Electricity and Magnetism
Motional EMF
Faraday's Law

Lana Sheridan

De Anza College

Mar 6, 2018

Last time

- relativity and electromagnetic fields

Overview

- motional emf
- induction
- Faraday's law
- Lenz's law

Magnetic Moment of Atoms

In atoms with many electrons, the electrons tend to cancel out each other's magnetic moments, but outer-shell, unpaired electrons can contribute a significant magnetic moment.

The particles in the nucleus also have magnetic moments, but they are much smaller.

Most of an atom's magnetic moment comes from unpaired electons.

These tiny magnetic moments add up to big effects in bulk materials.

Three Types of Bulk Magnetism

- ferromagnetism
- paramagnetism
- diamagnetism

Ferromagnetism

Atoms of ferromagnetic materials have non-zero magnetic moments.

Interactions between outer electrons in different atoms causes alignment of each atom's magnetic moment.

Magnetic moments reenforce each other and will tend to spontaneously align within domains.

Ferromagnetism

Atoms of ferromagnetic materials have non-zero magnetic moments.

Interactions between outer electrons in different atoms causes alignment of each atom's magnetic moment.

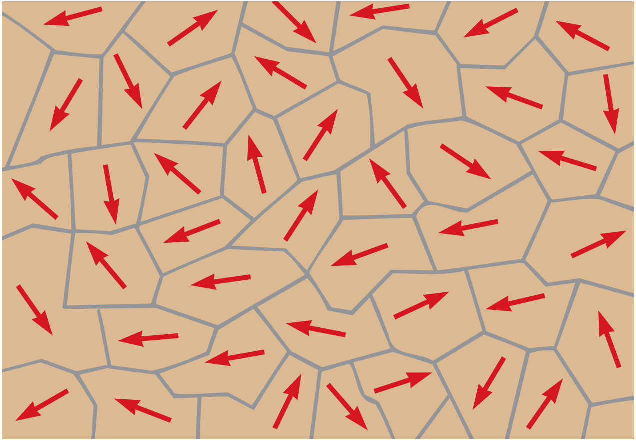
Magnetic moments reenforce each other and will tend to spontaneously align within domains.

Examples of ferromagnetic materials:

- iron
- nickel
- cobalt
- gadolinium
- dysprosium

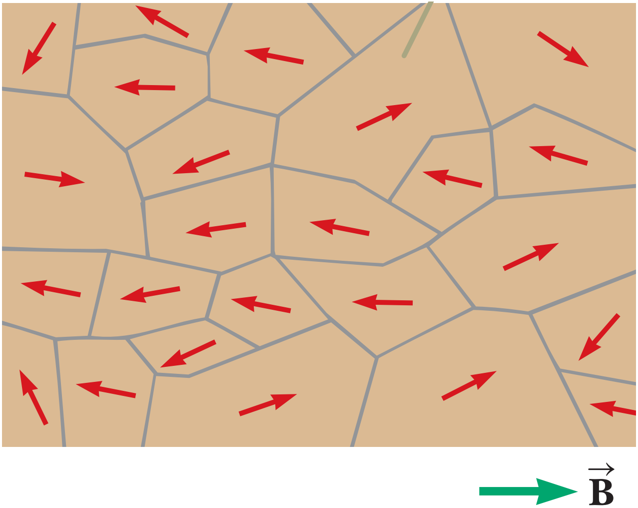
Ferromagnetism

No external B-field



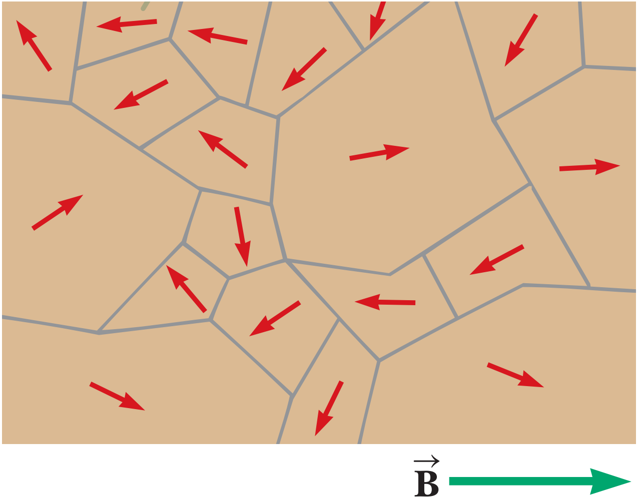
Ferromagnetism

Applied external B-field



Ferromagnetism

Strong external B-field



Paramagnetism

Atoms of paramagnetic materials have non-zero dipole moments, but electrons of different atoms do not interact with each other.

