Electricity and Magnetism
Mutual Inductance
Oscillations in Circuits

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De Anza College

Mar 15, 2018

Last time

- energy stored in an inductor
- coaxial inductor
- mutual inductance

Overview

- mutual inductance applications
- LC circuits
- RLC circuits

Mutual Inductance

An inductor can have an induced emf from its own changing magnetic field.

It also can have an emf from an external changing field.

That external changing field could be **another inductor**.

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For self-inductance on a coil labeled 1:

$$N_1 \Phi_{B,1} = L_1 i_1$$

For mutual inductance:

$$N_1 \Phi_{B,2\to 1} = M_{21} i_2$$

The flux is in coil 1, but the current that causes the flux is in coil 2.

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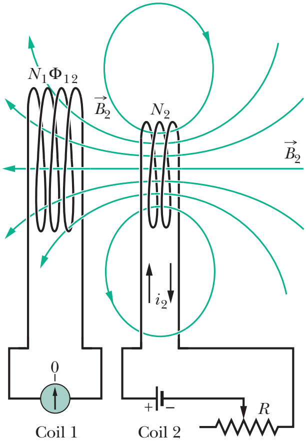
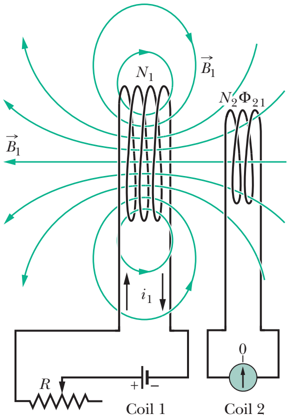


Mutual Inductance

mutual inductance

$$M = \frac{N_1 \Phi_{B,2\to1}}{i_2} = \frac{N_2 \Phi_{B,1\to2}}{i_1}$$



Mutual Inductance

$$N_1 \Phi_{B,2\to 1} = M_{21} i_2$$

Considering the rate of change of both sides with time, and using Faraday's Law $\mathcal{E} = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$,

$$\mathcal{E}_1 = -M \ \frac{\mathrm{di_2}}{\mathrm{dt}}$$

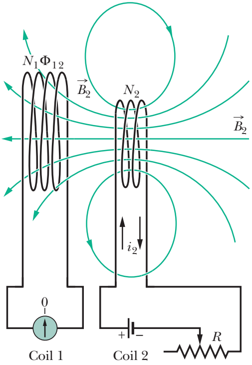
and

$$\mathcal{E}_2 = -M \ \frac{\mathrm{di_1}}{\mathrm{dt}}$$

A change of current in one coil causes a magnetic flux in the other.

Mutual Inductance

Imagine the two coils are moved closer together, with the orientation of both coils remaining fixed.



Because of this movement, the mutual induction of the two coils

(A) increases

(B) decreases

(C) is unaffected

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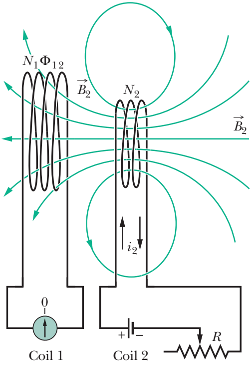

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Mutual Inductance Applications

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This can be used for **wireless charging**.

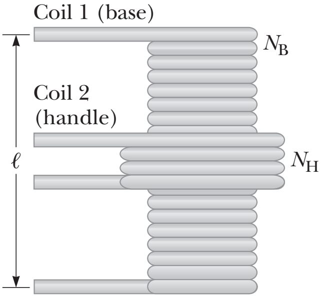
It is also used in **transformers**: devices that change the voltage and current of a power supply.

Other applications include **sensors**, particularly traffic light sensors and pulse induction metal detectors.

For any of these applications to work, there must be a constantly changing current.

Mutual Inductance Applications: Wireless Charger (Ex 32.5)

Electric toothbrush: Model the base of the charger as a solenoid of length ℓ, with N_B turns, carrying a current i, and having a cross-sectional area A. The handle coil contains N_H turns and completely surrounds the base coil. Find the mutual inductance of the system.



