Electricity and Magnetism
Electric Dipole
Continuous Distribution of Charge

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De Anza College

Jan 16, 2018

Last time

- electric field lines
- electric field from a point charge
- net electric field from many charges
- effect of fields on charges

Warm Up Questions

Which expression relating force to electric field is correct?

(A) $\mathbf{F} = m_0 \mathbf{E}$

(B) $\mathbf{E} = q_0 \mathbf{F}$

(C) $\mathbf{F} = q_0 \mathbf{E}$

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(C) Nm^2/C^2

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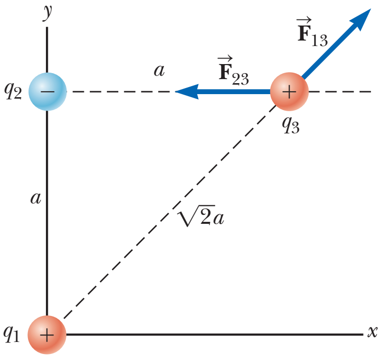
$q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, and $a = 0.100$ m.
The resultant force exerted on q_3 is $\mathbf{F}_{\text{net},3} = (-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N.

What is the electric field at the location of q_3 due to the other two charges?



(A) $(-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N

(B) $(-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N/C

(C) $(-0.208\ \mathbf{i} + 1.59\ \mathbf{j})$ MN/C

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[1] Figure from Serway & Jewett, pg 696, Ex 2.

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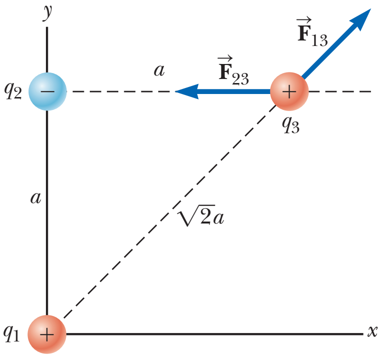

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Overview

- electric field of a dipole
- continuous distributions of charge

Electric Dipole

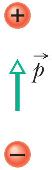
electric dipole

A pair of charges of equal magnitude q but opposite sign, separated by a distance, d.

dipole moment:

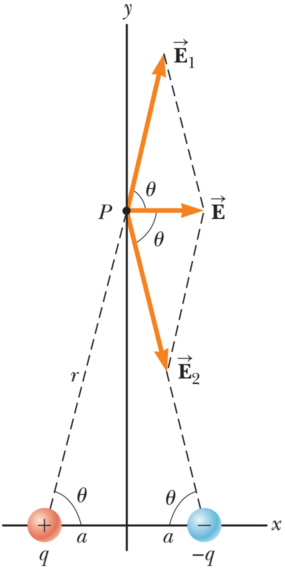
$$\mathbf{p} = qd\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing from the negative charge to the positive charge.

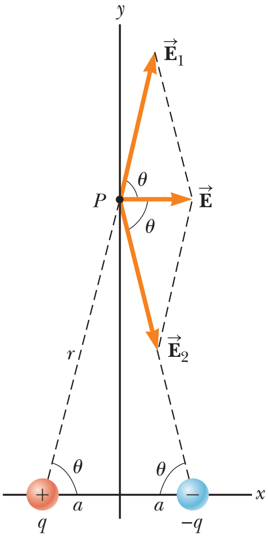


Electric Dipole (Example 23.6, B)

Evaluate the electric field from the dipole at point P, which is at position $(0, y)$.



Electric Dipole (Example 23.7)



The y-components of the electric field cancel out, $E_y = 0$.

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$$E_x = E_{1,x} + E_{2,x}$$

Also $E_{1,x} = E_{2,x}$

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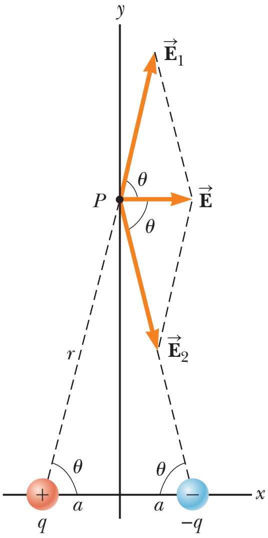

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$$\begin{aligned} E_x &= 2\left(\frac{k_e\, q}{r^2}\cos\theta\right) \\ &= \frac{2k_e\, q}{(a^2+y^2)}\left(\frac{a}{\sqrt{a^2+y^2}}\right) \\ &= \frac{2k_e\, a\, q}{(a^2+y^2)^{3/2}} \end{aligned}$$