They can interact with a strong magnetic field, and will align with the field.

Paramagnetic effects tend to be much smaller than ferromagnetic ones.

Paramagnetism

Atoms of paramagnetic materials have non-zero dipole moments, but electrons of different atoms do not interact with each other.

They can interact with a strong magnetic field, and will align with the field.

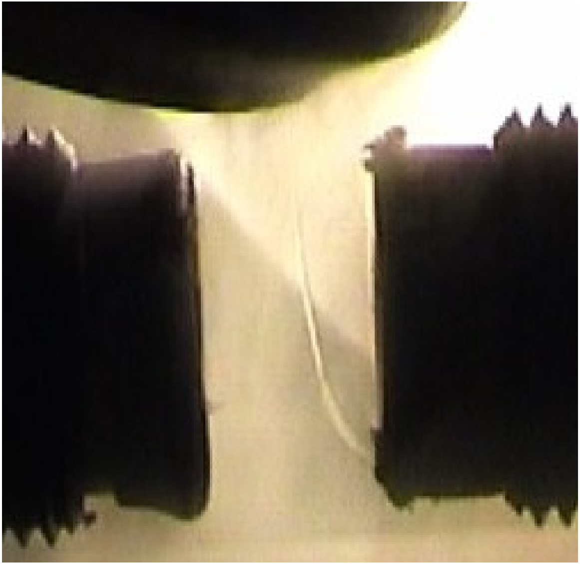
Paramagnetic effects tend to be much smaller than ferromagnetic ones.

Examples of paramagnetic materials:

- Tungsten
- Cesium
- Aluminium
- Lithium
- Magnesium
- Sodium

Paramagnetism

Liquid oxygen stream deflected in a strong magnetic field. The stream collects in the field.



[1]Image created by Pieter Kuiper.

Diamagnetism

Diamagnetism occurs in all materials, but is a weak effect, so it is "drowned out" if a material is ferro- or paramagnetic.

It is the dominant (but weak) effect when the net magnetic moment of a material's atoms is zero.

The field magnetizes the atoms and the resulting magnetic moments **oppose** the external magnetic field.

Diamagnetism

Diamagnetism occurs in all materials, but is a weak effect, so it is "drowned out" if a material is ferro- or paramagnetic.

It is the dominant (but weak) effect when the net magnetic moment of a material's atoms is zero.

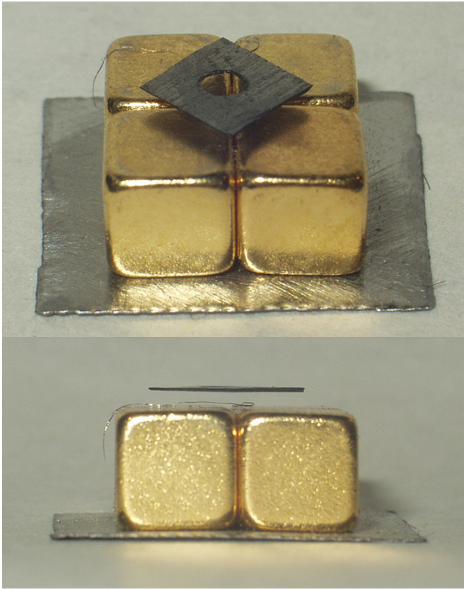
The field magnetizes the atoms and the resulting magnetic moments **oppose** the external magnetic field.

Examples of diamagnetic materials:

- Pyrolytic carbon
- Bismuth
- Mercury
- Silver
- diamond (form of Carbon)
- water

Also superconductors can be said to exhibit extreme diamagnetism.

Diamagnetism



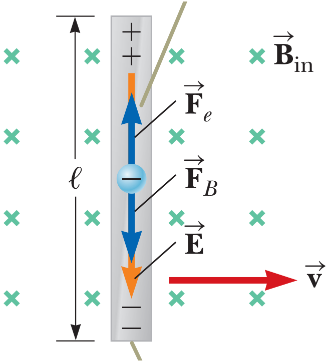
[1]Levitating pyrolytic carbon on neodymium magnets. Image by Splarka.