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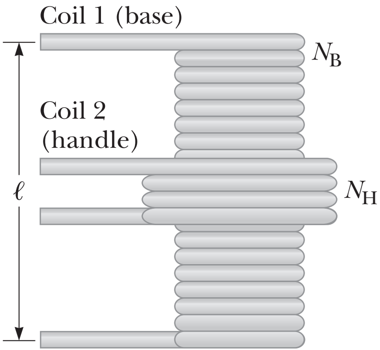

$$M = \frac{N_H \Phi_{B,H}}{i_B}$$

Must find an expression for $\Phi_{B,H}$.

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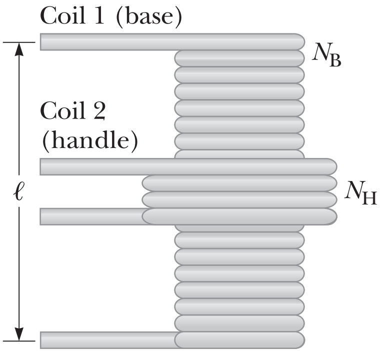

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$$\begin{aligned} \Phi_{B,H} &= \Phi_{B,B} \\ &= \mathbf{B} \cdot \mathbf{A} \\ &= \mu_0 i_b \frac{N_B}{\ell} A \end{aligned}$$

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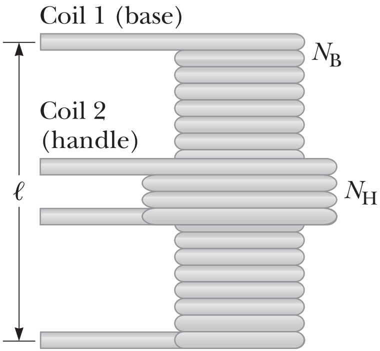

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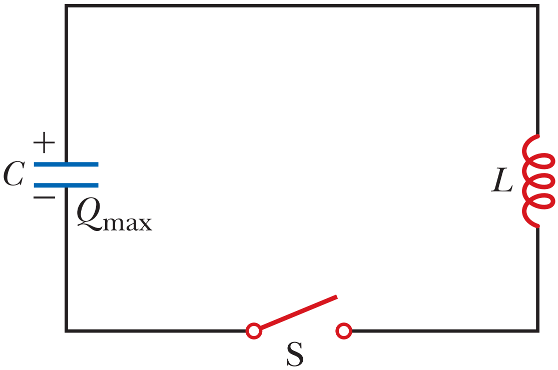
So,

$$M = \frac{\mu_0 N_H N_B A}{\ell}$$

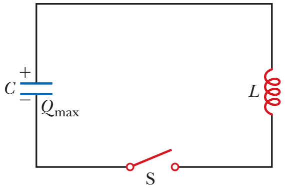
LC Circuits and Oscillations

Interesting time-dependent behavior of charge and current also occurs in circuits with **inductors and capacitors**.



The capacitor is first charged, then put into a circuit with the inductor.

LC Circuits and Oscillations



Assume the resistance of the wires is zero, then electromagntic energy is conserved in the circuit.

The energy is stored either in the E-field in the capacitor or the B-field in the inductor.

$$U_{\mathrm{tot}} = \frac{q^2}{2C} + \frac{Li^2}{2}$$

LC Circuits and Oscillations

Energy is conserved $\Rightarrow$ $\frac{\mathrm{dU_{tot}}}{\mathrm{dt}} = 0$.

$$\frac{q}{C}\frac{\mathrm{dq}}{\mathrm{dt}} + Li\frac{\mathrm{di}}{\mathrm{dt}} = 0$$

Remembering that $i = \frac{\mathrm{dq}}{\mathrm{dt}}$:

$$\frac{q}{C}\cancel{\frac{\mathrm{dq}}{\mathrm{dt}}} + L\cancel{\frac{\mathrm{dq}}{\mathrm{dt}}}\frac{\mathrm{d}^2q}{\mathrm{dt}^2} = 0$$

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This is a second order differential equation in q.

$$\frac{\mathrm{d}^2 q}{\mathrm{dt}^2} = -\frac{1}{LC}q$$

This is the equation for simple harmonic motion.

