Electricity and Magnetism
Faraday's Law Practice
Eddy Currents

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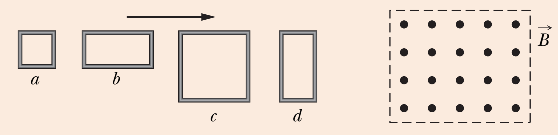
Last time

- motional emf
- induction
- Faraday's law

Warm Up: Loops in B-Fields Question

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field **B** (directed out of the page) at the same constant velocity.

Rank the four loops according to the maximum magnitude of the emf induced as they move into the field, greatest first.



A a, b, c, d

B (b and c), (a and d)

C (c and d), (a and b)

D (a and b), (c and d)

---

[1]Halliday, Resnick, Walker, page

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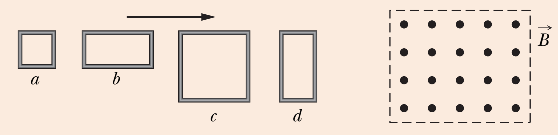

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Overview

- Lenz's law
- applying Faraday's law in problems
- technological applications

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

Faraday's Law for a conducting loop:

$$\mathcal{E}_F = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$$

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$$\mathcal{E}_F = -\frac{d\Phi_B}{dt}$$

Note: If there is a changing flux, there will be an induced emf, however, if $\frac{d\Phi_B}{dt} = 0$ there could still be emf in a wire from other effects. $(\mathcal{E}_{net} = \mathcal{E}_F + \mathcal{E}_{other})$

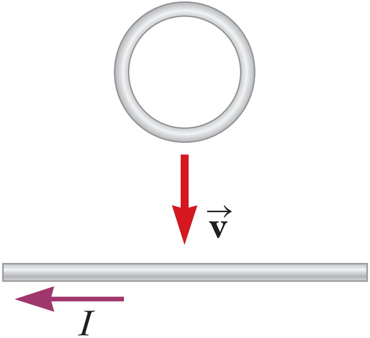
Changing Magnetic Flux

The magnetic flux might change for any of several reasons:

- the magnitude of $\mathbf{B}$ can change with time,
- the area A enclosed by the loop can change with time, or
- the angle θ between the field and the normal to the loop can change with time.

Faraday's and Lenz's Laws

What about this case? We found the current should flow counterclockwise.



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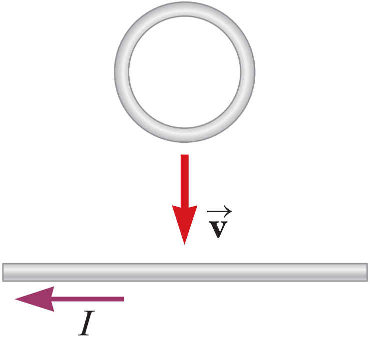


The flux from the wire is into the page and increasing.

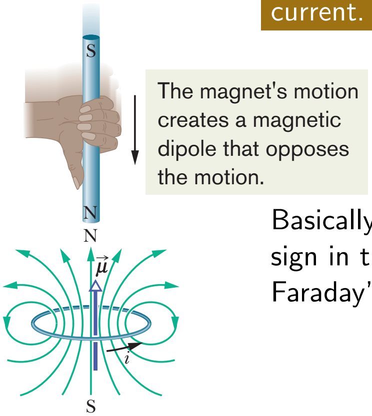
The field from the current in the loop is out of the page.

There is an upward resistive force on the ring. (*cf.* HW3, #3.)

Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



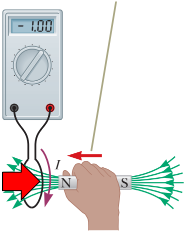
Basically, Lenz's law let's us interpret the minus sign in the equation we write to represent Faraday's Law.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

[1]Figure from Halliday, Resnick, Walker, 9th ed.

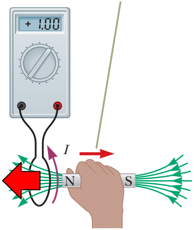
Direction of Current Flow

When the north pole of the magnet is moved towards the loop, the magnetic flux increases.



