Electricity and Magnetism
AC Circuits
L, C Circuits
Reactance

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De Anza College

Mar 20, 2018

Last time

- alternating current
- transformers
- rectifiers and filters

Overview

- phase offsets with inductors and capacitors
- reactance

AC and Types of circuits

The current and potential difference both change sinusoidally in AC circuits.

However, they do not necessarily reach their peaks at the same time across a particular circuit component.

It depends on the type of circuit.

AC and Types of circuits

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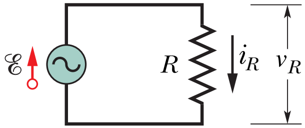
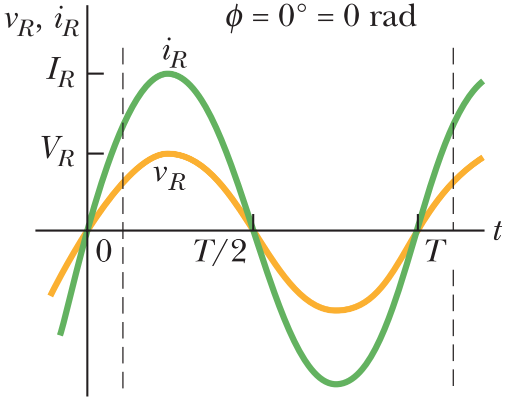
It depends on the type of circuit.

For every circuit we can choose to describe the dependance on time of the voltage and current as:

$$\Delta v(t) = \Delta v_{\max} \sin(\omega t)$$
$$i(t) = i_{\max} \sin(\omega t - \phi)$$

AC in Resistance-only Circuits



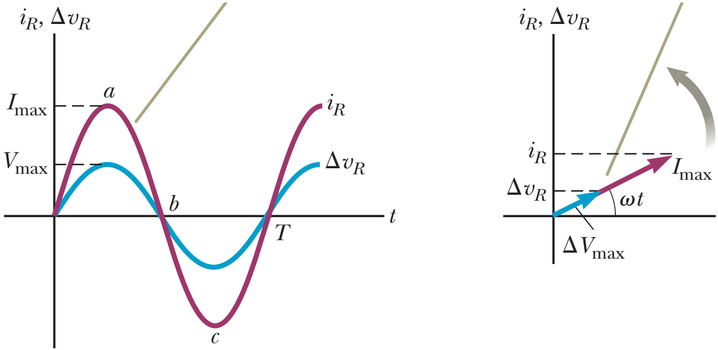


The voltage and current are in phase.

(Definition of resistance! $R = \frac{\Delta v}{i}$)

Resistive Circuits

On the right is a **phasor diagram**.



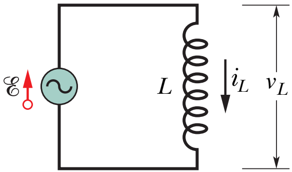
Instantaneous current and voltage are represented by vectors that rotate in time:

$$\Delta v(t) = \Delta v_{max} \sin(\omega t)$$

$$i(t) = i_{max} \sin(\omega t)$$

The voltage and current are in phase $\Rightarrow$ the angle between the vectors, $\phi = 0$.

AC in Inductance-only Circuits



Kirchoff's loop law holds at each instant in this circuit:

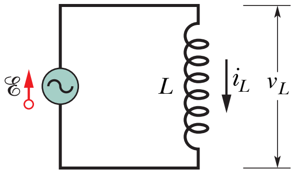
$$\Delta v - L\,\frac{\mathrm{di}}{\mathrm{dt}} = 0$$

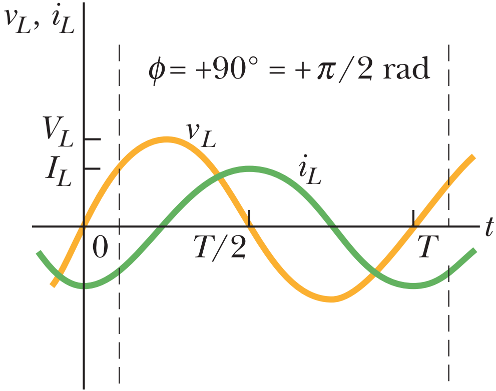
$$\Delta v = \Delta V_{\mathrm{max}} \sin(\omega t) = L\,\frac{\mathrm{di}}{\mathrm{dt}}$$

