Fluids, Thermodynamics, Waves, & Optics
Waves
Lab 6
Standing Waves

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Overview

- Purpose
- Theory
- Equipment & Setup
- Procedure

Purpose of the Lab

To explore the behavior of standing waves on a string.

You will use the “Wave on a String” PhET simulation to explore wave reflection and to experimentally determine the speed of a wave in two ways and check for agreement.

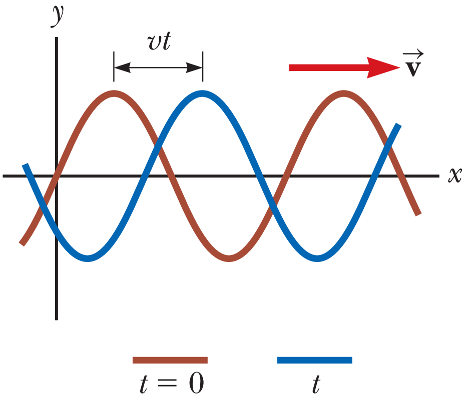
You will repeat this process with the string set to a different tension.

Theory: Sine Waves

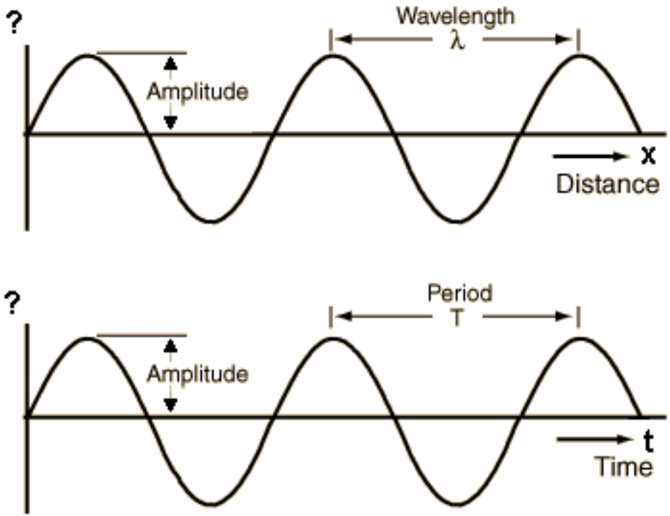
An important form of propagating function f is a sine or cosine wave. (All called "sine waves"). $y(x, t) = A \sin\left(B(x - vt) + C\right)$

This is the simplest periodic, continuous wave.

It is the wave that is formed by a (driven) simple harmonic oscillator connected to the medium.



Wave Quantities



Wave Quantities

wavelength, λ

the distance from one crest of the wave to the next, or the distance covered by one cycle.
units: length (m)

time period, T

the time for one complete oscillation.
units: time (s)

Sine Waves

Recall, the definition of frequency, from period T:

$$f = \frac{1}{T}$$

and

$$\omega = \frac{2\pi}{T} = 2\pi f$$

We also define a new quantity.

Wave number, k

$$k = \frac{2\pi}{\lambda}$$

units: m^{-1}

Wave speed

How fast does a wave travel?

$$\text{speed} = \tfrac{\text{distance}}{\text{time}}$$

It travels the distance of one complete cycle in the time for one complete cycle.

$$v = \frac{\lambda}{T}$$

But since frequency is the inverse of the time period, we can relate speed to frequency and wavelength:

$$v = f\lambda$$

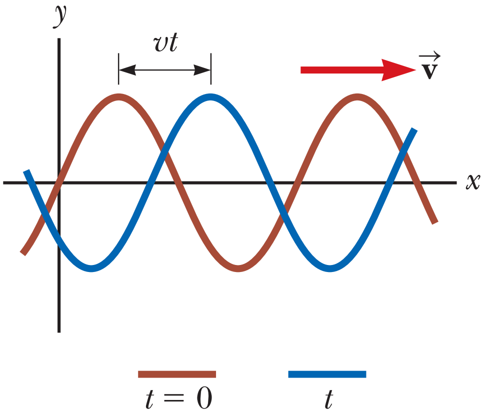
Wave speed

$$v = f\lambda$$

Since $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$:

$$v = \frac{\omega}{k}$$

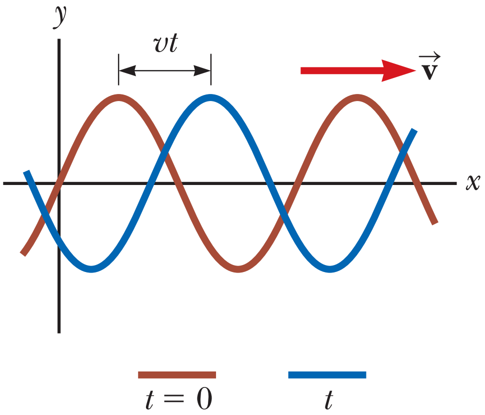
Sine Waves



$$y(x, t) = A \sin\left(\frac{2\pi}{\lambda}(x - vt) + \phi\right)$$

This is usually written in a slightly different form...