A current flows clockwise in the loop.
Magnetic moment of current loop point to the right.

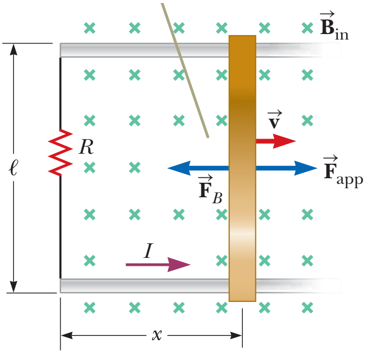
When the north pole of the magnet is moved away from the loop, the magnetic flux decreases.



A current flows counterclockwise in the loop.
Magnetic moment of current loop point to the left.

Faraday's and Lenz's Laws

Consider a conducting bar placed on conducting rails in a magnetic field, with a resistor (outside the field) completing the circuit.



Using the motional emf approach, what is the induced emf across the bar?

Using Faraday's law, what is the induced emf across the bar?

Faraday's and Lenz's Laws

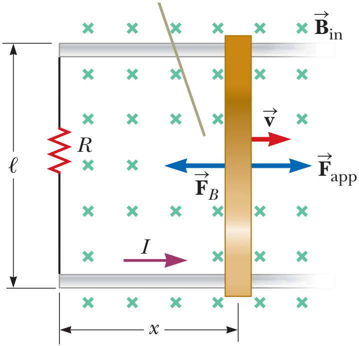
Motional emf:

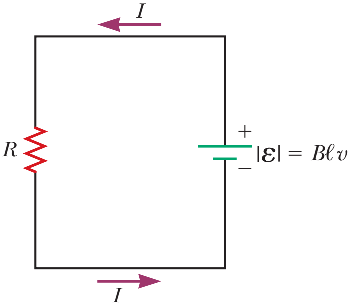
$$\mathcal{E} = Bv\ell$$

upwards

Faraday's Law:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -B\ell\frac{\mathrm{d}x}{\mathrm{d}t} = -Bv\ell$$



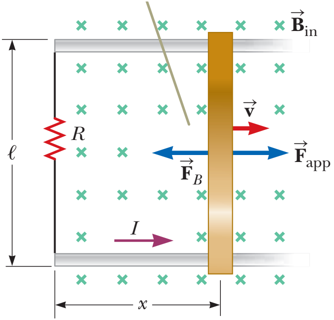


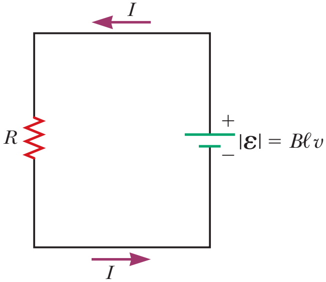
A counterclockwise current begins to flow as the rod moves. (Opposes the field.)

Faraday's and Lenz's Laws

Power is delivered to the resistor as current flows.

That power must come from the force needed to keep the rod in motion.





Prove they are equal.

Faraday's and Lenz's Laws

Power delivered to resistor:

$$P = \frac{\mathcal{E}^2}{R} = \frac{B^2 v^2 \ell^2}{R}$$

Power supplied by applied force needed to keep rod moving with constant velocity v:

$$\mathbf{F}_{\text{net}} = 0 \;\Rightarrow\; \mathbf{F}_{\text{app}} = \mathbf{F}_B$$

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$$\begin{aligned} P &= \mathbf{F}_{\text{app}} \cdot \mathbf{v} \\ &= (I\ell B)v \\ &= \frac{\mathcal{E}}{R} Bv\ell \\ &= \frac{B^2 v^2 \ell^2}{R} \end{aligned}$$