This means

$$i = \frac{\Delta V_{\mathrm{max}}}{\omega L} \sin(\omega t - \pi/2)$$

AC in Inductance-only Circuits

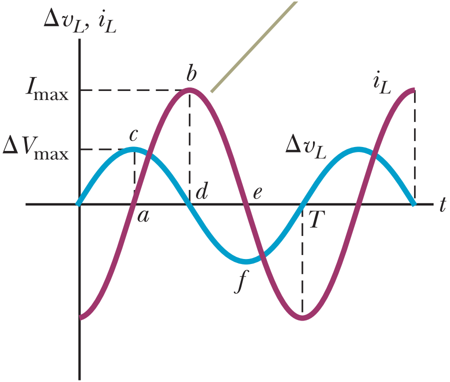


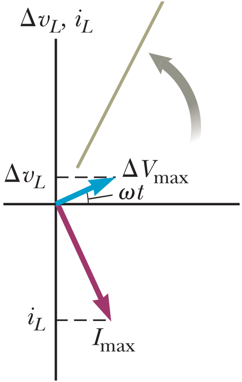


The voltage leads the current. Voltage peaks first.

The potential difference across the inductor is large when the change in current is large. ($V_L = -L\,\frac{\Delta i}{\Delta t}$)

AC in Inductance-only Circuits





Instantaneous current and voltage:

$$\Delta v(t) = \Delta v_{max} \sin(\omega t)$$

$$i(t) = i_{max} \sin(\omega t - \pi/2)$$

The voltage leads the current. The angle $i \rightarrow \Delta v$ is $\phi = \pi/2$.

AC in Inductance-only Circuits

From Kirchhoff's loop rule we derived:

$$i = \frac{\Delta V_{\text{max}}}{\omega L} \sin(\omega t - \pi/2)$$

Therefore

$$i_{\text{max}} = \frac{\Delta V_{\text{max}}}{\omega L}$$

The product ωL has units of Ohms and behaves effectively as a resistance. However, it depends on the frequency of the voltage.

We can think of the inductor "reacting" to the frequency of the voltage.

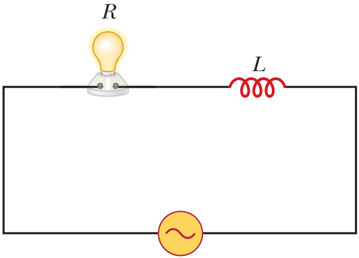
We define

Inductive Reactance

$$X_L = \omega L$$

Inductive Circuit Question

In the AC circuit shown, the frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest?



(a) It glows brightest at high frequencies.

(b) It glows brightest at low frequencies.

(c) The brightness is the same at all frequencies.

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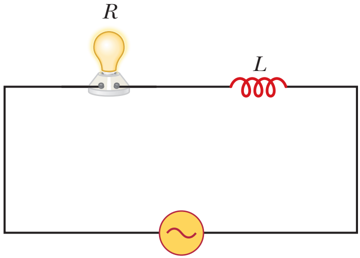

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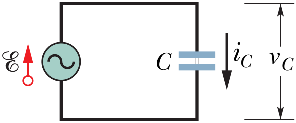
AC in Capacitance-only Circuits



