Electricity and Magnetism
Relativity and the Magnetic Field

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Overview

- questions about the magnetic field
- reference frames
- a “preferred frame” for the laws of EM?
- special relativity and frames
- field transformations
- recovering the Biot-Savart law

Magnetic Force

Why is the magnetic force on a particle at right angles to the B-field?

Why does the force depend on the velocity?

Why is there no magnetic force on a charged particle at rest?

Why does a moving charge create a magnetic field, but a stationary one does not?

Magnetic Field

Why does the magnetic field curl around a current, while the electric field points out from charges?

Why does the Biot-Savart law look so similar to the expression for electric field from Coulomb's law?

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2} \qquad \mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

Why are electricity and magnetism covered in the same course? How are they related?

Frames of Reference

A Principle: The laws of physics should be the same in all inertial frames.

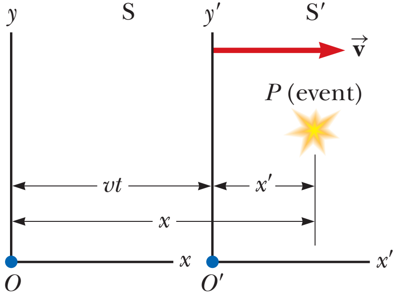
This was true of Newton's Laws, and is an attractive idea, because otherwise we must identify a "preferred frame" in which the laws hold.

It would seem this should be true for the laws of electricity and magnetism (Maxwell's laws) also.

Frames of Reference

How do we relate coordinates in different frames of reference?

Two frames S and S'



Galilean transformations:

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

EM and Frames: Illustrative Example

Lorentz force:

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Imagine a charged particle moving at velocity $\mathbf{v} = v\,\mathbf{i}$ in a uniform magnetic field $\mathbf{B} = B\,\mathbf{j}$ of infinite extent.

The force on the particle: $\mathbf{F} = q\mathbf{v} \times \mathbf{B} = qvB\,\mathbf{k}$

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Is there a force on the particle or not?

Maxwell's Laws in Different Frames

The laws of electromagnetism (Maxwell's laws) are **not** constant in different coordinate systems related by Galilean transformations.

By 1890 it was assumed that there was a "preferred frame" in which Maxwell's laws hold: the ether frame.

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Einstein had a radical suggestion: Maxwell's laws DO hold in all inertial frames, but inertial frames are **not** related by Galilean transformations.

Instead the speed of light c is constant in all frames:

$$c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = \text{const.}$$

Relativistic (Lorentz) Transformations

If c is constant in all frames, position and time coordinates become 'mixed'.

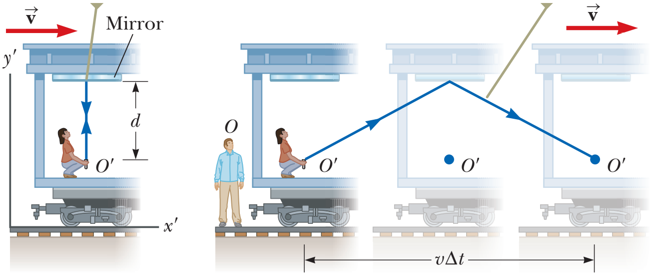
$$\begin{aligned} x' &= \gamma\,(x - v\,t) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{v}{c^2}\,x\right) \end{aligned}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

(Note: for $v = 0$, $\gamma = 1$, and as $v \rightarrow c$, $\gamma \rightarrow \infty$.)

Relativistic Transformations: Time Dilation



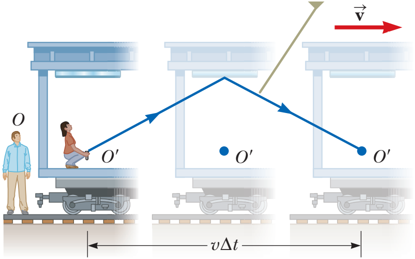
The observer in O' sees a light pulse going from the floor to the ceiling of the car as taking time:

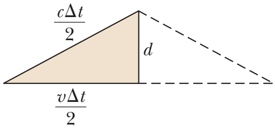
$$\Delta t' = \frac{2d}{c}$$

Because it travels a distance:

$$2d = c\Delta t'$$

Relativistic Transformations: Time Dilation





The observe in O sees a pulse going from the floor to the ceiling of the car as taking time:

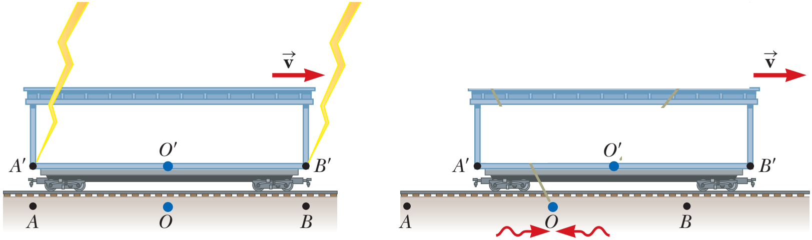
$$\Delta t = \frac{2\sqrt{d^2 + (v\Delta t/2)^2}}{c}$$

$$\Delta t = \frac{1}{\sqrt{1 - v^2/c^2}}\Delta t' = \gamma \Delta t'$$

So $\Delta t > \Delta t'$! (“Time dilation”)

Relativistic Transformations: Length Contraction

How long does the rail car appear to observer O?



Observer in O sees both ends of the car pass posts separated by a distance Δx at the same time.

The car length is therefore Δx.

An observer in O' sees the front end of the car pass the first post **before** the back end of the car passes the second post.

Meaning: $\Delta x < \Delta x'$! ("Length contraction")

Relativistic (Lorentz) Transformations

Again, generally:

$$
\begin{aligned}
x' &= \gamma\,(x - v\,t) \\
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t' &= \gamma\left(t - \frac{v}{c^2}\,x\right)
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where

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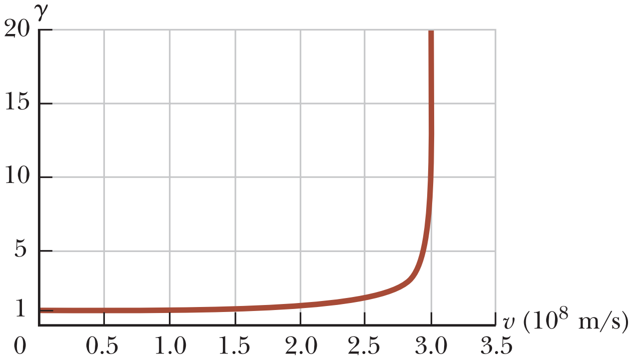
Relativistic Transformations

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

For everything in this course, you can assume $\gamma \approx 1$.



[1]Graph from Serway & Jewett, page 1201.

Approximate Transformations

For $\gamma \approx 1$:

$$\begin{aligned} x' &\approx (x - v\,t) \\ y' &= y \\ z' &= z \\ t' &\approx \left(t - \frac{v}{c^2}\,x\right) \end{aligned}$$

Field Transformations

$$
\begin{aligned}
E'_x &= E_x \\
E'_y &= \gamma\,(E_y + v\,B_z) \\
E'_z &= \gamma\,(E_z + v\,B_y)
\end{aligned}
$$

$$
\begin{aligned}
B'_x &= B_x \\
B'_y &= \gamma\left(B_y - \frac{v}{c^2}\,E_z\right) \\
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\end{aligned}
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The electric and magnetic fields become mixed!

Field Transformations

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\end{aligned}
$$

for $\gamma \approx 1$, ie. $v << c$.

What does it mean?

Different observers moving with different velocities do not agree about what kind of EM-field they are seeing.

A field that looks like a pure magnetic field in one frame looks like a mixture of E- and B-fields in other frames.

A field that looks like a pure electric field in one frame looks like a mixture of E- and B-fields in other frames.

Previous Example

Lorentz force:

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The force on the particle: $\mathbf{F} = qvB\,\mathbf{k}$

Previous Example

The force on the particle: $\mathbf{F} = qvB\,\mathbf{k}$

Suppose we move to the frame where the particle is at rest. How does the B-field appear in this frame?

It is not a pure magnetic field in this frame. It has an electric-field component!

$E_z' = \gamma v B_y \approx vB.$

Therefore: $\mathbf{F}' = q(\mathbf{E}' + \mathbf{0} \times \mathbf{B}') = q\mathbf{E}' = qvB\,\mathbf{k}.$

The force is the same in both frames!

Maxwell's Laws in Different Frames

- Maxwell's laws do not change under Lorentz (Relativistic) transformations.
- Relativity is now a well-tested theory, with Lorentz transforms giving the relation between different inertial frames.
- The laws of EM hold in all inertial frames.
- A field that looks like a pure B-field (or pure E-field) in one frame looks like a mixture of E- and B-fields in other frames.

