Electricity and Magnetism
Inductance

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Last time

- relativity and fields

Overview

- inductors and inductance
- resistor-inductor circuits

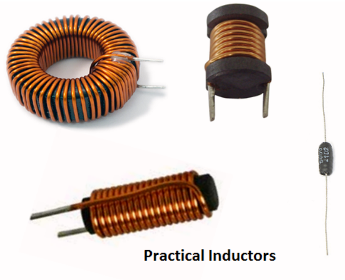
Inductors

A **capacitor** is a device that stores an electric field as a component of a circuit.

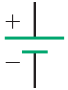
inductor

a device that stores a magnetic field in a circuit

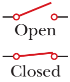
It is typically a coil of wire.



Circuit component symbols

battery V 

capacitor C 

switch S 

resistor R 

inductor L 

Inductance

Just like capacitors have a capacitance that depends on the geometry of the capacitor, inductors have an inductance that depends on their geometry.

Capacitance is defined as being the constant of proportionality relating the charge on the plates to the potential difference across the plates

$$Q = C\,(\Delta V)\,.$$

Inductance is defined in a similar way.

Inductance

The magnetic flux linkage is $N\Phi_B$.

Inductance, L

the constant of proportionality relating the *magnetic flux linkage* ($N\Phi_B$) to the current, I:

$$N\Phi_B = L\,I \qquad ; \qquad L = \frac{N\Phi_B}{I}$$

Φ_B is the magnetic flux through the coil, and I is the current in the coil.

Units: henries, H.

1 henry = 1 H = 1 T m^2 / A

Inductance

Just like capacitors have a capacitance that depends on the geometry of the capacitor, inductors have an inductance that depends on their structure.

For a solenoid inductor:

$$L = \mu_0 n^2 A \ell$$

where n is the number of turns per unit length, A is the cross sectional area, and ℓ is the length of the inductor.

(Doesn't depend on current or flux, only geometry of device.)

Value of μ_0: New units

The magnetic permeability of free space μ_0 is a constant.

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m / A}$$

It can also be written in terms of henries:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H / m}$$

(Remember, $1 \text{ H} = 1 \text{ T m}^2 / \text{A}$)

Inductance of Solenoid Inductors

Suppose now that the only source of magnetic flux in the solenoid is the flux produced by a current in the wire.

Then the field produced within the solenoid is:

$$B = \mu_0 I n$$

where n is the number of turns per unit length.

That means the flux will be:

$$\Phi_B = BA\cos(0^\circ) = BA = \mu_0 I n A$$

where A is the cross sectional area of the solenoid.

Inductance of Solenoid Inductors

$$L = \frac{N\Phi_B}{I}$$

Replacing $N = n\ell$, $\Phi_B = \mu_0 I n A$:

$$L = \frac{n\ell(\mu_0 I n)A}{I}$$

So we confirm our expression for a solenoid inductor:

$$L = \mu_0 n^2 A\ell$$

Induction from an external flux vs Self-Induction

Previously, we considered the effect of a changing magnetic field from some **external source** causing and emf and current flow in a wire loop.

However, a changing current **in the solenoid itself** can be causing the changing field that affects the flow of the current.

[1] In this textbook, and most sources, L is the self-inductance.

Induction from an external flux vs Self-Induction

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However, a changing current **in the solenoid itself** can be causing the changing field that affects the flow of the current.

This inductance[1], L, is the **self-inductance** of the coil.

$$N\Phi_B = L\,I$$

We are assuming the flux Φ_B is entirely due to the B-field resulting from the current in the solenoid and there is no external source of magnetic field adding to the flux.

[1] In this textbook, and most sources, L is the self-inductance.

Self-Induction

When the current in the solenoid circuit is changing there is a (self-) induced emf in the coil.

From Faraday's Law, we have

$$\mathcal{E} = -\frac{\mathrm{d}(\mathrm{N}\Phi_{\mathrm{B}})}{\mathrm{dt}}$$

Since L is a constant for a particular inductor and $Li = N\Phi_B$,

$$\mathcal{E}_L = -L\,\frac{\mathrm{di}}{\mathrm{dt}}$$

$\mathcal{E}_L$ is the self-induced emf.

The emf opposes the change in current.

Inductors vs. Resistors

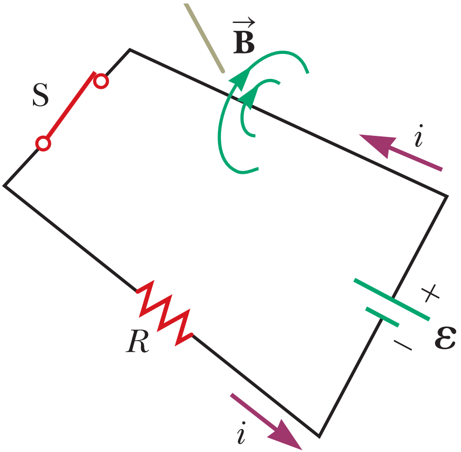
Inductors are a bit similar to resistors.