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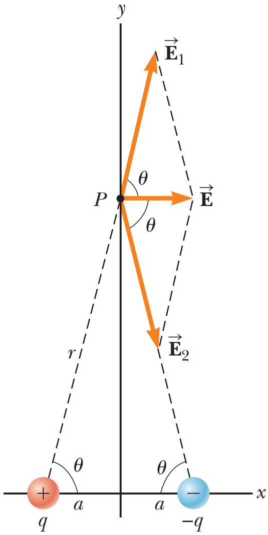

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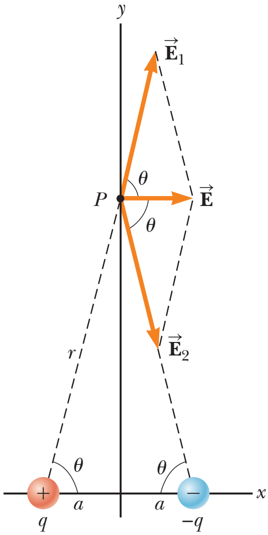

What happens as we move infinitely far from the dipole? ($y >> a$)

The constant a in the denominator has less and less affect on the function. We can see that the field function approaches $E_{\text{far}} = \frac{2k_e\, a\, q}{y^3}$

$$\lim_{y\to\infty}\left[\frac{E}{E_{\text{far}}}\right] = \lim_{y\to\infty}\left[\frac{\frac{2k_e\, a\, q}{(a^2+y^2)^{3/2}}}{\frac{2k_e\, a\, q}{y^3}}\right]$$

$$= \lim_{y\to\infty}\left[\frac{\frac{2k_e\, a\, q}{y^3\left(\left(\frac{a}{y}\right)^2+1\right)^{3/2}}}{\frac{2k_e\, a\, q}{y^3}}\right]$$

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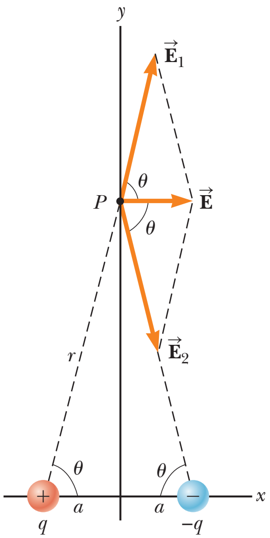

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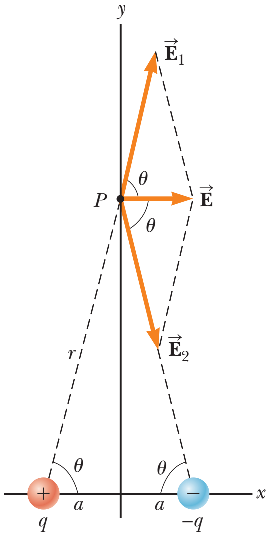
Big-O Notation (Example 23.7)



$y >> a$

Recall that $f(x) = O(g(x))$ if $\left|\frac{f(x)}{g(x)}\right| \leqslant C$ $\forall\ x > k$.

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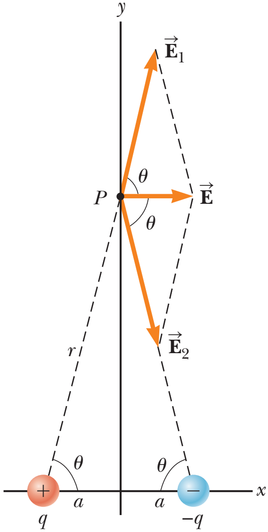

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$$\left|\frac{E}{E_{\text{far}}}\right| = \left|\frac{\frac{2k_e\,a\,q}{(a^2+y^2)^{3/2}}}{\frac{2k_e\,a\,q}{y^3}}\right|$$

$$= \left|\left(\left(\frac{a}{y}\right)^2 + 1\right)^{-3/2}\right|$$

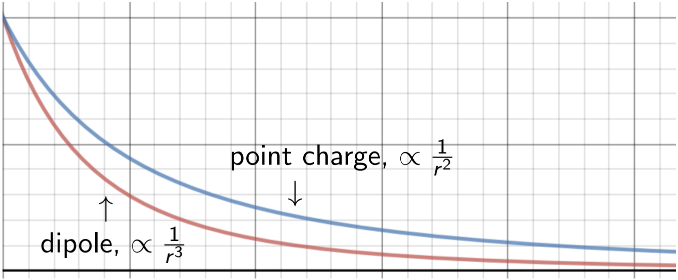
Choosing $k = a$ we can see:

$$\left|\frac{E}{E_{\text{far}}}\right| \leqslant \frac{1}{2\sqrt{2}} \quad \forall\ y > a$$

Therefore, $E = O\left(\frac{2k_e\,a\,q}{y^3}\right)$ or simply $O(y^{-3})$.

