Electricity and Magnetism
Electric Flux
Gauss's Law

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Last time

- conductors in electric fields
- electric flux

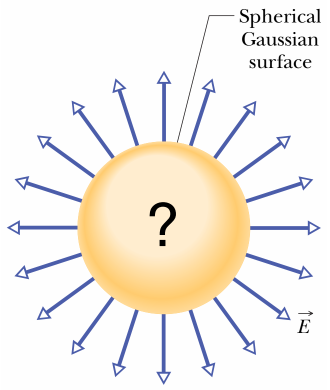
Overview

- electric flux
- Gauss's law
- Gauss's law applied to various cases

Gauss's Law basic idea

The electric field around a charge is stronger for charges with larger magnitudes.

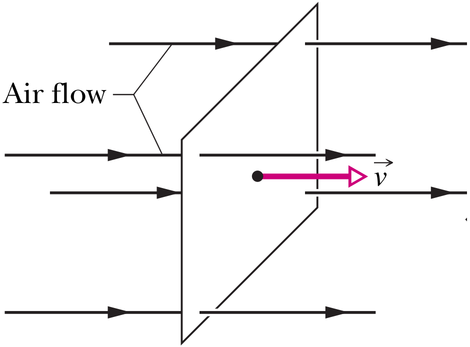
To make this observation useful, we need to quantify it.



Flux

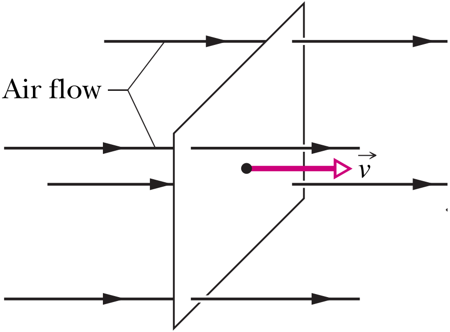
Flux is a quantity that makes the idea of the “electric field through some region” precise.

Flux is a flow rate through an area.



Flux

Imagine air blowing directly through a square loop of wire of area A.

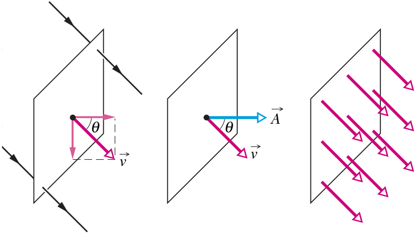


The volume of air that passes through in 1 s is $V = A \times v \times (1 \text{ s})$, where v is the speed of the air.

The *rate* of flow would be $\frac{\mathrm{dV}}{\mathrm{dt}} = Av$.

Flux

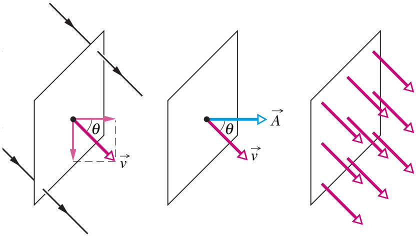
Now consider a more general situation: the air does not blow directly through the loop, but at some angle θ.



If $\theta = 90^\circ$, what is the flow rate (flux) through the loop?

Flux

Now consider a more general situation: the air does not blow directly through the loop, but at some angle θ.

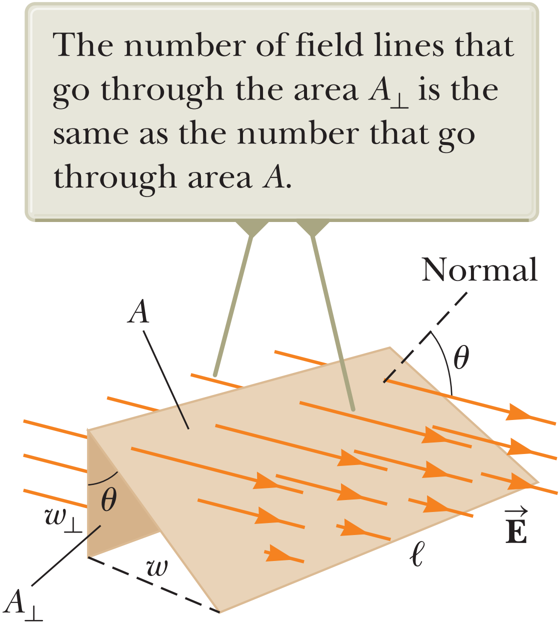


If $\theta = 90^\circ$, what is the flow rate (flux) through the loop? Zero!