Diamagnetism

[1]Magnet photo by Mai-Linh Doan, Wikipedia; Frog photo by Lijnis Nelemans/High Field Magnet Laboratory/Radboud University Nijmeg.

Motional EMF

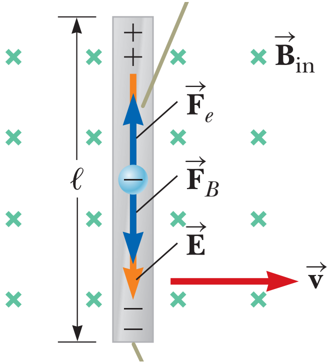
If a conductor moves through a magnetic field at an angle to the field, an emf is induced across the conductor.



There are two ways to see this:

1. force on conduction charges $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$
2. (relativity: in the rest frame of the conductor there is also an electric field as soon as the conductor moves)

Motional EMF



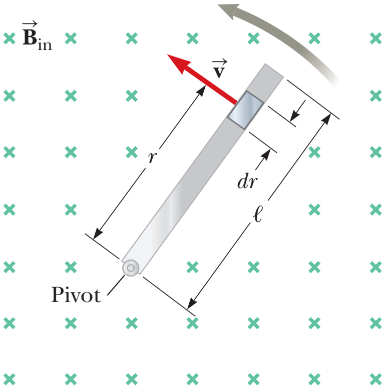
Once the charge distribution reaches equilibrium, the net force on each charge:

$$\mathbf{F}_{net} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

$$\begin{aligned} E &= vB \quad (\mathbf{v} \perp \mathbf{B}) \\ \frac{\mathcal{E}}{\ell} &= vB \\ \mathcal{E} &= vB\ell \end{aligned}$$

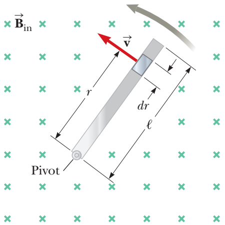
Motional emf in rotating bar (Ex 31.4)

A conducting bar of length ℓ, rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation. Find the motional emf induced between the ends of the bar.



Motional emf in rotating bar (Ex 31.4)

Each infinitesimal slice of the bar, dr, is moving at a different speed $v(r) = r\omega$.



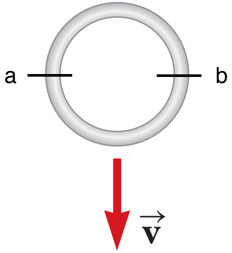
Using our previous result:

$$d\mathcal{E} = vB\,dr$$

$$\begin{aligned} \mathcal{E} &= \int_0^\ell \omega r B\,dr \\ &= \frac{1}{2}B\omega\ell^2 \end{aligned}$$

Motional emf and loops

Imagine a loop of wire that moves in a **uniform magnetic field**, **B**, directed **into** the page.

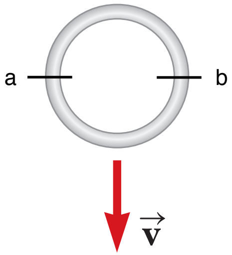


Imagine the loop is composed of a pair of curved rods cut along the lines shown.

Which way (left or right) is the emf directed in the top half? In the bottom? How do the magnitudes compare?

Motional emf and loops

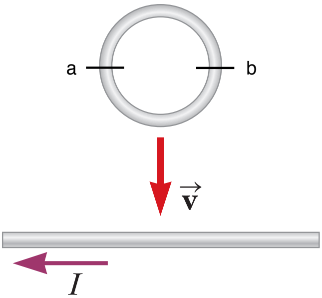
Imagine a loop of wire that moves in a **uniform magnetic field**, **B**, directed **into** the page.



In this case, part of the loop near *a* develops a negative charge and the part near *b* a positive charge, but overall no steady current flows around the loop.

Motional emf and loops

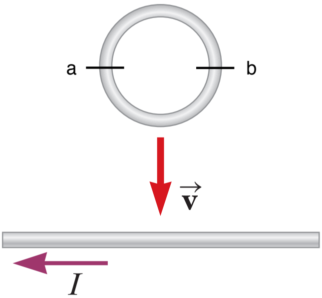
Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire.



