Electricity and Magnetism
Transformers and Alternating Current

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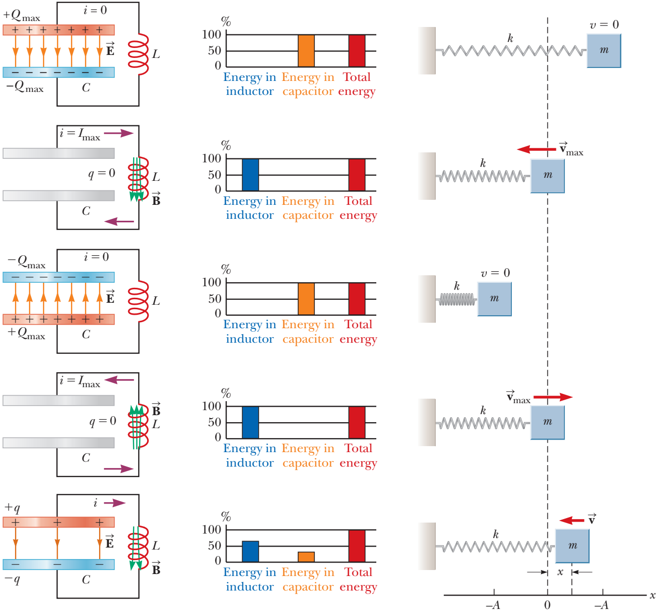
Last time

- mutual inductance
- LC circuits and oscillations

Overview

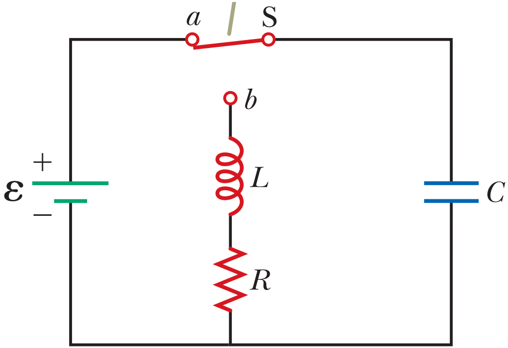
- LC circuits, mechanical analogy
- oscillations in RLC circuits
- alternating current

LC Circuits: Mechanical Analogy



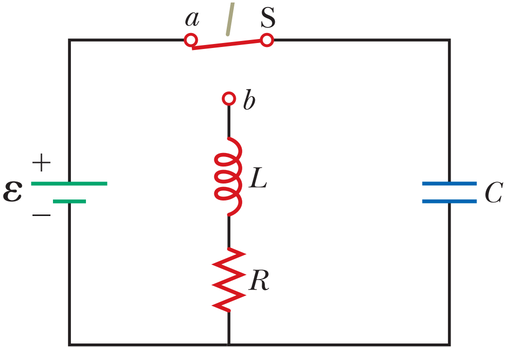
RLC Circuits

Of course, we can add resistors into an LC circuit.



RLC Circuits: Damped Oscillations

In RLC circuits, electromagnetic energy is “lost” as heat in the resistor.



$$\frac{dU}{dt} = -i^2 R$$

RLC Circuits: Damped Oscillations

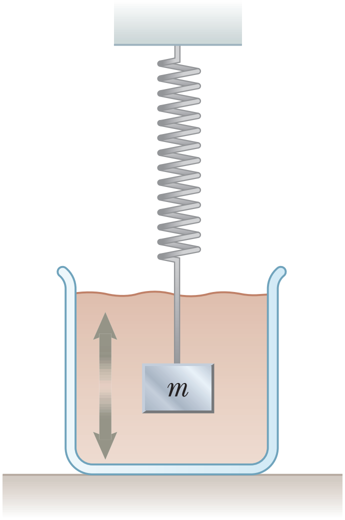
$$\frac{q}{C}\frac{\mathrm{d}q}{\mathrm{d}t} + L\frac{\mathrm{d}q}{\mathrm{d}t}\frac{\mathrm{d}^2q}{\mathrm{d}t^2} = -\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)^2 R$$

Giving,

$$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{LC}q = 0$$

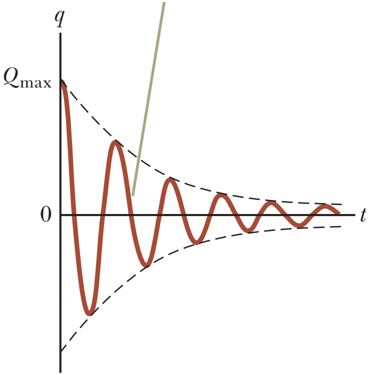
The equation for a damped oscillator!

RLC Circuits: Mechanical Analogy



RLC Circuits: Damped Oscillations

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$



Solution

$$q(t) = Q_{\max} e^{-Rt/2L} \cos(\omega_d t) \text{ , where } \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Mutual Inductance Applications

If there is a changing current in one coil, an emf can be induced in the other coil.

