Electricity and Magnetism
Maxwell's Laws
Electromagnetic Radiation

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De Anza College

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Last time

- alternating current (AC)
- impedance
- power and resonance

Overview

- Maxwell's equations
- Ampère-Maxwell law
- electromagnetic radiation

Maxwell's Laws

Amazingly, we can summarize the majority of the relations that we have talked about in this course in a set of just 4 equations.

These are together called Maxwell's equations.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{\text{enc}}$$

Gauss's Law for Magnetic Fields

The first of Maxwell's equations is Gauss's Law for E-fields:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\mathrm{enc}}}{\epsilon}$$

The second is for Gauss's Law for B-fields:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

Faraday's Law of Induction

Faraday's Law of Induction is the third of Maxwell's laws.

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

This tells us that a changing magnetic field will induce an electric field.

But what about the reverse? A changing electric field inducing a magnetic field?

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But what about the reverse? A changing electric field inducing a magnetic field?

It does happen!

Maxwell's Law of Induction

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

(A piece of the Ampère-Maxwell's Law.)

The Ampère-Maxwell's Law

The fourth and last of Maxwell's equations:

The Ampère-Maxwell's Law

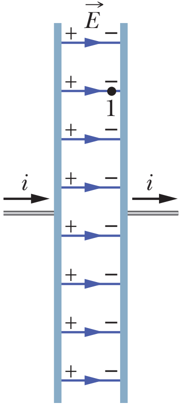
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{enc}$$

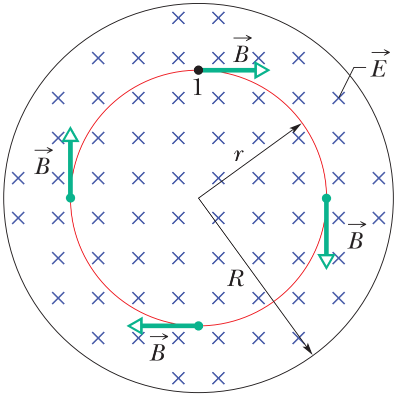
Differential form:

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

Maxwell's Law of Induction

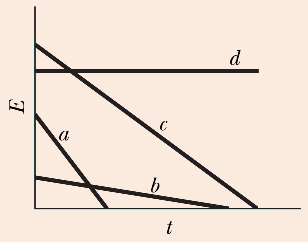
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$





Maxwell's Law of Induction question

The figure shows graphs of the electric field magnitude E versus time t for four uniform electric fields, all contained within identical circular regions as in the circular-plate capacitor. Rank the E-fields according to the magnitudes of the magnetic fields they induce at the edge of the region, greatest first.



A a, b, c, d

B a, c, b, d

C d, b, c, a

D d, c, a, b

[1] Halliday, Resnick, Walker, page 865.

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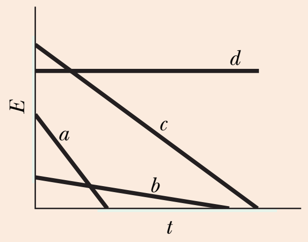


A a, b, c, d

B a, c, b, d ←

C d, b, c, a

D d, c, a, b

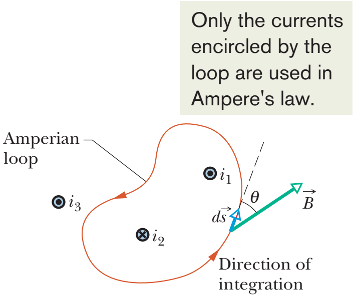
[1]Halliday, Resnick, Walker, page 865.

Reminder: Ampère's Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

(The other piece of the Ampère-Maxwell's Law.)

The line integral of the magnetic field around a closed loop is proportional to the current that flows *through* the loop.[1]



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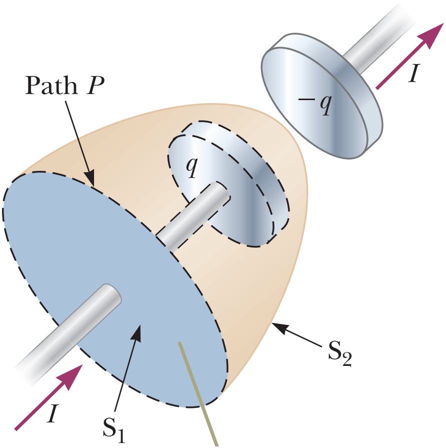
[1] That is, the current that flows through **any surface bounded by the loop.**

Maxwell's Law of Induction



Surfaces S_1 and S_2 have different currents flowing through them!

