Electricity and Magnetism
Energy of the Magnetic Field
Mutual Inductance

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Last time

- inductors
- resistor-inductor circuits

Overview

- wrap up resistor-inductor circuits
- energy stored in an inductor
- coaxial inductor
- mutual inductance

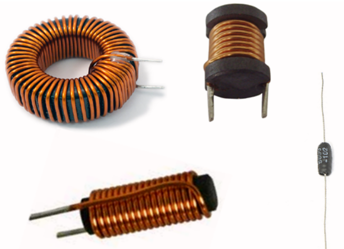
Reminder: Inductors

A **capacitor** is a device that stores an electric field as a component of a circuit.

inductor

a device that stores a magnetic field in a circuit

It is typically a coil of wire.



Practical Inductors

Reminder: Inductance

inductance

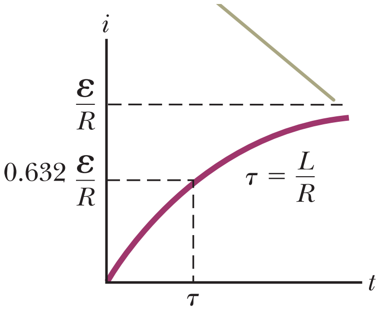
the constant of proportionality relating the *magnetic flux linkage* ($N\Phi_B$) to the current:

$$N\Phi_B = L\,I \qquad ; \qquad L = \frac{N\Phi_B}{I}$$

Φ_B is the magnetic flux through the coil, and I is the current in the coil.

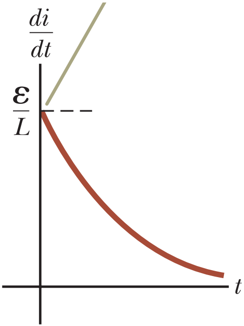
RL Circuits: current rising

Current in loop:



$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

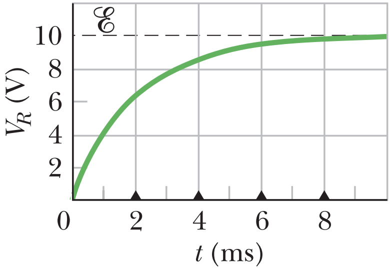
Derivative of current:



$$\frac{\mathrm{di}}{\mathrm{dt}} = \frac{\mathcal{E}}{L} e^{-Rt/L}$$

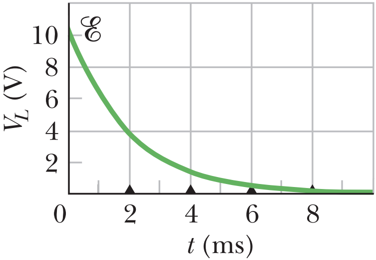
RL Circuits: current rising

Potential drop across resistor:



$$V_R = iR$$

emf across inductor:



$$|\mathcal{E}_L(t)| = L\,\frac{\mathrm{di}}{\mathrm{dt}}$$

RL Circuits: Time Constant

$$\tau_L = L/R$$

τ_L is called the **time constant** of the circuit.

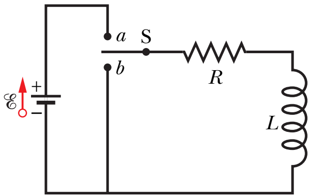
(Notice that it is defined differently for RL circuits as opposed to RC circuits.)

This gives the time for the current to reach $(1 - e^{-1}) = 63.2\%$ of its final value.

Alternatively, it is the time for the potential drop across the inductor to fall to $1/e$ of its initial value.

It is useful for comparing the "relaxation time" of different RL-circuits.

RL Circuits: current falling



Battery switched out - switch to *b*.

Loop rule:

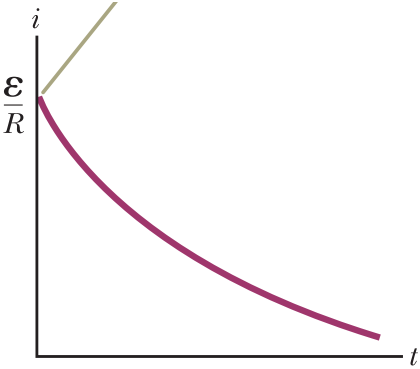
$$-\mathcal{E}_L - iR = 0$$

$$-L\,\frac{\mathrm{di}}{\mathrm{dt}} - iR = 0$$

Solution:

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} \quad ; \qquad \tau_L = \frac{L}{R}$$

RL Circuits: current falling



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Energy Stored in a Magnetic Field in an Inductor

Power delivered to inductor:

$$P = I\,|\mathcal{E}_L|$$

Energy stored in inductor carrying current i:

$$U_B = \int P\,\mathrm{dt} = \int_0^i i'\left(L\,\frac{\mathrm{di'}}{\mathrm{dt}}\right)\mathrm{dt}$$

$$U_B = \frac{1}{2}Li^2$$

Compare with $U_E = \frac{Q^2}{2C}$ (or $U_E = \frac{1}{2}CV^2$) for the energy stored in a capacitor.

