x^2)^{3/2}} (2\pi a)\, \mathbf{i} \\ &= \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}\, \mathbf{i} \end{aligned}$$

Similar to the E-field of an electric dipole...

Magnetic field from a circular loop of wire

Very far from the wire,

$$\mathbf{B} = \frac{\mu_0 I a^2}{2\pi x^3}\,\mathbf{i}$$

In terms of the magnetic moment $\mu = I\pi a^2$

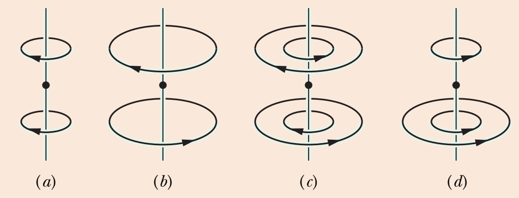
$$\mathbf{B} = \frac{\mu_0}{2\pi}\frac{\mu}{x^3}\,\mathbf{i}$$

Far from an electric dipole, along the axis of the dipole:

$$\mathbf{E} = \frac{1}{2\pi\epsilon_0}\frac{p}{x^3}\,\mathbf{i}$$

Current Loop Question

Consider the four arrangements of circular loops of radius r or $2r$, centered on vertical axes (perpendicular to the loops) and carrying **identical currents** in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.



A a, b, c, d

B b, c, d, a

C d, a, (b and c)

D (b and c), d, a

[1] Halliday, Resnick, Walker, pg 779.

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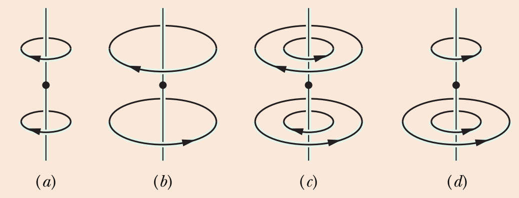

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[1] Halliday, Resnick, Walker, pg 779.

Magnetic Moment for a Current Loop

For a current loop, we can define the **magnetic moment** of the loop as

$$\boldsymbol{\mu} = I\mathbf{A}$$

And for a coil N turns (loops) of wire carrying a current:

$$\boldsymbol{\mu} = NI\mathbf{A}$$

Then the expression for the torque can be written

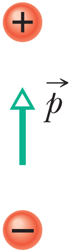
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Reminder: Electric Dipole Moment

Recall our definition for the Electric dipole moment:
dipole moment:

$$\mathbf{p} = q\,\mathbf{d}$$

where **d** is a vector pointing from the negative charge to the positive charge, and its magnitude d is the separation of the charges and each charge in the dipole has magnitude q.



Torque on a electric dipole in an electric field:

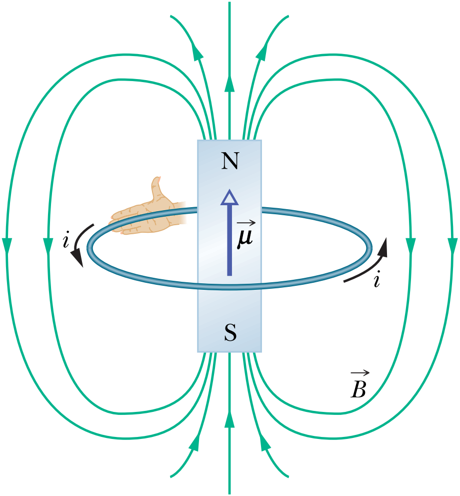
$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

Potential energy:

$$U = -\mathbf{p} \cdot \mathbf{E}$$

Current Loop vs Bar Magnet

A loop of wire with a current in it produces a magnetic field somewhat similar to a bar magnet.



Magnetic Dipole Moment

magnetic dipole moment, μ

The quantity relating an external magnetic field that a magnet or coil of wire is in to the torque on the magnet or coil due to that field.

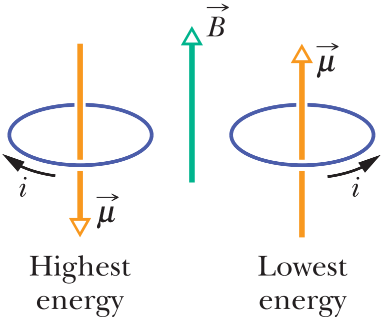
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

For a magnet, it is a vector pointing from the south pole of a magnet to the north pole, that is proportional to the strength of the B-field produced by the magnet itself.