In that case there is no flow through the loop. The air goes around the loop.

The flux depends on the angle that the flow makes to the loop / area.

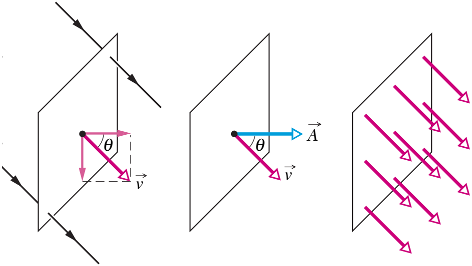
Flux


The number of field lines that go through the area $A_\perp$ is the same as the number that go through area A.
Normal
A
θ
θ
$w_\perp$
w
ℓ
$\vec{\mathbf{E}}$
$A_\perp$

Flux

The area $A_{\perp} = A\cos\theta$.

For other values of θ the flux of air that move through is $vA\cos\theta$.



We can define flux:

$$\Phi = vA\cos\theta$$

Electric Flux

The electric flux, Φ_E, through an area A is

$$\Phi_E = EA\cos\theta$$

where θ is the angle between the electric field vector at the surface and the normal vector to the surface.

This can be written:

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

The direction of $\mathbf{A}$ is $\perp$ to the surface, and the magnitude is the area of the surface.

Questions

What are the units of electric flux?

(A) $\mathrm{N\ m^2/C}$

(B) $\mathrm{N/C}$

(C) $\mathrm{N\ C^{-1}\ m^{-2}}$

(D) $\mathrm{N\ m^2}$

Questions

What are the units of electric flux?

(A) N m^2/C $\leftarrow$

(B) N/C

(C) N C^{-1} m^{-2}

(D) N m^2

Questions

A surface has the area vector $\mathbf{A} = (2\mathbf{i} + 3\mathbf{j})\ \text{m}^2$. What is the flux of a uniform electric field through the area if the field is:

$\mathbf{E} = 4\,\mathbf{i}\ \text{N/C}$?

(A) $0\ \text{Nm}^2/\text{C}$

(B) $2\ \text{Nm}^2/\text{C}$

(C) $4\ \text{Nm}^2/\text{C}$

(D) $8\ \text{Nm}^2/\text{C}$

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(A) 0 Nm^2/C

(B) 2 Nm^2/C

(C) 4 Nm^2/C

(D) 8 Nm^2/C $\leftarrow$

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A surface has the area vector $\mathbf{A} = (2\mathbf{i} + 3\mathbf{j})\ \mathrm{m}^2$. What is the flux of a uniform electric field through the area if the field is:

$\mathbf{E} = 4\,\mathbf{k}\ \mathrm{N/C}$?

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(A) 0 Nm^2/C $\leftarrow$

(B) 2 Nm^2/C

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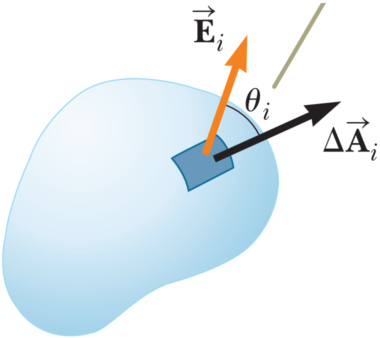
(D) 8 Nm^2/C

Electric Flux

What about the electric flux through an arbitrary curved surface?

The angle θ between the surface normal and the field varies along the surface.