Maxwell's Law of Induction

Maxwell realized that there should be another term in Ampère's law.

He introduced the notion of a **displacement current**:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Law of Induction

Maxwell realized that there should be another term in Ampère's law.

He introduced the notion of a **displacement current**:

$$I_d = \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

Note: The displacement "current" is not a current and has nothing to do with displacement. However, it does have units of Amps.

This completes Ampere's law as:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = \mu_0 (I_{enc} + I_d)$$

Ampere-Maxwell Law and Displacement "Current"

displacement "current"

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

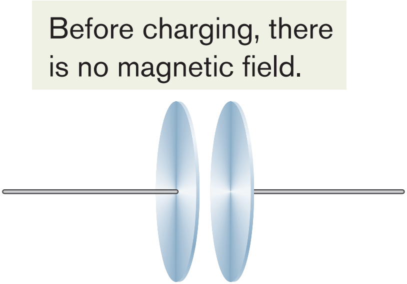
This lets us rewrite the Ampere-Maxwell law as:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_d + \mu_0 I_{enc}$$

Looking at it this way can give us some insights.

B-field around a charging capacitor

Suppose a capacitor is being charged with a constant current, i.



How does i relate to i_d the "current" from the E-field between the plates?

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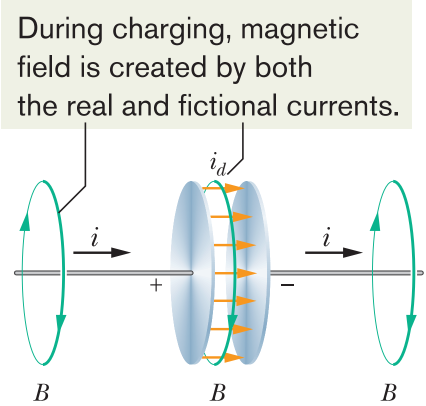

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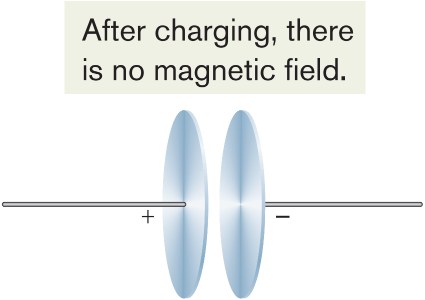


How does i relate to i_d the "current" from the E-field between the plates?

B-field around a charging capacitor

$$i_d = \epsilon_0 \frac{\mathrm{d}\Phi_\mathrm{E}}{\mathrm{dt}} = \epsilon_0 A \frac{\mathrm{dE}}{\mathrm{dt}}$$

Gauss's law allows us to relate q, the charge on one plate of the capacitor to the flux:

$$\frac{q}{\epsilon_0} = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = EA$$

The current is the rate of flow of charge:

$$i = \frac{\mathrm{dq}}{\mathrm{dt}} = \epsilon_0 A \frac{\mathrm{dE}}{\mathrm{dt}}$$

So, $i_d = i$!

The B-field outside a circular capacitor looks the same as the B-field around the wire leading up to the capacitor.

Magnetic Field around a circular capacitor

Can be calculated just like the field around a wire!

Magnetic Field around a circular capacitor

Can be calculated just like the field around a wire!

Outside the capacitor at radius r from the center:

$$B = \frac{\mu_0 i_d}{2\pi r}$$

Inside the capacitor (plates have radius R) at radius r from the center:

$$B = \frac{\mu_0 i_d}{2\pi R^2} r$$

But remember: i_d is not a current. No current flows across the gap between the plates.

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\epsilon}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

[1] Strictly, these are Maxwell's equations in a vacuum.

Maxwell's Equations Differential Form

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}
\end{aligned}$$

Another Implication of Maxwell's Equations

Using all 4 equations (in their differential form) it is possible to reach a pair of wave equations for the electric and magnetic fields:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

with wave solutions:

$$\mathbf{E} = E_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

$$\mathbf{B} = B_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

where $c = \frac{\omega}{k}$.

Another Implication of Maxwell's Equations

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

The constant c appears as the wave speed and

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$c = 3.00 \times 10^8$ m/s, is the speed of light.

The values of ϵ_0 and μ_0 together predict the speed of light!

Another Implication of Maxwell's Equations

Wave solutions:

$$\begin{aligned} \mathbf{E} &= E_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) \\ \mathbf{B} &= B_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) \end{aligned}$$

where $c = \frac{\omega}{k}$.

These two solutions are in phase. There is no offset in the angles inside the sine functions.

The two fields peak at the same point in space and time.