Faraday's and Lenz's Laws

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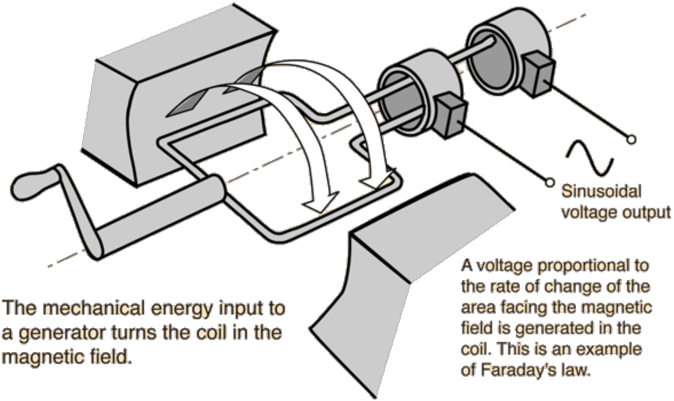
Faraday's and Lenz's Laws

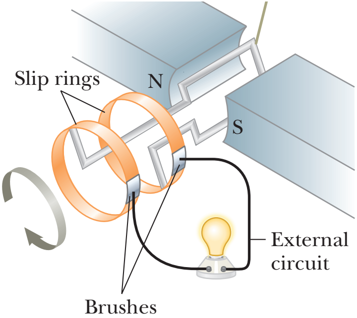
Implication: it is possible to turn mechanical power into electrical power.

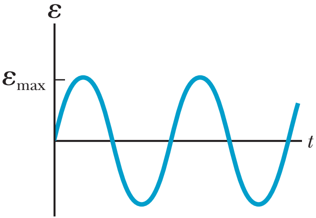
Electric Generators



[1] Figure from hyperphysics.phys-arstr.gsu.edu

Electric Generators





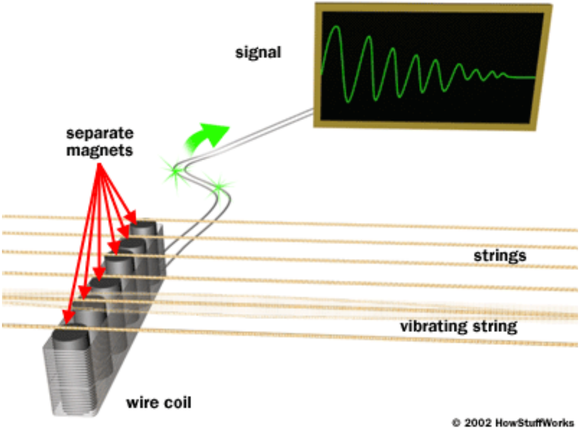
$$\Phi_B = BA\cos(\omega t)$$

$$\mathcal{E} \quad = \quad -N\,\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}} = \mathcal{E}_{\mathrm{max}}\sin(\omega t)$$

where $\mathcal{E}_{\mathrm{max}} = NBA\omega$.

[1] Figure from Serway & Jewett.

Electric Guitar Pickups

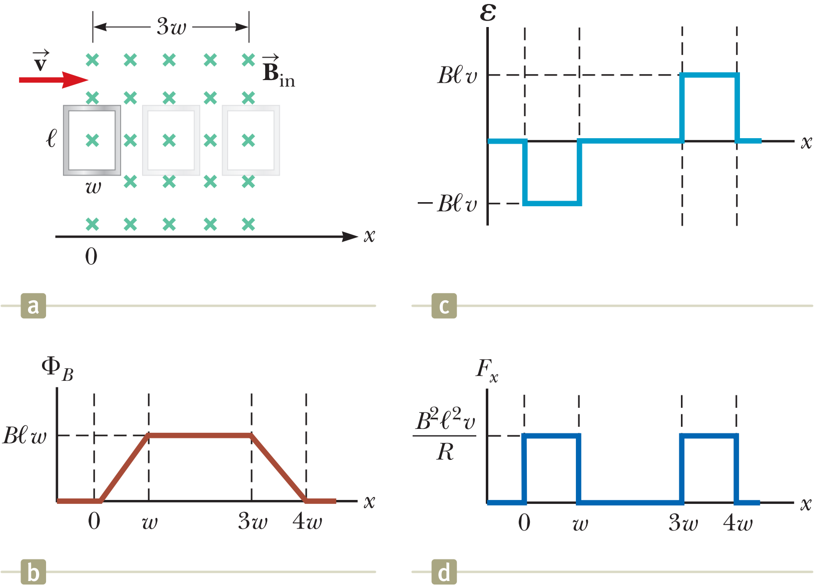


Strings are made of ferrous metal: steel (iron) or nickel, which become magnetized by the permanent magnets.