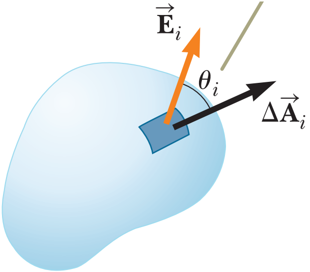


Solution: break up surface into small areas ΔA_i and add up all the contributions

$$\Phi_E = \sum_i E_i \left(\Delta A_i\right) \cos\theta_i$$

Electric Flux

To makes this approximation exact, take the limit as the areas $\Delta A_i \to 0$.



Total flux through the surface:

$$\Phi_E = \int_A \mathbf{E} \cdot \mathrm{d}\mathbf{A}$$

The electric flux Φ_E through a surface is proportional to the net number of electric field lines passing through that surface.

Gaussian Surface

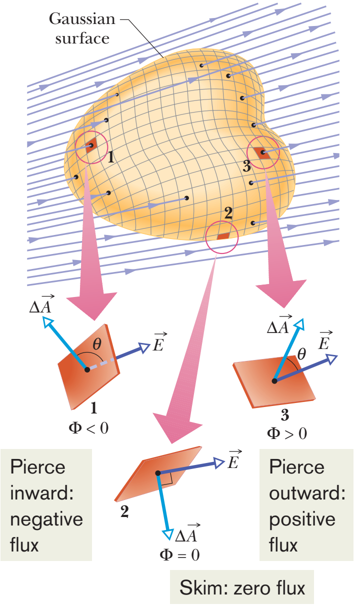
Gaussian surface

An imaginary boundary (closed surface) drawn around some region of space in order to study electric charge and field.

The surface can be any shape you like, but must be closed (have an interior and exterior).

It is just a tool for calculating charge or field.

Electric Flux through Gaussian Surfaces



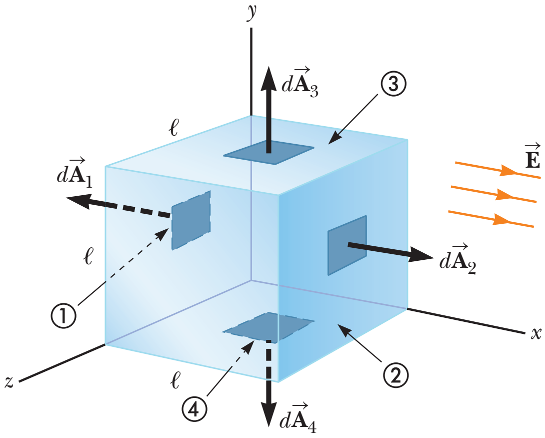
- The flux is **positive** where the field vector points out of the surface.
- The flux is **negative** where the field vector points into the surface.

For a closed surface:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$$

Electric Flux through Gaussian Surface Example

Consider a uniform electric field $\mathbf{E} = E\,\mathbf{i}$ in empty space. A cube of edge length ℓ, is placed in the field, oriented as shown. Find the net electric flux through the surface of the cube.



Electric Flux through Gaussian Surface Example

Find the net electric flux through the surface of the cube.

$$\Phi_E = \sum_i E\left(\Delta A_i\right)\cos\theta_i$$

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For sides 3, 4, 5, and 6, $\boldsymbol{\Delta A}_i \perp \mathbf{E}$, so $\Phi_{E,i} = 0$.

For side 1:

$$\Phi_{E,1} = E\,(\ell^2)\cos(180) = -E\ell^2$$

For side 2:

$$\Phi_{E,2} = E\,(\ell^2)\cos(0) = E\ell^2$$

Electric Flux through Gaussian Surface Example

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For side 1:

$$\Phi_{E,1} = E\left(\ell^2\right)\cos(180) = -E\ell^2$$

For side 2:

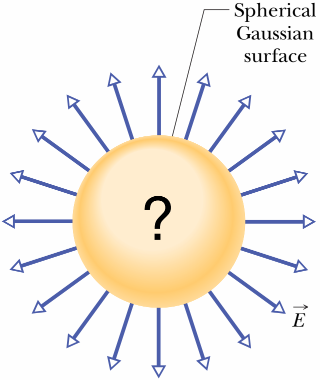
$$\Phi_{E,2} = E\left(\ell^2\right)\cos(0) = E\ell^2$$

In total:

$$\Phi_E = \sum_i \Phi_{E,i} = 0$$

Gauss's Law

Gauss's law relates the electric field across a closed surface (the flux) to the amount of net charge enclosed by the surface.