Electric Dipole (Example 23.7)

As we move away from the dipole (red line, r^{-3}) the E-field falls off faster than it does for a point charge (blue line, r^{-2}).



The negative charge partially shields the effect of the positive charge and vice versa.

Continuous distribution of charge

In previous example, we added up the field from each point charge.

But what about the case of a charged object, like a plate or a wire?

In just -1 Coulomb of charge, there are more than a **quintillion** excess electrons!

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Solution: treat the charge as a **continuous distribution** with some charge density.

Charge Density

charge density

The amount of charge in per unit 'volume' of an object.

(Here 'volume' could be volume, area, or length)

By convention, different symbols can be used in different cases:

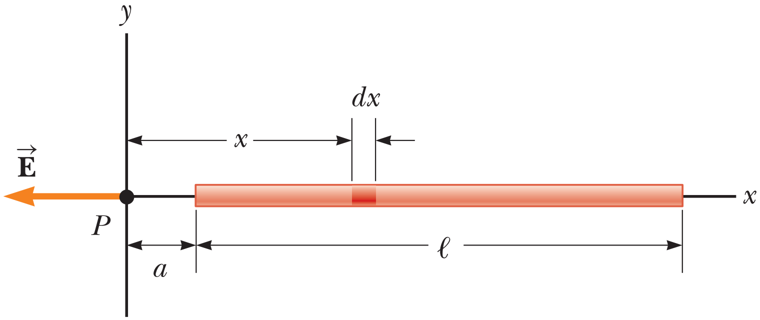
symbol	description	SI units
λ	charge per unit length	C m^{-1}
σ	charge per unit area	C m^{-2}
ρ	charge per unit volume	C m^{-3}

For a wire, usually use charge per length.
For a plate, charge per area.

Continuous distribution of charge (Ex. 23.7)

A rod of length ℓ, has a uniform positive charge per unit length λ and a total charge Q.

Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



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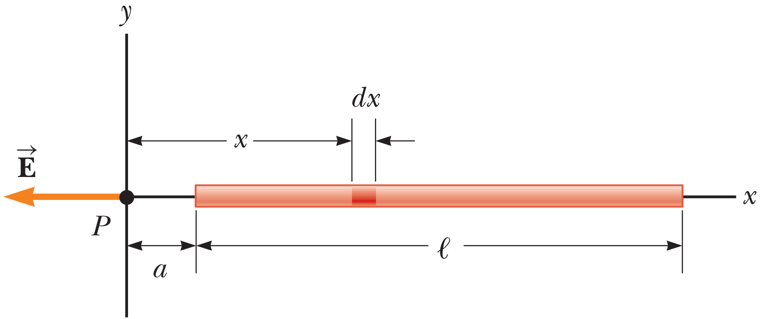
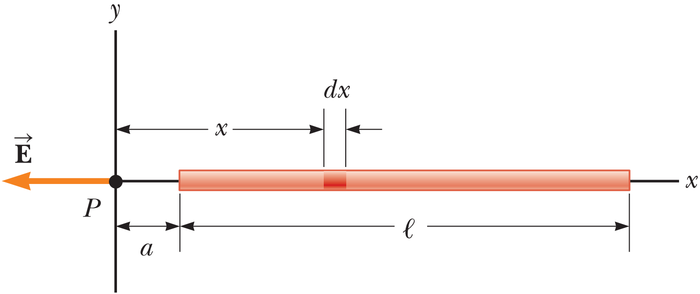


We need to add up the charge of each little "particle" dx. Each has charge $\lambda\,dx$.

