

1 Objective

- To experimentally determine Boltzmann's constant using Boyle's, Charles's, and Gay-Lussac's Laws and a simulated ideal gas.

2 Introduction

A gas is often described in terms of its *state variables*: pressure (P), temperature (T), and volume (V) in what is known as an equation of state. In general, equations of state are complicated but in the case of a gas with sufficiently low pressure the ideal gas law is a good model. The ideal gas law is as follows:

$$PV = Nk_B T \quad (1)$$

Where P, V, and T are defined above, N is the number of particles in the sample, and $k_B = 1.38 \times 10^{-23} \text{ J/K}$ is Boltzmann's constant.

For historical reasons (different aspects of the ideal gas law were discovered at different times), the ideal gas law is often expressed as three separate relationships. Boyle's law states that for constant temperature, the pressure of an ideal gas is inversely proportional to its volume.

$$p \propto \frac{1}{V} \quad (\text{For } T = \text{constant}) \quad (2)$$

Charles's Law states that the volume of a gas is proportional to its temperature when the pressure is held constant.

$$V \propto T \quad (\text{For } P = \text{constant}) \quad (3)$$

And finally, Gay-Lussac's Law states that for constant volume, pressure and temperature are directly proportional.

$$P \propto T \quad (\text{For } V = \text{constant}) \quad (4)$$

Taken together, and inserting the proper constants of proportionality, these three laws are equivalent to the ideal gas law.

This lab will be done virtually using a *PhET* simulation created at the University of Colorado at Boulder. Click [here](#) to get started with the simulation.

3 Procedure

3.1 Boyle's Law

In this part of the lab you will vary the volume of a container of ideal gas and observe the effect it has on pressure to experimentally determine Boltzmann's constant.

1. Open the Gas Properties simulation and choose the *Ideal* option.
2. In the bottom right corner of the page you will see three dots next to the acronym PhET. Click the dots and then *Options...* and deselect the *Pressure Noise* option. This will remove the random fluctuations in the pressure readings, making them easier to read.
3. Add between 50 and 150 particles to the container by raising and lowering the handle on the pump and/or clicking the plus sign next to *particles* and clicking the arrows. You can add heavy or light particles or a combination of the two. Record N , the total number of particles.

4. The temperature of the gas is arbitrary but you can adjust it by toggling the switch between *Heat* and *Cool* on the bucket beneath the container of gas. Record T , the temperature.
5. Under *Hold Constant* select *Temperature*
6. Select *Width* from the options on the right of the screen to display the width of the container. The height ($h = 8.75 \text{ nm}$) and depth ($d = 4.0 \text{ nm}$) are fixed and the width is variable (notice the handle on the left of the container).
7. With the temperature held constant, the volume of the container can be modified and the effect on the pressure of the gas observed. Start by sliding the handle on the container as far to the right as possible ($w = 5.0 \text{ nm}$) and record the width and pressure (in kPa) in a table such as Table 1.
8. Move the handle to the left in 1.0 nm increments until it reaches 15.0 nm and record the width and pressure each time for a total of 11 data points.
9. The container is a rectangular prism so its volume is $V = \ell wh$ where ℓ is length, w is width, and h is height. You will be plotting pressure vs. inverted volume ($\frac{1}{V}$) so you will need to calculate the volume of the container (at each incremental increase of width) to obtain the value $\frac{1}{V}$. My advice is to do this all in a spreadsheet program (Excel, Google Sheets, etc.) but you may write it down on paper if you prefer.
10. Plot P (vertical axis) vs. $\frac{1}{V}$ (horizontal axis) in your spreadsheet program and generate a linear trendline. The plot will be turned in so please
 - Label your axes (include units), and
 - Display equation on chart
11. Compare Boyle's Law (Equation 2) with the ideal gas law (Equation 1) to determine the constant of proportionality between P and $\frac{1}{V}$. Compare this expression with the slope of the trendline to determine the experimental value of Boltzmann's constant (k_B).
12. Calculate the percent error between the experimental value of k_B and the accepted value of $k_B = 1.38 \times 10^{-23} \text{ J/K}$
13. For this section your writeup should include your data table and any calculations you did to make your plot (which may be part of your table), the plot with trendline and equation, and your calculation of Boltzmann's constant and the percentage error.

$w(\text{nm})$	$P(kPa)$

Table 1: Sample data table for section 3.1 (your table will require more rows). You may also want to do your calculation of $1/V$ as part of this table.

3.2 Charles's Law or Gay-Lussac's Law

Your goal in this section is to devise a method of determining Boltzmann's constant using either Charles's Law or Gay-Lussac's Law. You may use the same tools as in part 3.1: the ideal gas simulation, spreadsheet software, and the equations that appear in the introduction to this lab. Your solution should involve a plot and a minimum of 8 data points.

1. Experimentally determine Boltzmann's constant. The following notes on the simulation may be helpful:

- Under *Hold Constant*, the option *Pressure* $\updownarrow V$ will hold pressure constant by adjusting the volume of the container as needed. You will be allowed to vary the temperature and the volume will change automatically.
 - The option *Pressure* $\updownarrow T$ is similar. Selecting this option will allow you to vary the volume of the container while the temperature automatically adjusts to keep pressure constant.
2. Calculate the percent error between your experimental value and the accepted value of k_B .
 3. Write up what you did: briefly (3-4 sentences) describe the method you used in this section to experimentally determine the Boltzmann constant. Include the following information:
 - Which law did you use?
 - Which state variable(s) did you hold constant, and which did you vary?
 - What did you plot and how did you use your plot to solve for k_B ?

You do *not* need to write things like “We held the temperature constant by clicking on *temperature* under *hold constant*”. Simply writing, “We held the temperature constant” is sufficient. (Keep it brief.)

4. As in section 3.1, your writeup for this section should include your data table and any calculations you did to make your plot (which may be part of your table), the plot with trendline and equation, and your calculation of Boltzmann’s constant and the percentage error.