Gauss's Law

The **net flux through** a surface is directly proportional to the **net charge** enclosed by the surface.

$$\epsilon_0 \Phi_E = q_{\text{enc}}$$

This can also be written:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

This is called the "integral form" of Gauss's Law.

Gauss's Law

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\mathrm{enc}}}{\epsilon_0}$$

General definition of divergence of $\mathbf{u}$ at point p:

$$\nabla \cdot \mathbf{u} = \lim_{V \to \{p\}} \frac{1}{V} \oint \mathbf{u} \cdot \mathrm{d}\mathbf{A}$$

Differential form of Gauss's Law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

where ρ is the charge density.

Divergence

Differential form of Gauss's Law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

where ρ is the charge density.

Divergence of a vector field at a point $\mathbf{v} = [v_x, v_y, v_z]$:

$$\nabla \cdot \mathbf{v} = \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z$$

Intuitively, the divergence is a measure of the "outgoingness" of a vector field at each point.

Electrical Permittivity

Gauss's Law[1] indicates what the permittivity, ϵ, is:

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon}$$

It relates the amount of charge required to generate one unit of electric flux in the vacuum or in a particular medium.

[1] assuming the material is homogeneous, isotropic, and linear, and the field is static

Electrical Permittivity

Gauss's Law[1] indicates what the permittivity, ϵ, is:

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon}$$

It relates the amount of charge required to generate one unit of electric flux in the vacuum or in a particular medium.

Different materials have different values of ϵ, depending on how they become polarized in response to an electric field.

Confusingly, a larger permittivity indicates a larger "resistance" to an electric field.

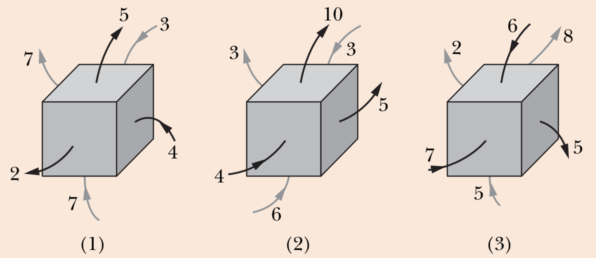
$$\epsilon = \kappa\epsilon_0$$

where κ is the *dielectric constant* or *relative permittivity* of the material (see Ch. 26) and ϵ_0 is the vacuum permittivity.

[1] assuming the material is homogeneous, isotropic, and linear, and the field is static

Gauss's Law Question

Three Gaussian cubes sit in electric fields. The arrows and the values indicate the directions of the field lines and the magnitudes (in $\mathrm{Nm^2/C}$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.)



The cube (1) encloses:

(A) positive charge

(B) negative charge

(C) zero charge

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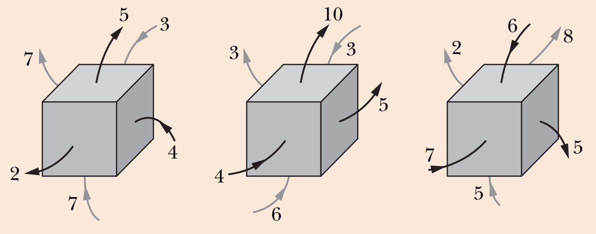

(1) (2) (3)

The cube (1) encloses:

(A) positive charge

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(C) zero charge $\longleftarrow$

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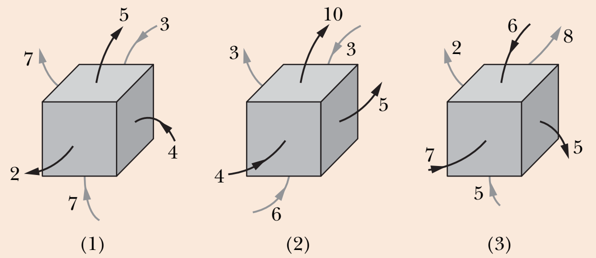