For a coil, it is defined according the the right hand rule for current in a wire loop and is proportional to the coil area and current.

Potential Energy of a Dipole in a B-Field

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

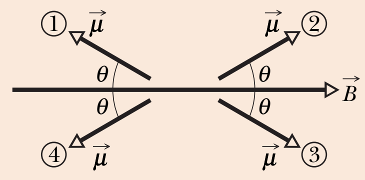


The energy can be found by integrating the torque over the angle of rotation, choosing $U(\pi/2) = 0$. (See Lecture 13 for derivation.)

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Question

The figure shows four orientations, at angle θ, of a magnetic dipole moment **μ** in a magnetic field. Rank the orientations according to the magnitude of the torque on the dipole, greatest first.



(A) (1 and 2), (3 and 4)

(B) (1 and 4), (2 and 3)

(C) 3, 2, 1, 4

(D) all the same

[1] Halliday, Resnick, Walker, 9th ed, page 745.

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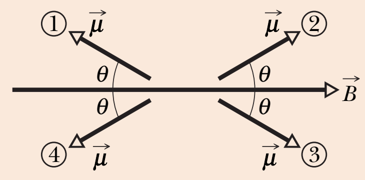

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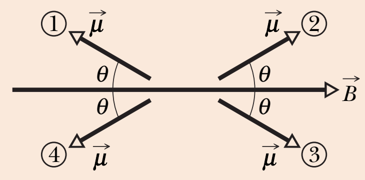

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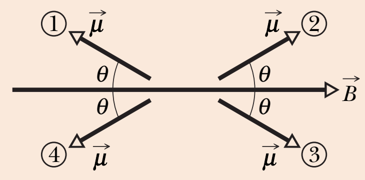

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Electric Dipole and Magnetic Dipole

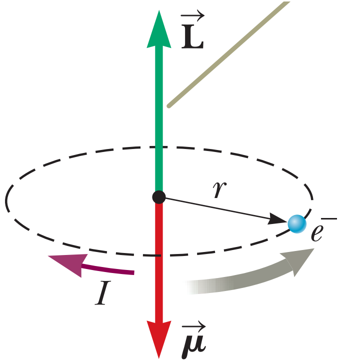
	electric dipole	magnetic dipole
torque $\boldsymbol{\tau}$	$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
potential energy U	$U = -\mathbf{p} \cdot \mathbf{E}$	$U = -\boldsymbol{\mu} \cdot \mathbf{B}$

Magnetism in Matter: Magnetic Moment of Atoms

Atoms and subatomic particles also have magnetic moments!

Why? Consider a classical model of a hydrogen atom. One electron orbits the nucleus.

$$\mu = IA$$



Magnetic Moment of Atoms

The current is the rate of charge flow with time:

$$I = \frac{-e}{T} = -e\,\frac{v}{2\pi r}$$

assuming an orbital radius of r, speed v.

$$\begin{aligned}
\boldsymbol{\mu} &= I\mathbf{A} \\
&= -e\,\frac{v}{2\pi r}(\pi r^2 \hat{\mathbf{n}}) \\
&= -\frac{evr}{2}\hat{\mathbf{n}}
\end{aligned}$$

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Recall that for a particle of mass m orbiting at a radius r, velocity v, the angular momentum is:

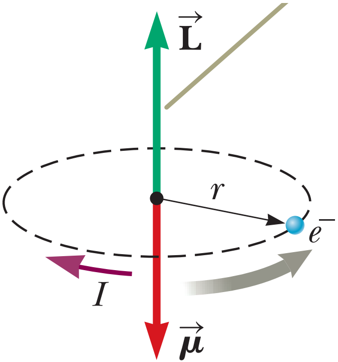
$$L = mvr$$

$$\boldsymbol{\mu} = -\frac{e}{2m_e}\mathbf{L}$$

Magnetism in Matter: Magnetic Moment of Atoms

Orbital magnetic moment

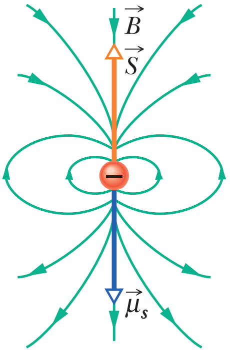
$$\boldsymbol{\mu} = -\frac{e}{2m_e}\mathbf{L}$$



Electron Spin Angular Momentum

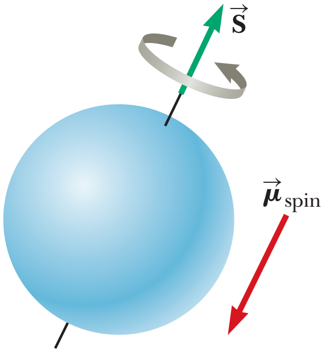
Electrons also have another kind of angular momentum: *intrinsic angular momentum*. This is also called "spin".