Sine Waves



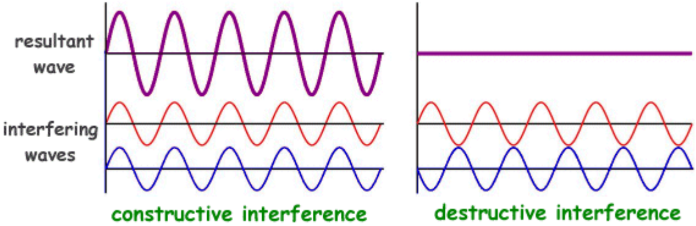
$$y(x, t) = A \sin (kx - \omega t + \phi)$$

where ϕ is a phase constant.

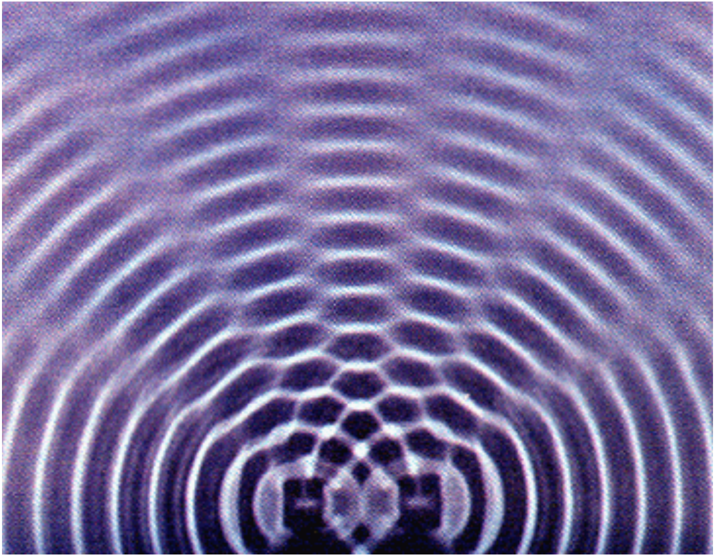
Theory: Interference of Waves

When two wave disturbances interact with one another they can amplify or cancel out.

Waves of the same frequency that are "in phase" will reinforce, amplitude will increase; waves that are "out of phase" will cancel out.



Interference of Waves



Interference of Waves

Waves that exist at the same time in the same position in space add together.

superposition principle

If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

This works because the wave equation we will be studying is *linear*.

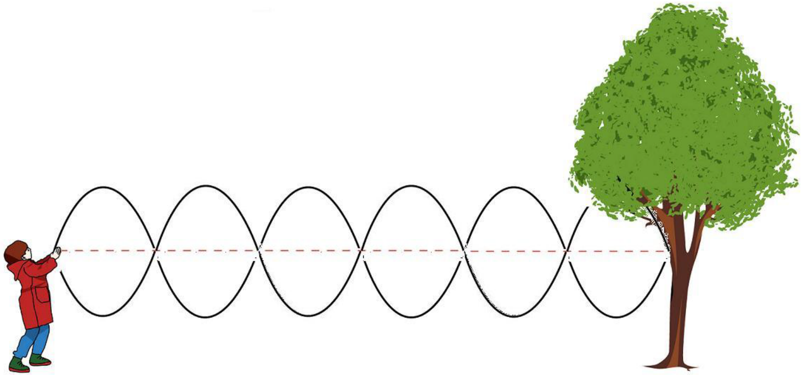
This means solutions to the wave equations can be added:

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

y is the resultant wave function.

Theory: Standing Waves

Standing waves are formed from sine waves that are traveling in opposite directions.



Notice that there are a whole number of half wavelengths between the child and the tree.

Standing Waves

The incoming wave:

$$y_1(x, t) = A\sin(kx - \omega t)$$

Reflected wave:

$$y_2(x, t) = A\sin(kx + \omega t)$$

Using the trig identity:

$$\sin(\theta \pm \psi) = \sin\theta\cos\psi \pm \cos\theta\sin\psi$$

The resultant wave is:

$$y = \underset{\substack{\uparrow \\ \textbf{Amplitude at } x}}{[2A\sin(kx)]} \;\; \underset{\substack{\uparrow \\ \textbf{SHM oscillation}}}{\cos(\omega t)}$$