The cube (2) encloses:

(A) positive charge

(B) negative charge

(C) zero charge

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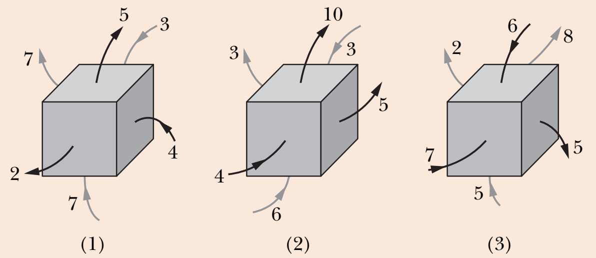

The cube (2) encloses:

(A) positive charge $\longleftarrow$

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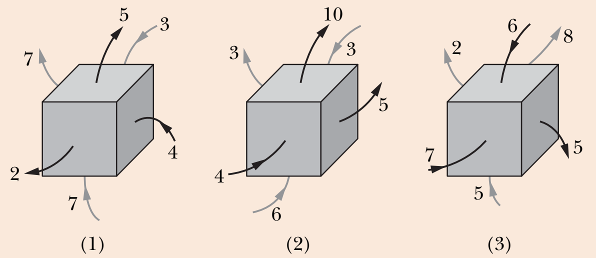

The cube (3) encloses:

(A) positive charge

(B) negative charge

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Gauss's Law Question

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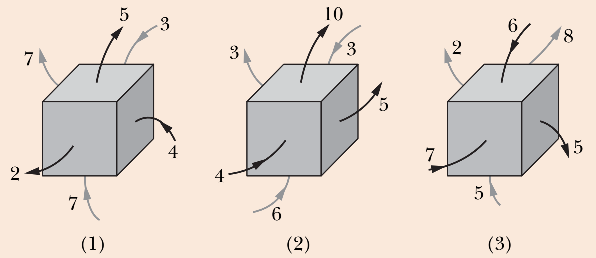


The cube (3) encloses:

(A) positive charge

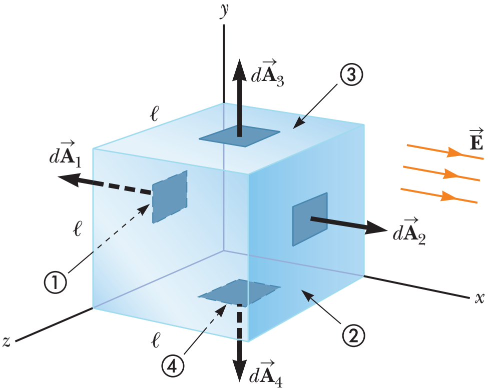
(B) negative charge $\leftarrow$

(C) zero charge

Example: uniform field

Return to this example:

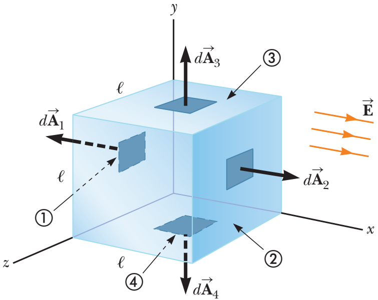
Consider a uniform electric field $\mathbf{E} = E\,\mathbf{i}$ in empty space. A cube of edge length ℓ, is placed in the field, oriented as shown. Find the net electric flux through the surface of the cube.



Electric Flux through Gaussian Surface Example

We found the net electric flux through the surface of the cube:

$$\Phi_E = \sum_i \Phi_{E,i} = 0$$



From Gauss's Law $\epsilon_0 \Phi_E = q_{\text{enc}}$, we know:

$$q_{enc} = 0$$

This is always true for any Gaussian surface in a uniform electric field.

Summary

- electric flux
- Gauss's law

Quiz tomorrow.

Homework

- Collected homework 1, posted online, due on Monday, Jan 22.

Serway & Jewett:

- Ch 24, Obj Qs: 3; Conc. Qs: 1, 5; Probs: 3, 7, 17, 21