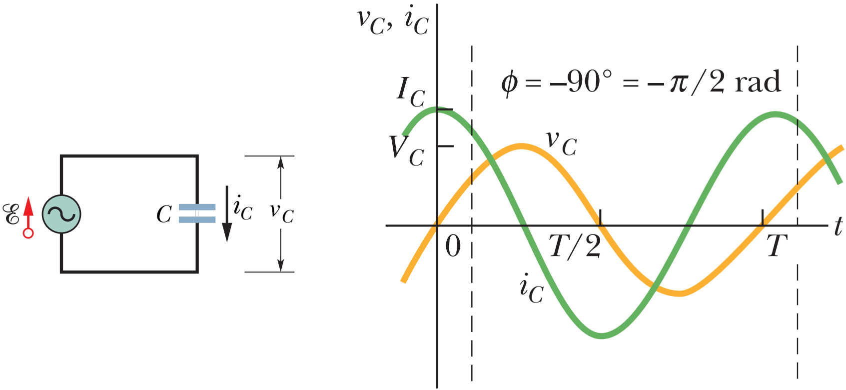
Kirchoff's loop law holds at each instant in this circuit:

$$\Delta v - \frac{q}{C} = 0$$

$$i = \frac{\mathrm{dq}}{\mathrm{dt}} = C(\Delta V_{\mathrm{max}})\,\frac{\mathrm{d}}{\mathrm{dt}}\sin(\omega t)$$

$$i = \omega C(\Delta V_{\mathrm{max}})\sin(\omega t + \pi/2)$$

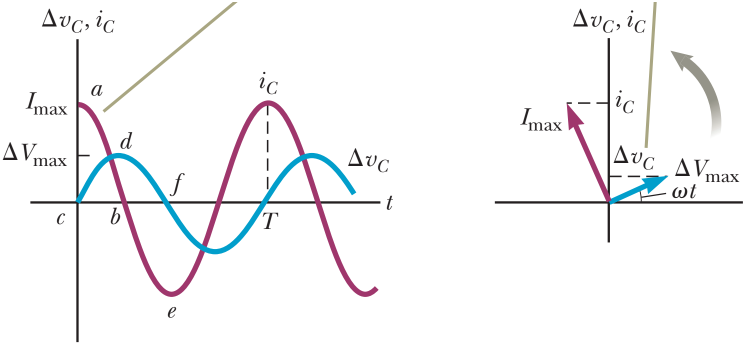
AC in Capacitance-only Circuits



The current leads the voltage. Current peaks first.

The potential across the capacitor is only high once the capacitor has built up some charge.

AC in Capacitance-only Circuits



Instantaneous current and voltage:

$$\Delta v(t) = \Delta v_{max} \sin(\omega t)$$
$$i(t) = i_{max} \sin(\omega t + \pi/2)$$

The current leads the voltage. The angle $i \to \Delta v$ is $\phi = -\pi/2$.

AC in Capacitance-only Circuits

From Kirchhoff's loop (capacitance circuit) rule we derived:

$$i = \omega C(\Delta V_{\text{max}}) \sin(\omega t + \pi/2)$$

Therefore

$$i_{\text{max}} = \frac{\Delta V_{\text{max}}}{(1/\omega C)}$$

The expression $\frac{1}{\omega C}$ also has units of Ohms and behaves effectively as a resistance. Again, it depends on the frequency of the voltage.

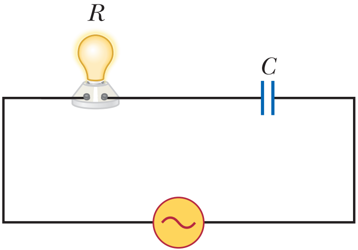
We define

Capacitive Reactance

$$X_C = \frac{1}{\omega C}$$

Capacitive Circuit Question

In the AC circuit shown, the frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest?



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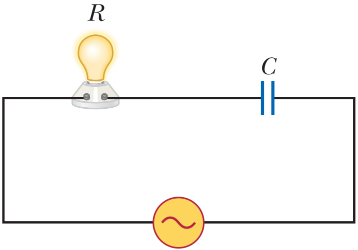

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Summary

- phase offsets with inductors and capacitors
- reactance

Collected Homework 4! due Thursday.

Final Exam Tuesday, Mar 27, 9:15-11:15am, S35 (here).

Homework Serway & Jewett:

- Ch 33, onward from page 1021. Problems: 9, 15, 19