Plucked strings create a changing magnetic field that produces a current in the pickup coil.

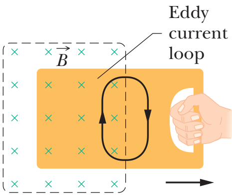
[1]Figure from HowStuffWorks.

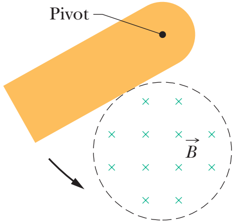
Loop moving into and out of a B-field



Eddy Currents

If the wire is replaced by a solid conducting plate, circulations of current form in the plate.





Since the cross section of the plate is larger than that of a similar wire, the resistance will be low, but the current can be high.

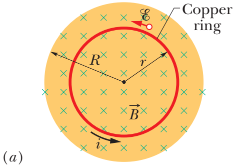
The plate will heat.

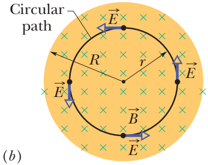
Induced Electric Fields

If moving a conductor in a magnetic field causes a current to flow, if must be because the process has created an electric field across the conductor.

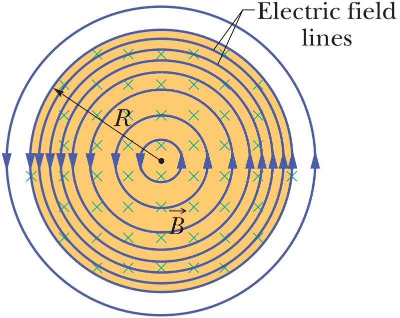
The fact that in a conducting plate circulations of current appear tells us that the electric field lines must also makes these circles.

Another way to cause a current and electric field is to change the flux by increasing or decreasing the magnetic field.





Induced Electric Fields



The circulation E-field occurs whether or not a conductor is present: it is the direct result of the changing magnetic flux.

Faraday's Law of Induction (in words)

A changing magnetic field gives rise to an electric field.

Induced emf and the Electric Field

For a closed path, **s**,

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$

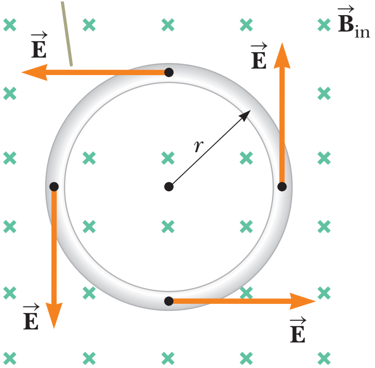
Notice that by definition $\Delta V = -\oint \mathbf{E} \cdot d\mathbf{s} = 0$. Emf does not have this property.

When a charge is moved around a *closed path* in an **electrostatic** electric field ($\mathbf{E} = -\nabla V$) the work done is zero:

$$W_E = -q(\Delta V) = 0$$

Induced emf and the Electric Field

For the induced E-field from a changing magnetic flux, the associated force $\mathbf{F} = q\mathbf{E}$ is **not conservative**.