Standing Waves

$$y = [2A\sin(kx)]\ \cos(\omega t)$$

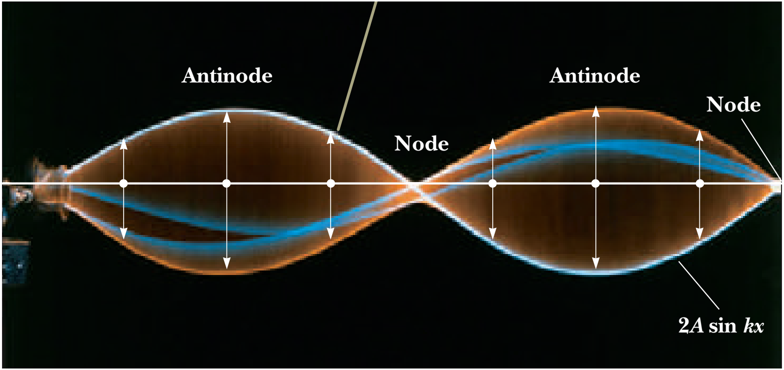
This does not correspond to a traveling wave!

It is a **standing wave**.

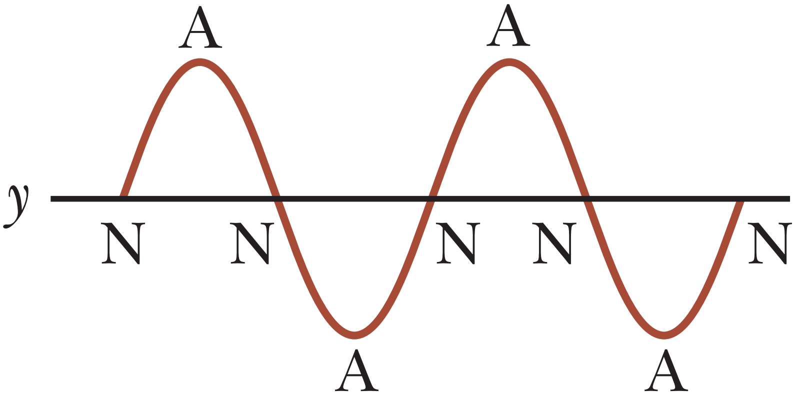
Points where $\sin kx = 0$ are called **nodes**. At these points the medium does not move.

Points where $\sin kx = \pm 1$ are called **antinodes**. At these points particles in the medium undergo their largest displacement.

Nodes and Antinodes



Nodes and Antinodes

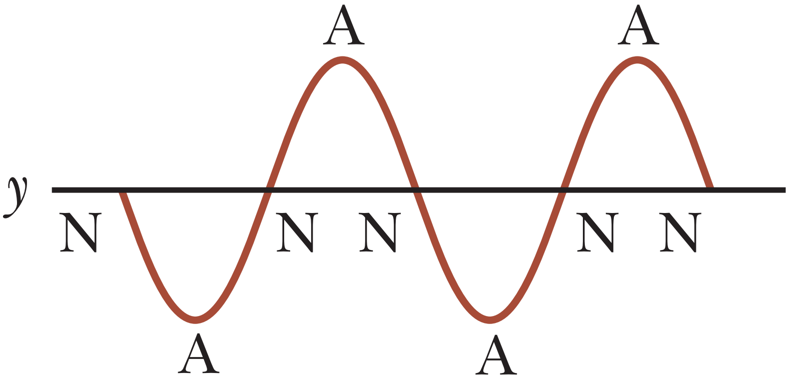


$t = 0$

Nodes and Antinodes

$$t = T/4$$

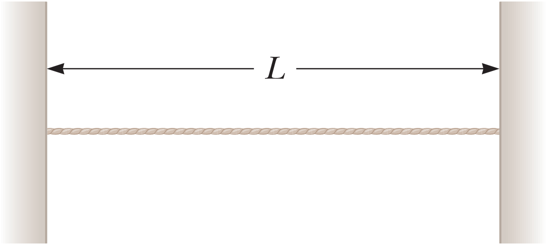
Nodes and Antinodes



$$t = T/2$$

Standing Waves and Resonance on a String

For a given string, fixed at both ends, only some wavelengths can correspond to standing waves.



The boundary conditions are now

$$y(x = 0, t) = y(x = L, t) = 0$$

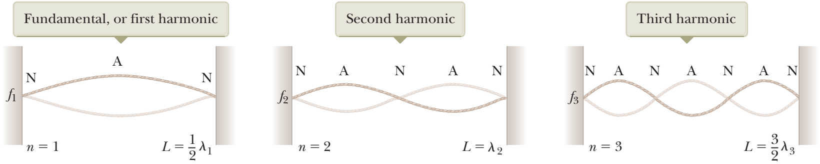
$x = 0$ and $x = L$ must be the positions of nodes.