Implications

Every charge has a surrounding electric field.

In a frame where the charge is moving, the charge must appear to have a magnetic field as well.

$\rightarrow$ the Biot-Savart law.

Biot-Savart law

Charge q sits at the origin of frame S.

At a point, P, on the y-axis at $y = -r$, the field is pure electric:

$$E_y = -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

All other components are zero: $E_x = E_z = B_x = B_y = B_z = 0$

Biot-Savart law

Consider a frame S' moving in the positive x-direction of frame S.

At a point, P' on the y-axis $y = -r$, the field is mixed:

$$E'_y = -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\begin{aligned} B'_z &= B_z - \frac{v}{c^2}E_y \\ &= 0 - \frac{v}{c^2}\left(-\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right) \\ &= \frac{1}{4\pi\epsilon_0 c^2}\frac{q\,v}{r^2} \end{aligned}$$

Doesn't look like the Biot-Savart Law yet... What is $(\epsilon_0\mu_0)^{-1/2}$?

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$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

Biot-Savart law

$B'_z = \frac{1}{4\pi\epsilon_0 c^2}\frac{q\,v}{r^2}, \quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

So,

$$B'_z = \frac{\mu_0}{4\pi}\frac{q\,v}{r^2}$$

This is at right angles to $\mathbf{v} = -v\mathbf{i}'$ of the particle according to S', and at right angles to $\mathbf{r}$.

In general:

$$\mathbf{B}' = \frac{\mu_0}{4\pi}\frac{q\,\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$$

Another example

Consider a charge moving parallel to a current carrying wire.
Assume the wire is neutrally charged overall.

Does it experience a force?

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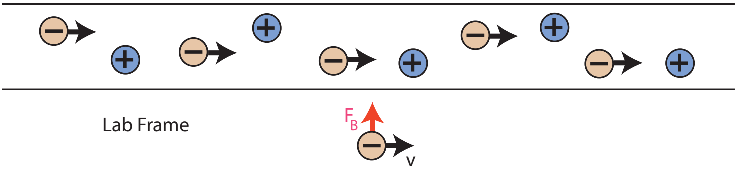
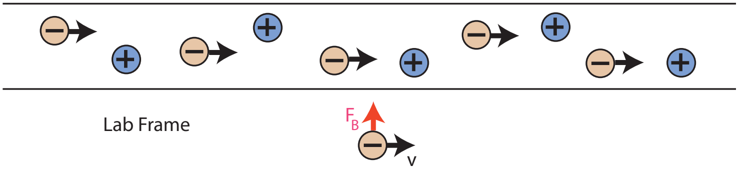


Yes, the wire has a magnetic field, and the charge is moving in the field.

Another example

To make the situation concrete: let the charge be an electron, moving alongside the wire with a velocity equal to the drift velocity of the electrons making up the wire's current.

In that case, the force on the electron will be towards the wire.



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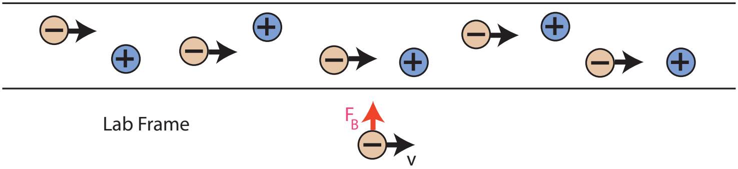
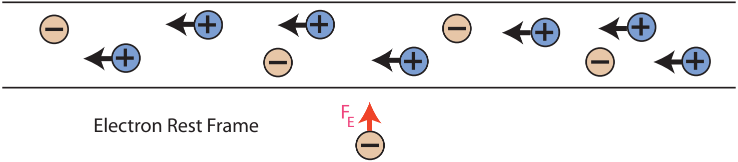
What would this look like in the frame of the electron?





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What would this look like in the frame of the electron?

The electron would “see” an electric field radiating out from the wire that would attract it to the wire. Why?

In the rest frame of the wire, the conduction electrons in the wire are also stationary, but the ions making up the rest of the wire are moving at the drift velocity.

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This moving wire of ions is just slightly length-contracted. **It’s apparent charge density increases.**

The wire appears to have a net positive charge in the electron’s rest frame. The electron is attracted to it.

Another example

We can either understand the electron's attraction to the wire as a magnetic **or** an electrostatic interaction depending on the frame!

Summary

- transformations between frames
- Maxwell's laws transform according to relativistic transformations.

Homework Serway & Jewett:

- read chapter 32