The current can be transferred to a whole different circuit that is no directly connected.

This can be used for wireless charging and transformers.

For either of those applications to work, there must be a constantly changing current.

Alternating Current (AC)

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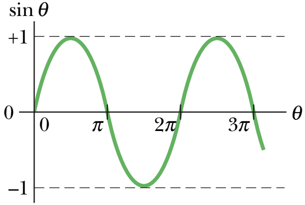
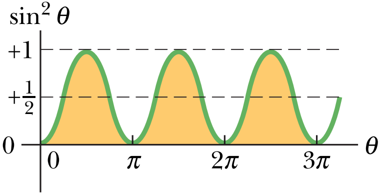
In an alternating current supply, the voltage and current vary sinusoidally with time:





$$\Delta v = \Delta V_{\max} \sin(\omega t)$$

The power delivered to a resistive load fluctuates as $P = P_{\max} \sin^2(\omega t)$.

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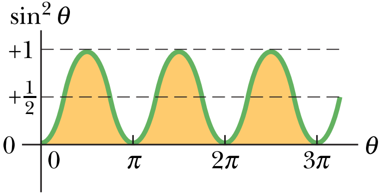

Integrating to find the shaded area we see that the average power delivered over a cycle will be:

$$P_{\text{avg}} = \frac{P_{\text{max}}}{2}$$

Alternating Current (AC)

We describe the amount of current and potential difference across the circuit by its root-mean-square (RMS) value.

The RMS voltage supplied is

$$\Delta V_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{max}}}{\sqrt{2}}$$

The RMS current supplied is

$$I_{\mathrm{rms}} = \frac{I_{\mathrm{max}}}{\sqrt{2}}$$

Average power (resistive circuit):

$$P = I_{\mathrm{rms}}\left(\Delta V_{\mathrm{rms}}\right) = \frac{1}{2} P_{\mathrm{max}}$$

Alternating Current (AC) Example 33.1

The voltage output of an AC source is given by the expression $\Delta v = 200 \sin(\omega t)$, where Δv is in volts.

Find the rms current in the circuit when this source is connected to a 100 Ω resistor.

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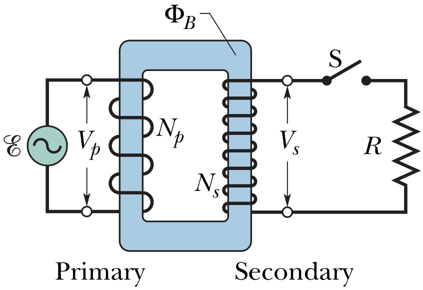
$$\Delta V_{\text{rms}} = \frac{200}{\sqrt{2}} = 141 \text{ V}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = 1.41 \text{ A}$$

Transformers

Transformers change ΔV_{rms} and I_{rms} simultaneously, while keeping the average power $P_{\text{avg}} = I_{\text{rms}}\Delta V_{\text{rms}}$ constant (conservation of energy).



This works via mutual inductance. If the current in the first coil did not constantly change (AC) this would not work.

$$\Delta v_s = \Delta v_p \frac{N_s}{N_p}$$

Summary

- RLC circuits
- alternating current

Collected Homework 4 due Thursday, Mar 22.

Homework

Serway & Jewett:

- NEW: Ch 32, Probs: 45, 53, 57, 59
- NEW: Ch 33, onward from page 1021. Obj. Qs: 12, 13; Conc. Qs.: 8; Probs: 1, 3, 5, 49, 51, 57

Appendix: Damped Oscillations Solution Derivation

$$\frac{\mathrm{d}^2x}{\mathrm{dt}^2} + \frac{b}{m}\frac{\mathrm{dx}}{\mathrm{dt}} + \frac{k}{m}x = 0$$

Suppose an exponential function is the solution to this equation:

$$x = B\,e^{rt}$$

r and B are constants.
Then

$$B\,e^{rt}(r^2 + \frac{b}{m}r + \frac{k}{m}) = 0$$

The exponential function is not zero for any finite t, so the other factor must be zero. We must find the roots for r that make this equation true.

Appendix: Damped Oscillations Solution Derivation

This is called the characteristic equation

$$r^2 + \frac{b}{m} r + \frac{k}{m} = 0$$

The roots are:

$$r = \frac{-b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

This means the solutions are of the form:

$$x = e^{-b/(2m)t} \left(B_1 e^{i\omega t} + B_2 e^{-i\omega t}\right)$$

where

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

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Recall that $\cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$. (If you haven't seen this, try to prove it using the series expansions of cosine and the exponential function.)

We can write the solution as

$$x = A\,e^{-(b/2m)t}\cos(\omega t + \phi)$$

where $B_1 = A\,e^{i\phi}$ and $B_2 = A\,e^{-i\phi}$.