Spin is an inherent property of all electrons. It cannot be understood with classical mechanics, but also contributes a magnetic moment.



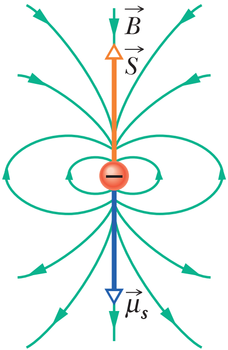
Electron Spin Angular Momentum

You might imagine an electron as a rigid charge sphere spinning on an axis through its center...



...but really, it's not.

Electron Spin Angular Momentum



Electron's spin magnetic dipole moment:

$$\mu_s = -\frac{g\,e}{2m_e}\mathbf{S}$$

where $g \approx 2$.

Magnetic Moment of Atoms

In atoms with many electrons, the electrons tend to cancel out each other's magnetic moments, but outer-shell, unpaired electrons can contribute a significant magnetic moment.

The particles in the nucleus also have magnetic moments, but they are much smaller.

Most of an atom's magnetic moment comes from unpaired electons.

These tiny magnetic moments add up to big effects in bulk materials.

Three Types of Bulk Magnetism

- ferromagnetism
- paramagnetism
- diamagnetism

Ferromagnetism

Atoms of ferromagnetic materials have non-zero magnetic moments.

Interactions between outer electrons in different atoms causes alignment of each atom's magnetic moment.

Magnetic moments reenforce each other and will tend to spontaneously align within domains.

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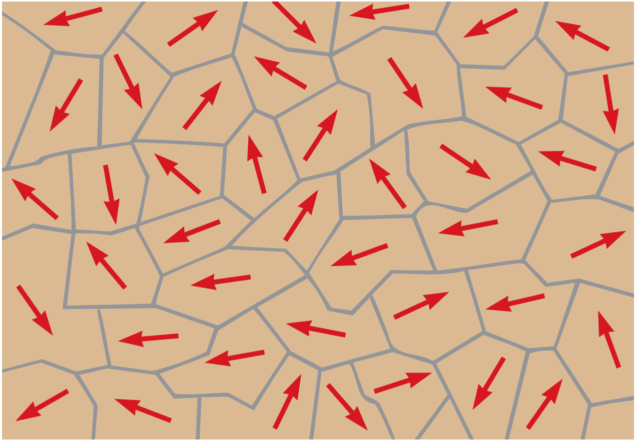
Examples of ferromagnetic materials:

- iron
- nickel
- cobalt
- gadolinium
- dysprosium

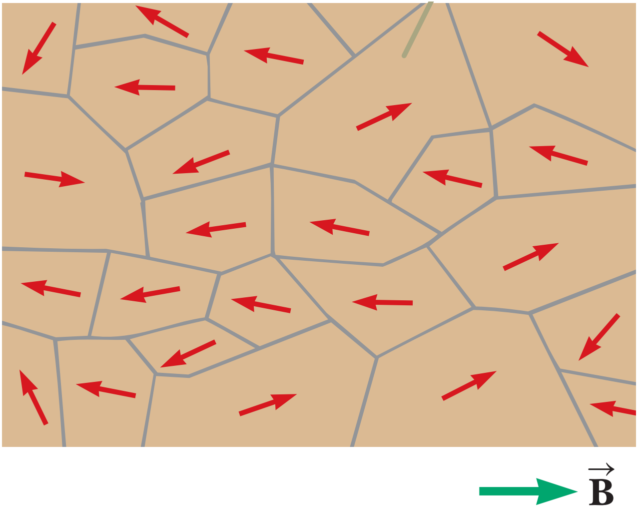
Ferromagnetism

No external B-field



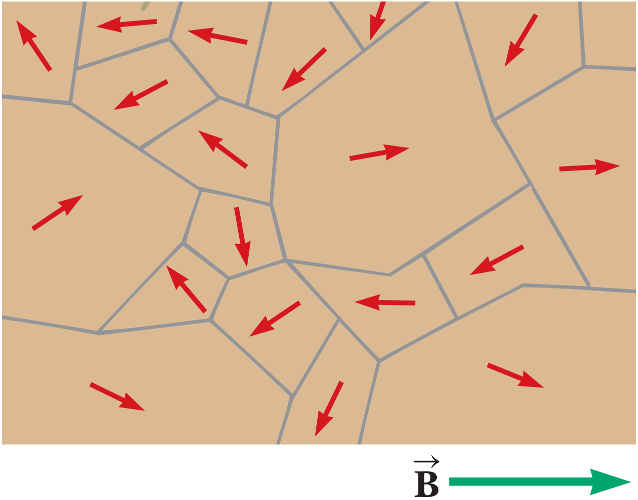
Ferromagnetism

Applied external B-field



Ferromagnetism

Strong external B-field



Paramagnetism

Atoms of paramagnetic materials have non-zero dipole moments, but electrons of different atoms do not interact with each other.

They can interact with a strong magnetic field, and will align with the field.

Paramagnetic effects tend to be much smaller than ferromagnetic ones.

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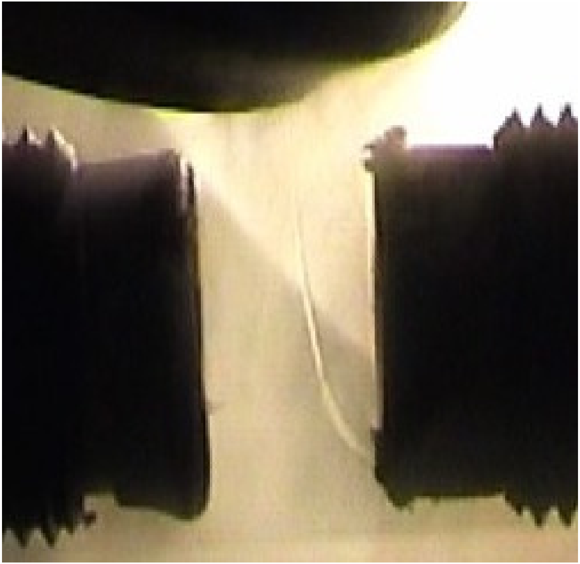
Examples of paramagnetic materials:

- Tungsten
- Cesium
- Aluminium
- Lithium
- Magnesium
- Sodium

Paramagnetism

Liquid oxygen stream deflected in a strong magnetic field. The stream collects in the field.



[1]Image created by Pieter Kuiper.

Diamagnetism

Diamagnetism occurs in all materials, but is a weak effect, so it is "drowned out" if a material is ferro- or paramagnetic.

It is the dominant (but weak) effect when the net magnetic moment of a material's atoms is zero.

The field magnetizes the atoms and the resulting magnetic moments **oppose** the external magnetic field.

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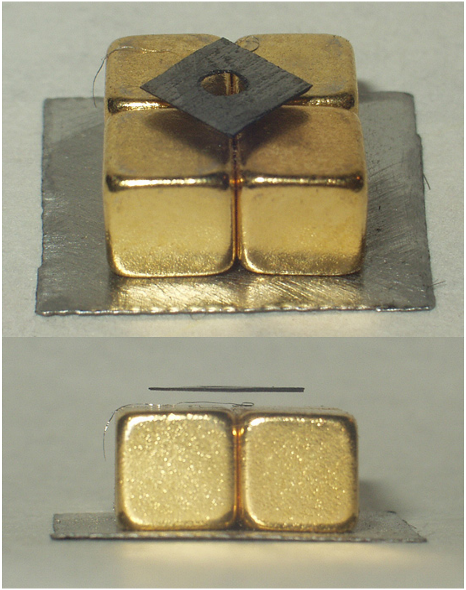
Examples of diamagnetic materials:

- Pyrolytic carbon
- Bismuth
- Mercury
- Silver
- diamond (form of Carbon)
- water

Also superconductors can be said to exhibit extreme diamagnetism.

Diamagnetism



[1]Levitating pyrolytic carbon on neodymium magnets. Image by Splarka.

Diamagnetism

[1]Magnet photo by Mai-Linh Doan, Wikipedia; Frog photo by Lijnis Nelemans/High Field Magnet Laboratory/Radboud University Nijmeg.

Summary

- B-field near a current loop
- more about magnetic dipoles
- magnetism of matter

3rd Test Friday, March 9.

Homework Serway & Jewett:

- PREVIOUS: Ch 29, Problems: 47, 53 (dipole energy)
- NEW: Ch 30, Problems: 7, 49