Standing Waves and Resonance on a String

The wavelengths of these normal modes are given by the constraint $\sin(0) = \sin(kL) = 0$:

$$\lambda_n = \frac{2L}{n}$$

where n is a positive natural number (1, 2, 3...).



$\lambda_1 = 2L$ $\qquad$ $\lambda_2 = L$ $\qquad$ $\lambda_3 = \frac{2L}{3}$

Standing Waves and Resonance on a String

$$v = f\lambda \qquad ; \qquad \lambda_n = \frac{2L}{n}$$

The frequencies that correspond to these wavelengths are called the **natural frequencies**:

$$f_n = \frac{nv}{2L} = n\, f_1$$

where n is a positive natural number.

where v is the speed of the wave.

Theory

A constant speed can be found by taking the distance traveled by a disturbance over the time taken:

$$v = \frac{\Delta x}{\Delta t}$$

For a string of density μ under tension T, the wave speed is constant

$$v = \sqrt{\frac{T}{\mu}}$$

This should equal to the product of the measured frequency and wavelength.

$$v = f\lambda$$

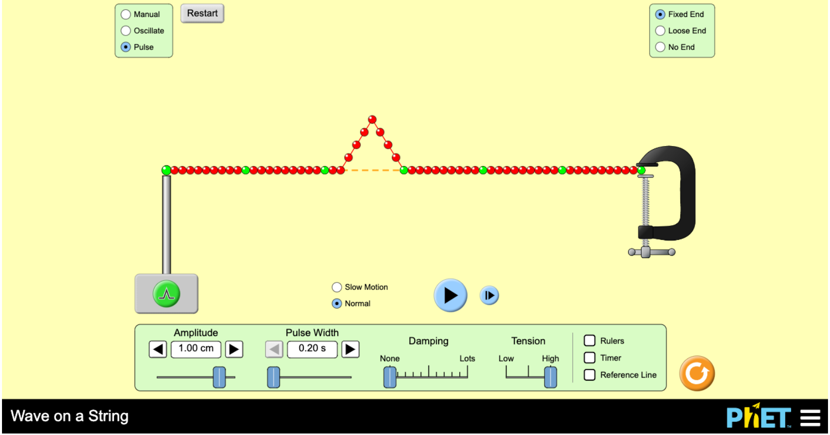
Lab Activity

First, explore wave reflections on the string.

Then, measure the speed of the wave on the string.

Simulation: Pulse Mode

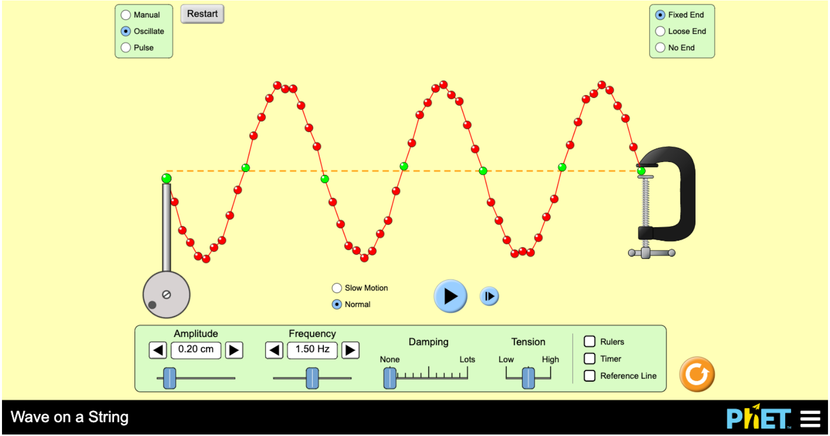
Use the *Pulse* setting to measure the wave speed using distance-over-time.



Lab Activity

Use the *Oscillation* setting to find the resonant (natural) frequencies.

Simulation: Oscillation Mode



Lab Activity

1. Repeat: slowly increase / change the frequency and find the next resonance.

2. Try to record $\lambda_n,\ f_n$ for $n \in \{1, 2, 3, 4, 5, 6\}$. You may find higher modes first; you can record those also.

3. Find $v = f\lambda$ for each pair of λ, f values. (You should have at least 6 values.)

4. Calculate the average wave speed, $\bar{v}$.

5. Calculate the **standard error** of the sample mean using this formula:

$$\text{s.e.}(v) = \sqrt{\frac{\sum_{i=1}^{N}(v_i - \bar{v})^2}{N(N-1)}}$$

Lab Activity

Does the uncertainty range for the value for the speed from distance-over-time overlap with the uncertainty for the speed from the frequency and wavelength measurements?

Repeat for a different tension. Can you find a relationship between the tensions (high and medium)?