How do the magnitudes of the emfs in the top and bottom compare?

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire.

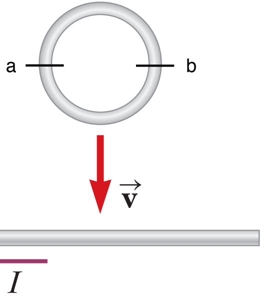


How do the magnitudes of the emfs in the top and bottom compare?

They are not the same! A current can flow.

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire. (Quiz 31.3)



What is the direction of the induced current in the loop of wire?

(A) clockwise

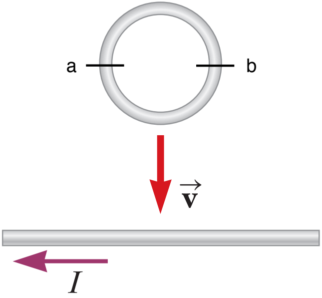
(B) counterclockwise

(C) zero

(D) impossible to determine

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire. (Quiz 31.3)



What is the direction of the induced current in the loop of wire?

(A) clockwise

(B) counterclockwise ←

(C) zero

(D) impossible to determine

Motional emf and loops

What was different in the two cases (uniform vs. non-uniform field)?

Motional emf and loops

What was different in the two cases (uniform vs. non-uniform field)?

$\rightarrow$ The field at different parts of the loop was different.

$\rightarrow$ The magnetic flux through the loop was changing.

Motional emf and loops

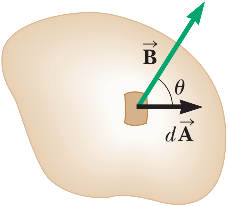
What was different in the two cases (uniform vs. non-uniform field)?

$\rightarrow$ The field at different parts of the loop was different.

$\rightarrow$ The magnetic flux through the loop was changing.

When we have a loop with a changing magnetic flux through it, we can predict the EMF around the loop (Faraday's Law).

Reminder: Magnetic Flux



Magnetic flux

The magnetic flux of a magnetic field through a surface **A** is

$$\Phi_B = \int \mathbf{B} \cdot \mathrm{d}\mathbf{A}$$

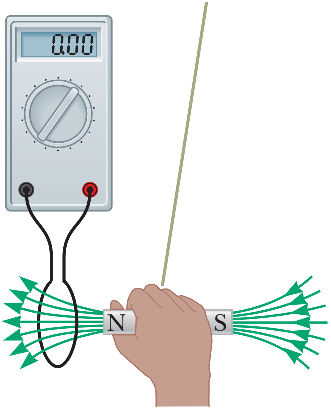
Units: Tm^2

If the surface is a flat plane and **B** is uniform, that just reduces to:

$$\Phi_B = \mathbf{B} \cdot \mathbf{A}$$

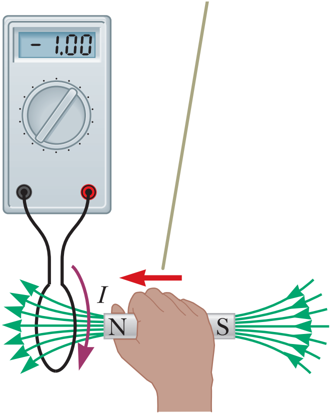
Changing flux and emf

When a magnet is at rest near a loop of wire there is no potential difference across the ends of the wire.



Changing flux and emf

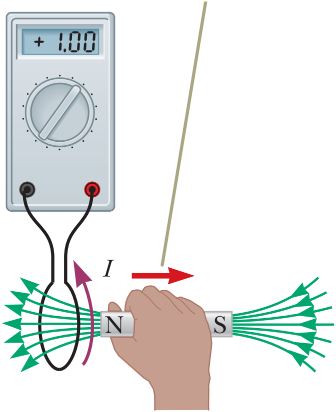
When the north pole of the magnet is moved towards the loop, the magnetic flux increases.



A current flows clockwise in the loop.
(Magnetic moment of current loop point to the right.)

Changing flux and emf

When the north pole of the magnet is moved away from the loop, the magnetic flux decreases.



A current flows counterclockwise in the loop.
(Magnetic moment of current loop point to the left.)