To be perfectly accurate, we would make the length of dx $\to 0$. This is an integral: $\sum \lambda\,\Delta x \to \int \lambda\,dx$

Continuous distribution of charge (Ex. 23.7)



$$
\begin{aligned}
E &= \int \frac{k_e \, \mathrm{dq}}{x^2} \\
&= k_e \int_a^{\ell+a} \frac{1}{x^2} \, \lambda \, \mathrm{dx}
\end{aligned}
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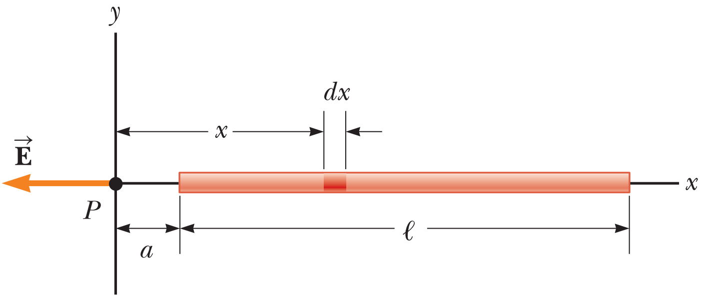

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&= k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell+a} \\
&= k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \frac{k_e Q}{a(\ell + a)}
\end{aligned}
$$

Question

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the **net electric field** at point P?

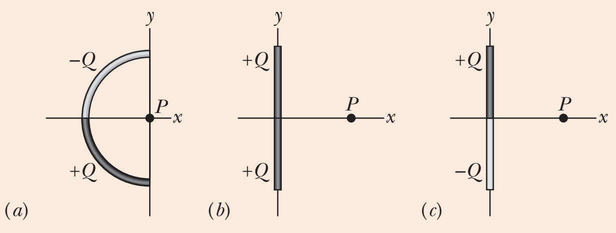
For (a) it is:

(A) up

(B) down

(C) left

(D) right



[1] Page 590, Halliday, Resnick, Walker.

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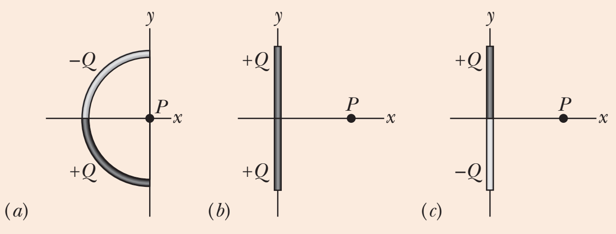

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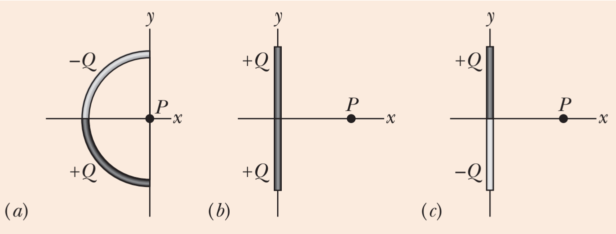

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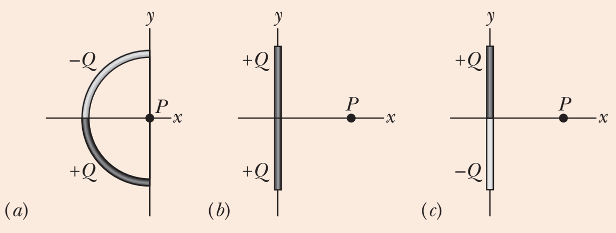

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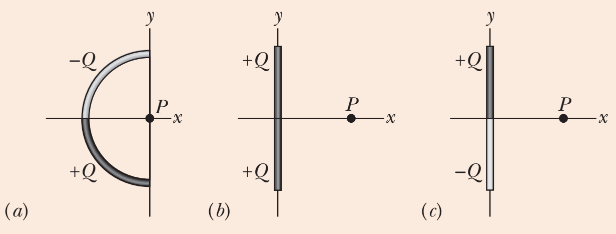

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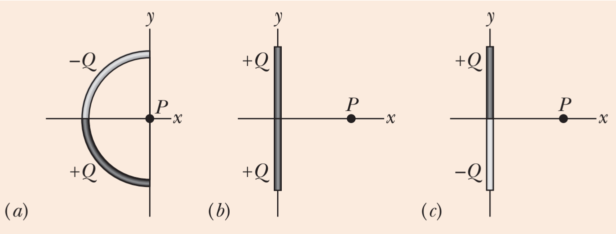

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Summary

- electric dipole field
- electric fields of charge distribution

Homework

- Collected homework 1, posted online, due on Monday, Jan 22.

Serway & Jewett:

- Ch 23, onward from page 716. Probs: 45, 71, 84