Energy Density of a Magnetic Field

The **energy density** of a magnetic field is the energy stored per unit volume.

$$u_B = \frac{U_B}{A\ell}$$

where A is the cross-sectional area of a solenoid and ℓ is the length.

$$u_B = \frac{L}{A\ell}\frac{i^2}{2}$$

Remember, $L = \mu_0 n^2 A\ell$ and $B = \mu_0 i n$.

Energy Density of a Magnetic Field

$$u_B = \frac{B^2}{2\mu_0}$$

Compare with $u_E = \frac{1}{2}\epsilon_0 E^2$ for electric fields.

Energy Density of a Magnetic Field Question

Which of the following adjustments increases the energy density in a solenoid?

(A) increasing the number of turns per unit length on the solenoid
(B) increasing the cross-sectional area of the solenoid
(C) increasing the current in the solenoid
(D) two of the above

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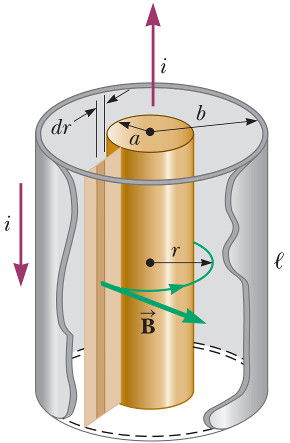
Coaxial Cable Inductor (Ex 32.4)

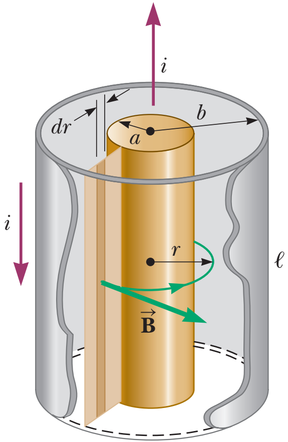
Model a long coaxial cable as a thin, cylindrical conducting shell of radius b concentric with a solid cylinder of radius a. The conductors carry the same current i in opposite directions.



Calculate the inductance L of a length ℓ, of this cable.

Coaxial Cable Inductor (Ex 32.4)

Need to relate the flux linkage, $N\Phi_B$, to the current. ($N = 1$.)



$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$, and area of rectangle is A:

$$dA = \ell \, dr$$

Coaxial Cable Inductor (Ex 32.4)

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$$\begin{aligned}
\Phi_B &= \int B \, \mathrm{dA} \\
&= \int_a^b \frac{\mu_0 i}{2\pi r} \, \ell \, \mathrm{dr} \\
&= \frac{\mu_0 i \ell}{2\pi} \int_a^b \frac{1}{r} \, \mathrm{dr} \\
&= \frac{\mu_0 i \ell}{2\pi} \ln\left(\frac{b}{a}\right)
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\end{aligned}
$$

Inductance of length ℓ:

$$
L = \frac{\Phi_B}{i} = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right)
$$

Mutual Inductance

An inductor can have an induced emf from its own changing magnetic field.

It also can have an emf from an external changing field.

That external changing field could be **another inductor**.

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For self-inductance on a coil labeled 1:

$$N_1 \Phi_{B,1} = L_1 i_1$$

For mutual inductance:

$$N_1 \Phi_{B,2\to 1} = M_{21} i_2$$

The flux is in coil 1, but the current that causes the flux is in coil 2.

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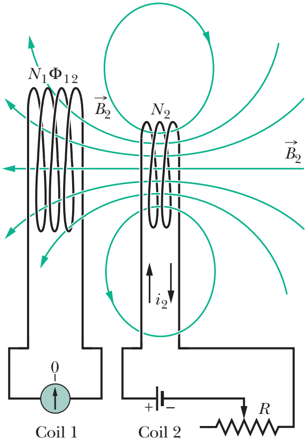
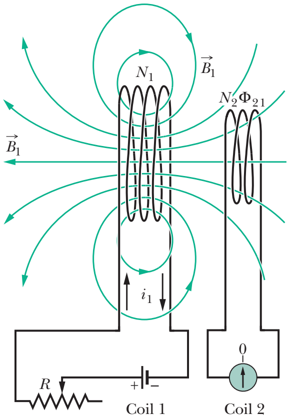


Mutual Inductance

mutual inductance

$$M = \frac{N_1 \Phi_{B,2\to1}}{i_2} = \frac{N_2 \Phi_{B,1\to2}}{i_1}$$



Mutual Inductance

$$N_1 \Phi_{B,2\to1} = M_{21} i_2$$

Considering the rate of change of both sides with time, and using Faraday's Law $\mathcal{E} = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$,

$$\mathcal{E}_1 = -M \ \frac{\mathrm{di}_2}{\mathrm{dt}}$$

and

$$\mathcal{E}_2 = -M \ \frac{\mathrm{di}_1}{\mathrm{dt}}$$

A change of current in one coil causes a magnetic flux in the other.

Summary

- energy stored in the magnetic field
- coaxial inductor
- mutual inductance
- applications of mutual inductance

4th Collected Homework due Thursday Mar 22.

Quiz this Friday.

Homework

Serway & Jewett:

- Ch 32, onward from page 988. Obj. Qs: 1; Conc. Qs.: 7; Probs: 11, 15, 19, 33, 41, 43