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

Faraday's Law for a conducting loop:

$$\mathcal{E}_F = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{d}t}$$

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

Faraday's Law for a conducting loop:

$$\mathcal{E}_F = -\frac{d\Phi_B}{dt}$$

Note: If there is a changing flux, there will be an induced emf, however, if $\frac{d\Phi_B}{dt} = 0$ there could still be emf in a wire from other effects. $(\mathcal{E}_{\text{net}} = \mathcal{E}_F + \mathcal{E}_{\text{other}})$

Faraday's Law

Faraday's Law for a coil of N turns:

$$\mathcal{E}_F = -N \frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$$

if Φ_B is the flux through a single loop.

Changing Magnetic Flux

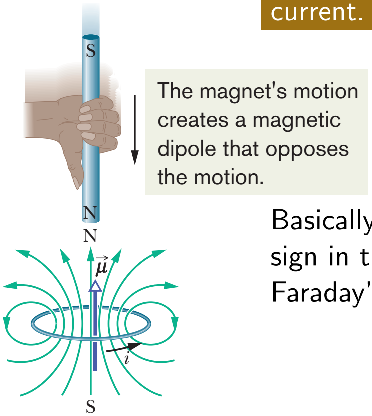
The magnetic flux might change for any of several reasons:

- the magnitude of **B** can change with time,
- the area A enclosed by the loop can change with time, or
- the angle θ between the field and the normal to the loop can change with time.

Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



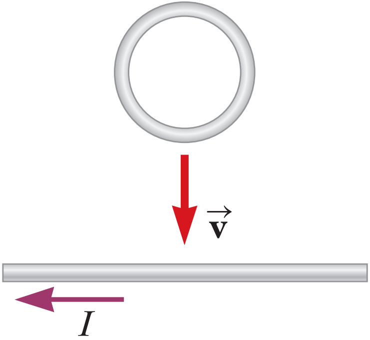
Basically, Lenz's law let's us interpret the minus sign in the equation we write to represent Faraday's Law.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

[1] Figure from Halliday, Resnick, Walker, 9th ed.

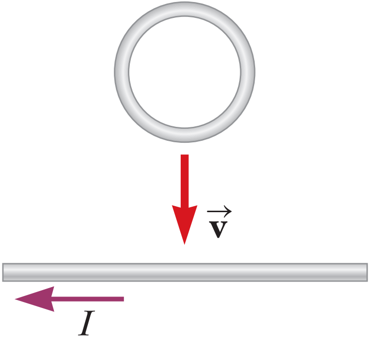
Faraday's and Lenz's Laws

What about this case? We found the current should flow counterclockwise.



Faraday's and Lenz's Laws

What about this case? We found the current should flow counterclockwise.



The flux from the wire is into the page and increasing.

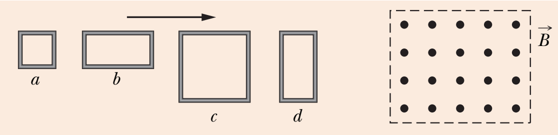
The field from the current in the loop is out of the page.

There is an upward resistive force on the ring. (*cf.* HW3, #3.)

Loops in B-Fields Question

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field **B** (directed out of the page) at the same constant velocity.

Rank the four loops according to the maximum magnitude of the emf induced as they move into the field, greatest first.



A a, b, c, d

B (b and c), (a and d)

C (c and d), (a and b)

D (a and b), (c and d)

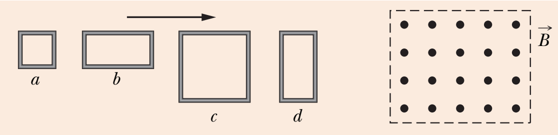
---

[1]Halliday, Resnick, Walker, page

Loops in B-Fields Question

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field **B** (directed out of the page) at the same constant velocity.

Rank the four loops according to the maximum magnitude of the emf induced as they move into the field, greatest first.



A a, b, c, d

B (b and c), (a and d)

C (c and d), (a and b) ←

D (a and b), (c and d)

---

[1] Halliday, Resnick, Walker, page

Summary

- motional emf
- Faraday's law
- Lenz's law
- applications

Next Test this Friday, Mar 9.

Homework

Serway & Jewett:

- NEW: Ch 31, Obj. Qs: 1, 3, 5, 7; Conc. Qs: 3, 5; Problems: 1, 5, 9, 13, 21, 27, 31, 33 (hint: use the Lorentz force on the electrons in the spinning disk)