Electricity and Magnetism
Eddy Currents
Faraday's Law and Electric Field

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Last time

- Lenz's law
- applying Faraday's law in problems
- technological applications

Overview

- eddy currents
- changing magnetic field and electric field

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

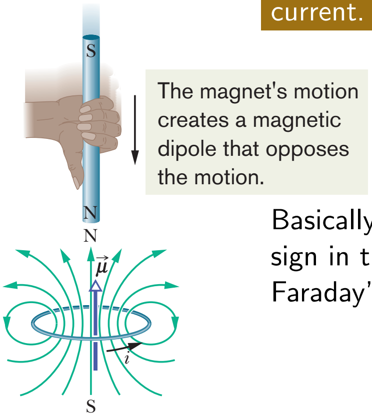
Faraday's Law for a conducting loop:

$$\mathcal{E}_F = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$$

Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



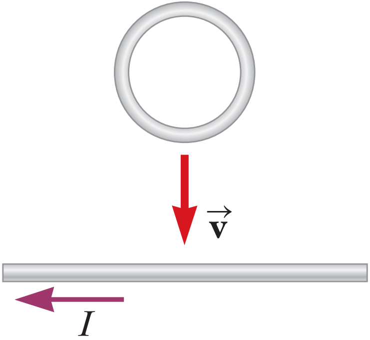
Basically, Lenz's law let's us interpret the minus sign in the equation we write to represent Faraday's Law.

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Faraday's and Lenz's Laws

What about this case? We found the current should flow counterclockwise.



Faraday's and Lenz's Laws

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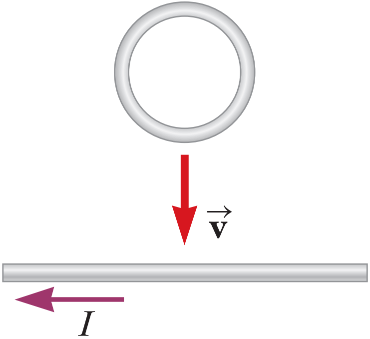
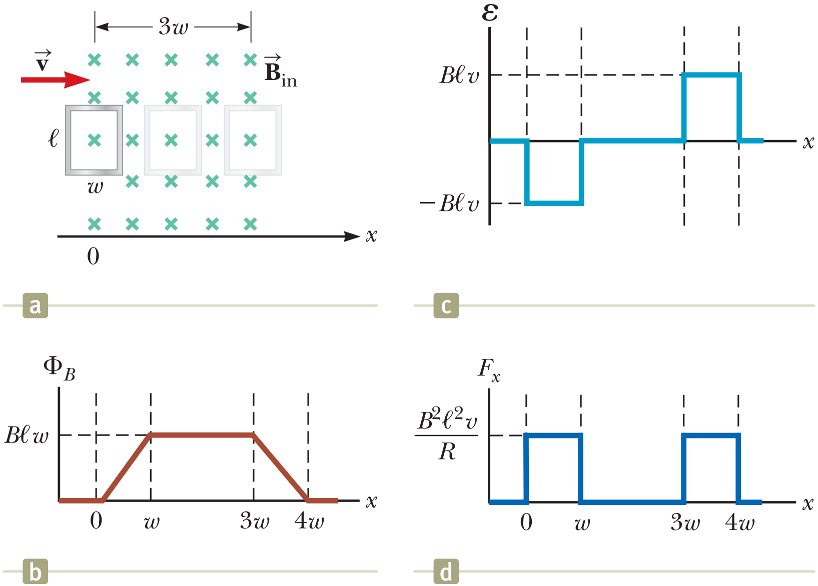


The flux from the wire is into the page and increasing.

The field from the current in the loop is out of the page.

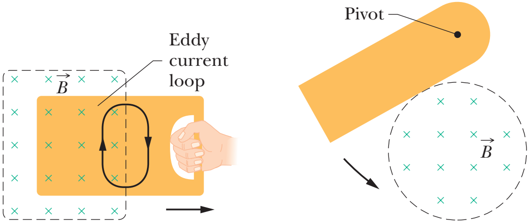
There is an upward resistive force on the ring. (*cf.* HW3, #3.)

Loop moving into and out of a B-field



Eddy Currents

If the wire is replaced by a solid conducting plate, circulations of current form in the plate.



Since the cross section of the plate is larger than that of a similar wire, the resistance will be low, but the current can be high.

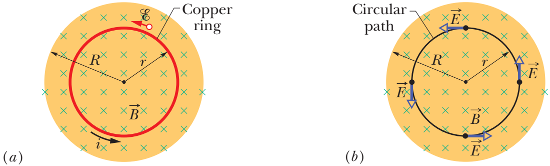
The plate will heat.

Induced Electric Fields

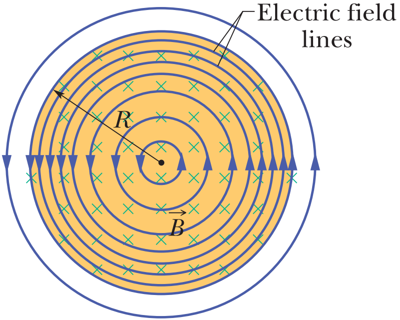
If moving a conductor in a magnetic field causes a current to flow, if must be because the process has created an electric field across the conductor.

The fact that in a conducting plate circulations of current appear tells us that the electric field lines must also makes these circles.

Another way to cause a current and electric field is to change the flux by increasing or decreasing the magnetic field.



Induced Electric Fields



The circulation E-field occurs whether or not a conductor is present: it is the direct result of the changing magnetic flux.

Faraday's Law of Induction (in words)

A changing magnetic field gives rise to an electric field.