At all times:

$$\frac{E}{B} = c$$

Relation between Electric and Magnetic Fields

These oscillating electric and magnetic fields make up light.

Faraday's Law of Induction

A changing magnetic field gives rise to an electric field.

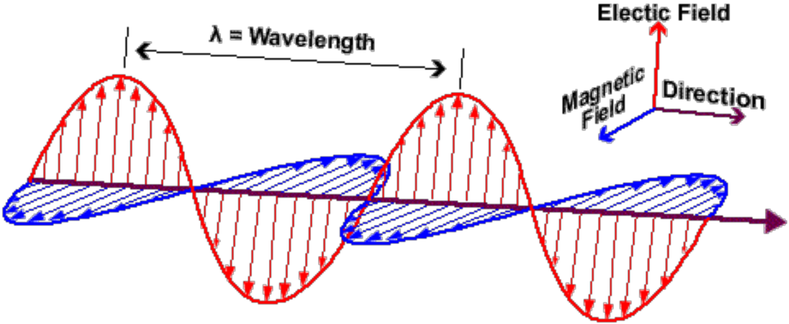
Ampere-Maxwell Law of Induction

A changing electric field gives rise to an magnetic field.

Light

Faraday's Law ⇒ a changing magnetic field causes an electric field.

Maxwell's Law ⇒ a changing electric field causes a magnetic field.



Light (Electromagnetic Radiation)

All light waves in a vacuum travel at the same speed, the speed of light, $c = 3.00 \times 10^8$ m s^{-1}.

Maxwell's equations possess the 'wrong' symmetry for Gallilean transformations between observers; they are Lorentz-invariant. This gave Einstein an important idea.

All observers, no matter how they move relative to one another all agree that any light wave travels at that same speed.

Light (Electromagnetic Radiation)

Light travels at this fixed speed, $c = 3.00 \times 10^8$ m s^{-1}.

For any wave, if v is the wave propogation speed:

$$v = f\lambda$$

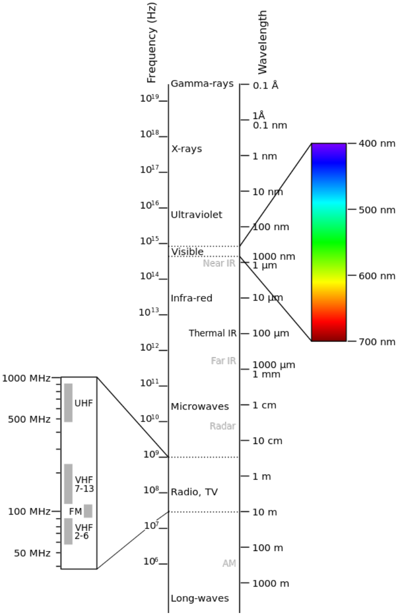
For light:

$$c = f\lambda$$

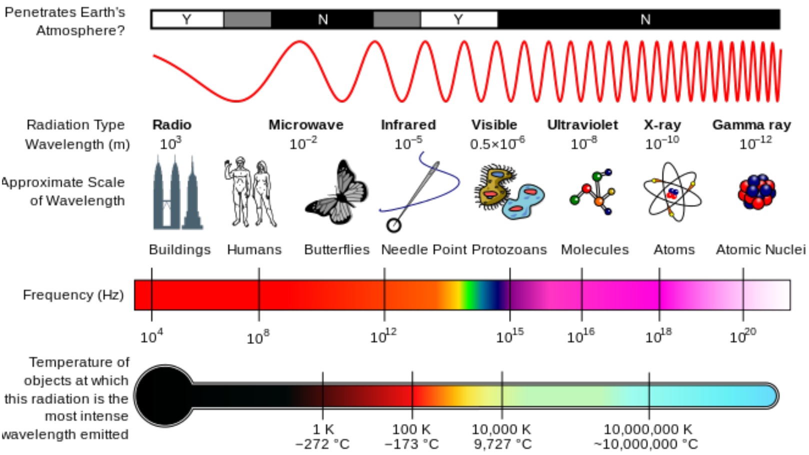
So, if the frequency of the light is given, you also know the wavelength, and vice versa.

$$\lambda = \frac{c}{f} \quad ; \quad f = \frac{c}{\lambda}$$

Electromagnetic spectrum



Electromagnetic spectrum



Summary

- Maxwell's equations
- Ampère-Maxwell law
- electromagnetic radiation

Final Exam Tuesday, Mar 27, 9:15-11:15am, S35 (here).

Homework

Serway & Jewett:

- NEW: Ch 34, onward from page 1048. Obj. Qs: 3; Probs: 1, 3, 5