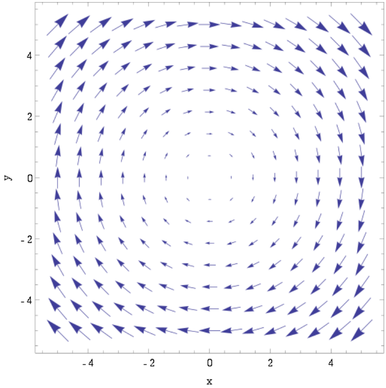


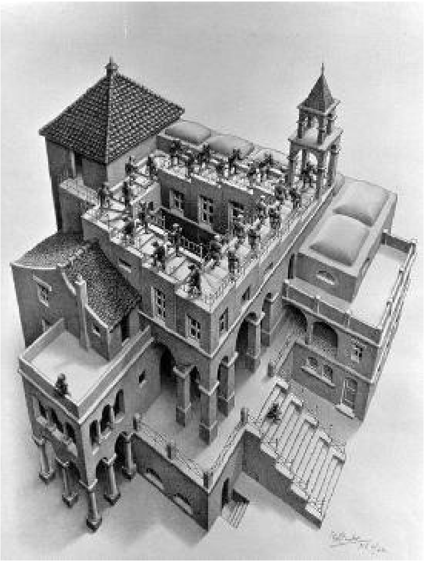
The work done tracing around the grey loop is not zero.

$$W_{\mathsf{E}} \neq 0$$

Non-Conservative Fields

Non-conservative vector fields cannot be represented with a topological map.





[1] Lithograph in the mathematically-inspired impossible reality style, by M.C. Escher.

Induced emf and the Electric Field

For the induced E-field from a changing magnetic flux, the associated force $\mathbf{F} = q\mathbf{E}$ is **not conservative**.

We say the E-field is **nonconservative**.

We now write the electric field in a more general way:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{a}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{a}$$

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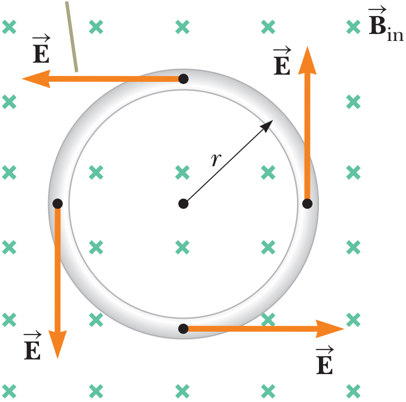
(Note that a transformation:

$$\begin{aligned} V &\rightarrow V - \frac{\partial \lambda}{\partial t} \\ \mathbf{a} &\rightarrow \mathbf{a} + \nabla \lambda \end{aligned}$$

where λ is any twice-differentiable function of position and time, does not change the **E** and **B** fields. "Gauge invariance".)

Induced emf and the Electric Field



We can also write Faraday's Law as:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Summary of Material in Ch 29-31

Chapter 29

Magnetic Fields

- force on a charge from a magnetic field
- motion of a charge in a magnetic field
- particle accelerators
- particle in crossed electric and magnetic fields
    - velocity selector
    - mass spectrometer / discovery of the electron
    - the Hall effect

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  - the Hall effect
- force on a wire carrying current in a B-field
- torque on a wire loop in a B-field
- magnetic moment
- torque and potential energy of a magnetic moment in a B-field

Summary of Material in Ch 29-31

Chapter 30

Sources of the Magnetic field

- B-field around a moving charge
- B-field around a steady current (Biot-Savart law)
- B-field from a long, straight wire
- B-field around a loop of wire

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- B-field around a moving charge
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- B-field from a long, straight wire
- B-field around a loop of wire
- Gauss's law
- Ampère's law
- B-field inside soleniods
- magnetism of bulk matter

Summary of Material in Ch 29-31

Chapter 31

Faraday's Law and Induction

- Motional emf
- Faraday's law
- Lenz's law
- generators and applications
- nonconservative electric field

Summary

- motional emf
- Faraday's law
- Lenz's law
- applications

Next Test this Friday, Mar 9.

Homework

Serway & Jewett:

- PREVIOUS: Ch 31, Obj. Qs: 1, 3, 5, 7; Conc. Qs: 3, 5; Problems: 1, 5, 9, 13, 21, 27, 31, 33
- NEW: Ch 31, Problems: 39, 40, 41, 45, 50 (for each eddy current show is it correct or incorrect?).