Resistors resist the flow of current.

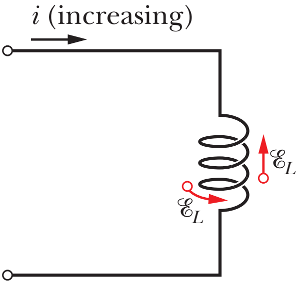
Inductors resist any **change** in current.

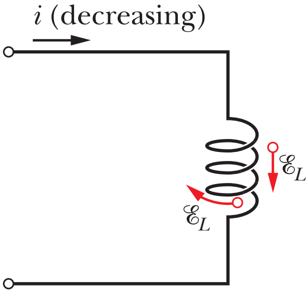
If the current is high and lowered, the emf acts to keep the current flowing. If the current is low and increased, the emf acts to resist the increase.

Self-Induction



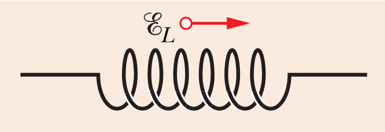
Self-Induction





Self-inductance question

The figure shows an emf $\mathcal{E}_L$ induced in a coil.



Which of the following can describe the current through the coil:

(A) constant and rightward

(B) increasing and rightward

(C) decreasing and rightward

(D) decreasing and leftward

Self-inductance question

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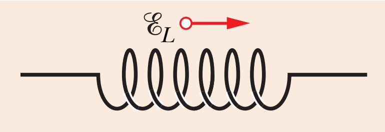

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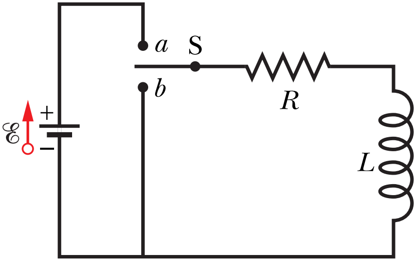
RL Circuits

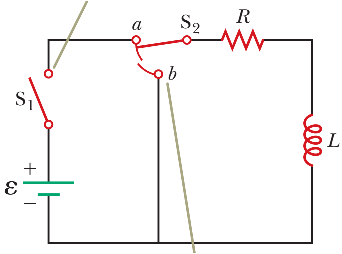
Just like circuits with capacitors and resistor, circuits with inductors and resistors have time-dependent behavior.



Initially, an inductor acts to oppose changes in the current through it.

A long time later, the current stabilizes and it acts like ordinary connecting wire.

RL Circuits Question



Consider the circuit shown with S_1 open and S_2 at position a. Switch S_1 is now thrown closed.

At the instant it is closed, across which circuit element is the voltage equal to the emf of the battery?

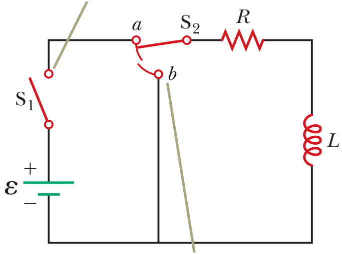
(A) the resistor

(B) the inductor

(C) both each of the inductor and the resistor

(D) neither

RL Circuits Question



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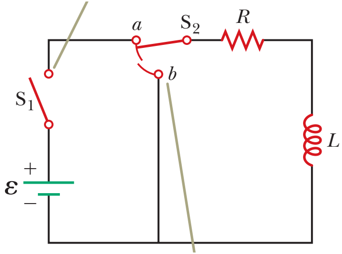

Consider the circuit shown with S_1 open and S_2 at position a. Switch S_1 is now thrown closed.

After a very long time, across which circuit element is the voltage equal to the emf of the battery?

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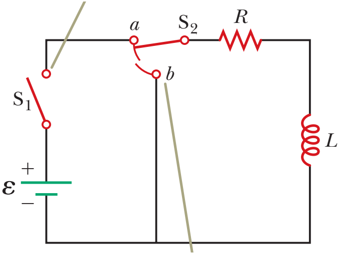
RL Circuits Question



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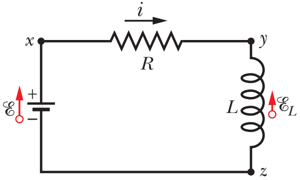
RL Circuits



Loop rule:

$$\mathcal{E} - \mathcal{E}_L - iR = 0$$

$$\mathcal{E} - L\,\frac{\mathrm{di}}{\mathrm{dt}} - iR = 0$$

This is a differential equation. Solution:

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \quad ; \qquad \tau_L = \frac{L}{R}$$

Current varies with time

$$\mathcal{E} - L\,\frac{\mathrm{di}}{\mathrm{dt}} - iR = 0$$

Rearranging:

$$\begin{aligned}
\frac{\mathrm{di}}{\mathrm{dt}} &= \frac{\mathcal{E}}{L} - i\frac{R}{L} \\
\frac{\mathrm{di}}{\mathrm{dt}} &= \frac{R}{L}\left(\frac{\mathcal{E}}{R} - i\right) \\
\int \frac{1}{\mathcal{E}/R - i}\,\mathrm{di} &= \int \frac{R}{L}\,\mathrm{dt}
\end{aligned}$$

The limits of our integral will be determined by the initial conditions for the situation we are considering.

