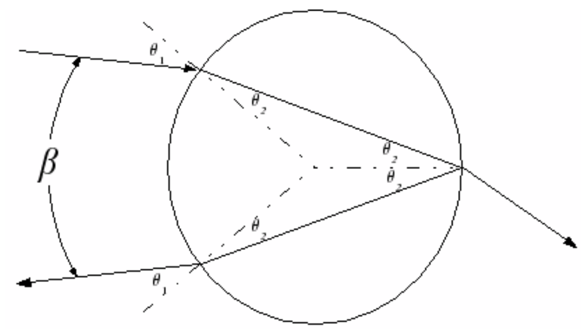
Physics 4C: Collected Homework 4

Solutions are not considered complete without the logical argument and/or full calculation.

1. The diagram shows sunlight being refracted as it shines into a spherical raindrop, being reflected once, and then being refracted again as it leaves the drop. The angle of incidence at the first surface is θ_1 and the angle of refraction at the first surface is θ_2. All of the angles labelled θ_2 are equal in magnitude (convince yourself using the law of reflection and the fact that the drop is a sphere). Assume that the refractive index of air is exactly 1. Take the refractive index of water to be n.



   (a) Using geometry and a clear diagram, find an expression for β (the rainbow angle) in terms of θ_1 and θ_2.

   (b) You can now write β as a function of θ_1 only, since θ_2 is a function of θ_1. Show

$$\beta = 4\sin^{-1}\left(\frac{\sin\theta_1}{n}\right) - 2\theta_1\,.$$

   Find an expression for the maximum value of β and find an expression for $\theta_{1,\text{max}}$, the value of θ_1 that achieves this maximum.

   (c) Sketch a plot the function $\beta(\theta_1)$ vs θ_1. You should see that for a range of values of θ_1 near $\theta_{1,\text{max}}$ the rainbow angle is nearly the same. This means that there is a concentration of rays at the rainbow angle for all the incident angles close to $\theta_{1,\text{max}}$.

   (d) For red light in water, $n = 1.3318$. Find the rainbow angle for red light.

   (e) For violet light in water, $n = 1.3435$. Find the rainbow angle for violet light.

2. **To get full credit your work must include large, clear ray diagrams.**

   (a) Considering two incident rays, show that a convex spherical mirror has a focal length of $f = \frac{R}{2}$, where R is the radius of curvature.

   (b) Prove that the mirror equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

   also holds for convex mirrors.