LC Circuits and Oscillations

$$\frac{d^2 q}{dt^2} = -\frac{1}{LC} q$$

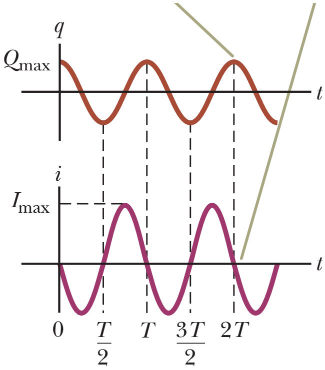
The solutions are oscillations in time:

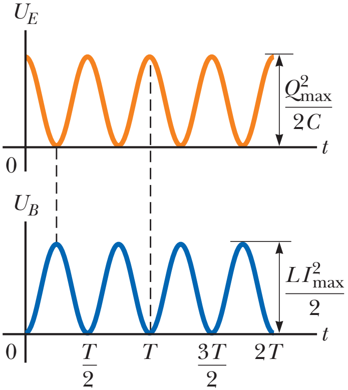
$$q(t) = Q_{max} \cos(\omega t + \phi)$$

where

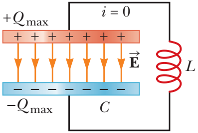
$$\omega = \frac{1}{\sqrt{LC}}$$

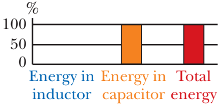
LC Circuits and Oscillations

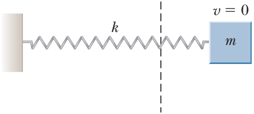




LC Circuits: Mechanical Analogy







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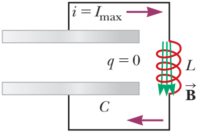

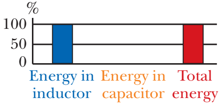

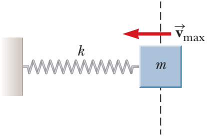

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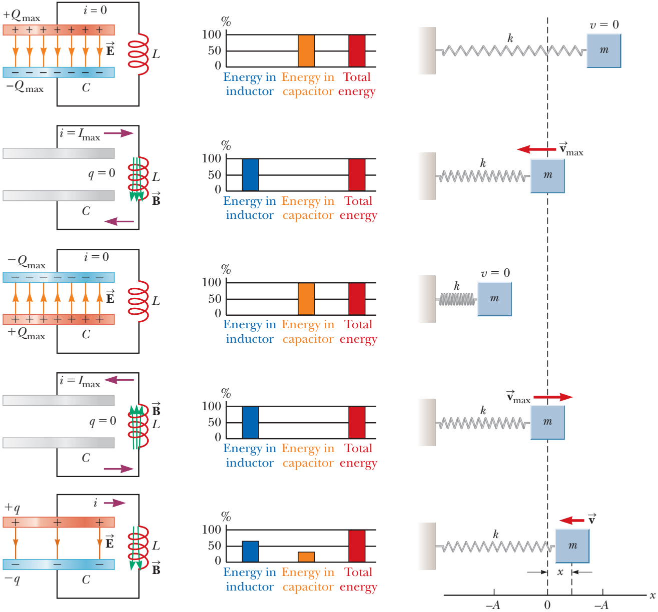

Summary

- mutual inductance
- applications of mutual inductance
- LC circuits

Collected Homework 4! due Thursday, Mar 22.

Homework

Serway & Jewett:

- PREV: Ch 32, onward from page 988. Obj. Qs: 1; Conc. Qs.: 7; Probs: 11, 15, 19, 33, 41, 43
- NEW: Ch 32, Probs: 49