Induced emf and the Electric Field

For a closed path, $\mathbf{s}$,

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$

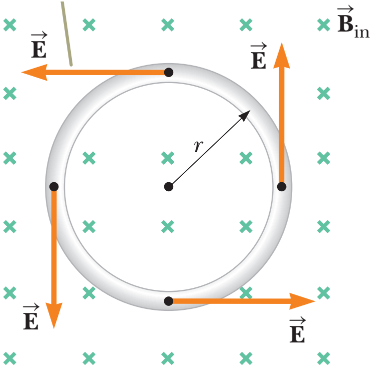
Notice that by definition $\Delta V = -\oint \mathbf{E} \cdot d\mathbf{s} = 0$. Emf does not have this property.

When a charge is moved around a *closed path* in an **electrostatic** electric field ($\mathbf{E} = -\boldsymbol{\nabla} V$) the work done is zero:

$$W_{\mathrm{E}} = -q(\Delta V) = 0$$

Induced emf and the Electric Field

For the induced E-field from a changing magnetic flux, the associated force $\mathbf{F} = q\mathbf{E}$ is **not conservative**.

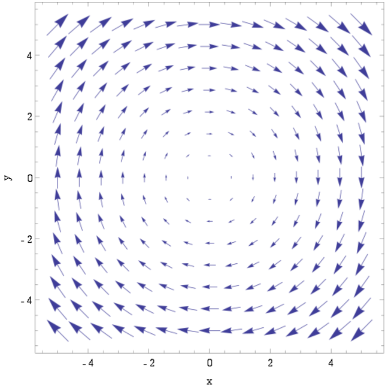


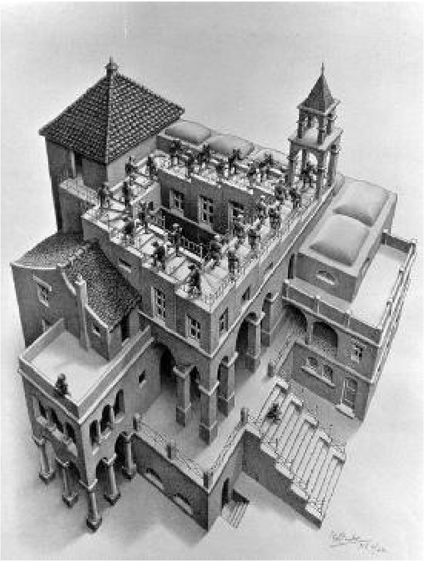
The work done tracing around the grey loop is not zero.

$$W_{\mathsf{E}} \neq 0$$

Non-Conservative Fields

Non-conservative vector fields cannot be represented with a topological map.





[1]Lithograph in the mathematically-inspired impossible reality style, by M.C. Escher.

Induced emf and the Electric Field

For the induced E-field from a changing magnetic flux, the associated force $\mathbf{F} = q\mathbf{E}$ is **not conservative**.

We say the E-field is **nonconservative**.

We now write the electric field in a more general way:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{a}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{a}$$

Induced emf and the Electric Field

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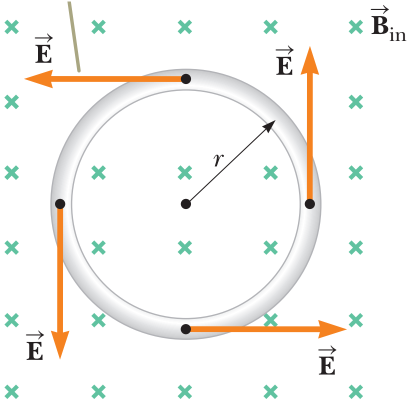
(Note that a transformation:

$$\begin{aligned} V &\rightarrow V - \frac{\partial \lambda}{\partial t} \\ \mathbf{a} &\rightarrow \mathbf{a} + \nabla \lambda \end{aligned}$$

where λ is any twice-differentiable function of position and time, does not change the $\mathbf{E}$ and $\mathbf{B}$ fields. "Gauge invariance".)

Induced emf and the Electric Field



We can also write Faraday's Law as:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Summary of Material in Ch 29-31

Chapter 29

Magnetic Fields

- force on a charge from a magnetic field
- motion of a charge in a magnetic field
- particle accelerators
- particle in crossed electric and magnetic fields
    - velocity selector
    - mass spectrometer / discovery of the electron
    - the Hall effect

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  - the Hall effect
- force on a wire carrying current in a B-field
- torque on a wire loop in a B-field
- magnetic moment
- torque and potential energy of a magnetic moment in a B-field

Summary of Material in Ch 29-31

Chapter 30

Sources of the Magnetic field

- B-field around a moving charge
- B-field around a steady current (Biot-Savart law)
- B-field from a long, straight wire
- B-field around a loop of wire

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- B-field around a moving charge
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- B-field from a long, straight wire
- B-field around a loop of wire
- Gauss's law
- Ampère's law
- B-field inside soleniods
- magnetism of bulk matter

Summary of Material in Ch 29-31

Chapter 31

Faraday's Law and Induction

- Motional emf
- Faraday's law
- Lenz's law
- generators and applications
- nonconservative electric field

Review

Your questions.

Summary

- motional emf
- Faraday's law
- Lenz's law
- applications

Next Test this Friday, Mar 9.

Homework

- study

Serway & Jewett:

- PREVIOUS: Ch 31, Obj. Qs: 1, 3, 5, 7; Conc. Qs: 3, 5; Problems: 1, 5, 9, 13, 21, 27, 31, 33
- PREVIOUS: Ch 31, Problems: 39, 40, 41, 45, 50 (for each eddy current show is it correct or incorrect?).