RL Circuits: current rising

When charging an initially uncharged capacitor: $i = 0$ at $t = 0$

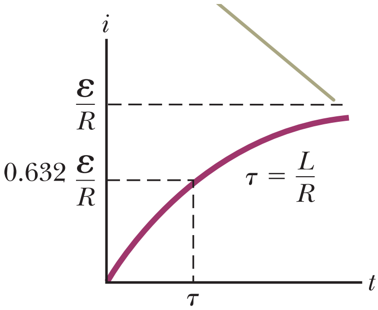
$$\begin{aligned}
\int_0^i \frac{1}{\mathcal{E}/R - i'}\, \mathrm{di'} &= \int_0^t \frac{R}{L}\, \mathrm{dt'} \\
-\ln(\mathcal{E}/R - i) + \ln(\mathcal{E}/R - 0) &= \frac{R}{L} t \\
\ln\left(\frac{\mathcal{E}/R}{\mathcal{E}/R - i}\right) &= \frac{Rt}{L} \\
\frac{\mathcal{E}/R}{\mathcal{E}/R - i} &= e^{Rt/L}
\end{aligned}$$

The solution is:

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

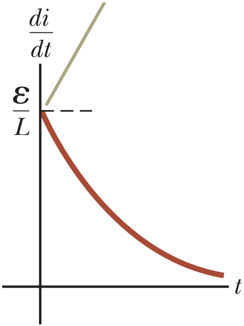
RL Circuits: current rising

Current in loop:



$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

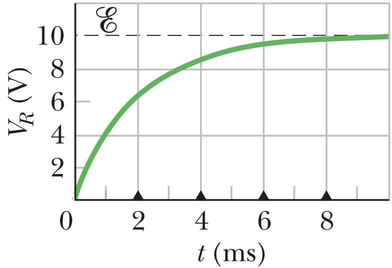
Derivative of current:



$$\frac{\mathrm{di}}{\mathrm{dt}} = \frac{\mathcal{E}}{L}e^{-Rt/L}$$

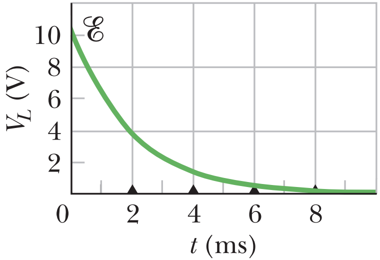
RL Circuits: current rising

Potential drop across resistor:



$$V_R = iR$$

emf across inductor:



$$|\mathcal{E}_L(t)| = L\,\frac{\mathrm{di}}{\mathrm{dt}}$$

RL Circuits: Time Constant

$$\tau_L = L/R$$

τ_L is called the **time constant** of the circuit.

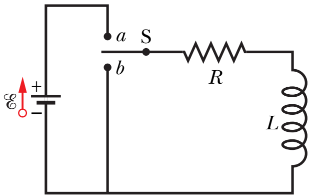
(Notice that it is defined differently for RL circuits as opposed to RC circuits.)

This gives the time for the current to reach $(1 - e^{-1}) = 63.2\%$ of its final value.

Alternatively, it is the time for the potential drop across the inductor to fall to $1/e$ of its initial value.

It is useful for comparing the "relaxation time" of different RL-circuits.

RL Circuits



Battery switched out - switch to *b*.

Loop rule:

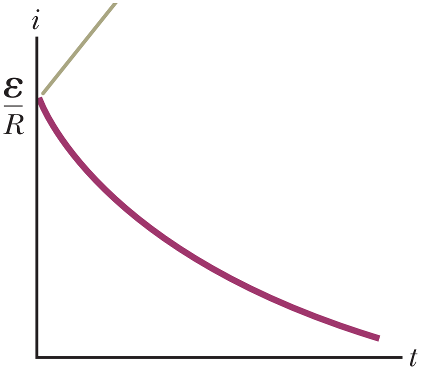
$$\mathcal{E}_L - iR = 0$$

$$-L\,\frac{\mathrm{di}}{\mathrm{dt}}\,-iR = 0$$

Solution:

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} \quad ; \qquad \tau_L = \frac{L}{R}$$

RL Circuits: current falling



$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} \quad ; \qquad \tau_L = \frac{L}{R}$$

Summary

- inductance
- RL circuits

Homework

Serway & Jewett:

- Ch 32, onward from page 988. Obj. Qs: 3, 5; Conc. Qs.: 5; Probs: 1, 3, 9