1. The string shown is driven sinusoidally at a frequency of 6.00 Hz. The amplitude of the motion is $A = 10.0$ cm, and the wave speed is $v = 25.0$ m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$.

   (a) Determine the angular frequency for this wave. [2 pts]

   (b) Determine the wave number for this wave. [2 pts]

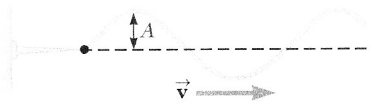
   (c) Write an expression for the wave function. [2 pts]

   (d) Calculate the maximum transverse speed of an element of the string. [3 pts]

   (e) Calculate the maximum transverse acceleration of an element of the string. [3 pts]

   (f) Considering the acceleration of an element of the string, show that the motion of each element is simple harmonic motion. [2 pts]

   (g) Suppose the string has a length 1.00 m and a mass of 20.0 g. What is the power of this wave? [4 pts]



a) Find ω.

$\omega = 2\pi f$

$= 2\pi (6\ Hz)$

$= 37.7\ Hz$

(or $37.7\ s^{-1}$)

b) Find k.

$k = \frac{2\pi}{\lambda}$

$k = \frac{\omega}{v} = \frac{37.7\ s^{-1}}{25.0\ m s^{-1}}$

$k = 1.51\ m^{-1}$

c) $y = A \sin(kx - \omega t)$

$y = (10.0\ cm) \sin((1.51\ m^{-1})x - (37.7\ s^{-1})t$

y in cm.
x in m.
t in s.

d) $v_y = \frac{\partial y}{\partial t}$

$= \frac{\partial}{\partial t}(A \sin(kx - \omega t))$

$= (-\omega) A \cos(kx - \omega t)$

this will reach max. value when $\cos(kx - \omega t) = -1$

$v_{y\,max} = \omega A$

$= (37.7\ s^{-1})(0.100\ m)$

$= 3.77\ m s^{-1}$

e) $a_y = \frac{\partial^2 y}{\partial t^2}$

$= \frac{\partial}{\partial t}(A(-\omega) \cos(kx - \omega t))$

$= -A(-\omega)^2 \sin(kx - \omega t)$

$= -\omega^2 y$

Reaches max. when $\sin(kx - \omega t) = -1$

$a_{y\,max} = \omega^2 A$

$= (37.7\ s^{-1})^2 (0.100\ m)$

$= 142\ m s^{-2}$

f) Definition of SHM:

$a_x = -(\text{const})\, x$

displac

$\rightarrow a_y = -\omega^2 y$

$\therefore$ Meets Def. of SHM

g) $\mu = \frac{m}{L}$

$= \frac{0.0200\ kg}{1.00\ m}$

$= 0.02\ kg/m$

$P = \frac{1}{2}\mu\omega^2 A^2 v$

$= \frac{1}{2}(0.0200)(0.100)^2(25)(37.7)^2$

$P = 3.55\ W$

1. A standing-wave pattern is observed in a thin wire with a length of 1.20 m. Let the point $x = 0$ be the left end of the wire. The wave function is

$$y = 3.00 \sin\left(\frac{10\pi}{3}x\right)\cos(180\pi t)$$

where x is in meters, y is in millimeters, and t is in seconds.

(a) What is the wavenumber of this wavefunction? [1 pt]

(b) What is the wave speed on this wire? [2 pts]

(c) How many loops does this pattern exhibit? [3 pt]

(d) Consider an element of the wire at a point $x = 0.05$ m. What is the maximum transverse displacement of this element? [3 pts]

(e) Show that the transverse motion of the element at $x = 0.05$ m is simple harmonic motion (SHM), by referring to the definition of SHM. [5 pts]

a) wavenumber has the symbol k.

$$\underline{k = \frac{10\pi}{3}\ m^{-1}}$$

b) $$v = \frac{\omega}{k} = \frac{180\pi}{\left(\frac{10\pi}{3}\right)}$$

$$\underline{v = 54\ m/s}$$

c) # loops = # of half-λs

$$= \frac{L}{(\lambda/2)} \qquad \lambda = \frac{2\pi}{k}$$

$$= \frac{Lk}{\pi} = \frac{(1.20m)\left(\frac{10\pi}{3}\right)}{\pi} = \underline{4\text{ loops}}$$

d) Transverse displacement is y.

@ $x = 0.05\ m$

$$y_{max}(x=0.05) = \overset{mm}{3.00}\sin\left(\underbrace{\frac{10\pi}{3}(0.05m)}_{\pi/6}\right) = \underline{1.50mm}$$

e) Def. of SHM

$$\frac{\partial^2 y}{\partial t^2} = -\underbrace{c}_{\text{some constant}}\, y$$

i.e. the acceleration of the oscillator is directly proportional to its displacement from equilibrium, but in the opposite direction. $\left(a_y = \frac{\partial^2 y}{\partial t^2}\right)$

at $x = 0.05\ m$

$$y = 1.50\cos(180\pi t)$$

we need to show this satisfies the definition.

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left((-180\pi)\,1.50\sin(180\pi t)\right)$$

$$= -(180\pi)^2\,1.50\cos(180\pi t)$$

$$= -\underbrace{(180\pi)^2}_{\text{a const.}}\, y$$

$\therefore$ satifies definition.

1. Two waves are described by the wave functions

$$y_1(x,t) = 6.00\sin(2\pi x - 10\pi t),$$

$$y_2(x,t) = 8.00\cos(2\pi x - 10\pi t + \pi)$$

where x, y_1, and y_2 are in meters and t is in seconds.

(a) What are the wavelength, frequency, and speed of these waves? [6 pts]

(b) Show that the wave resulting from their superposition can be expressed as a single sine function, and give this resultant wave explicitly, including the amplitude and phase angle giving units. [6 pts]

a) The wavelength, frequency, and speed will be the same for both waves.

waves are of the form:

$y = A\sin(kx - \omega t + \phi)$

$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi}$

wavelength, $\underline{\lambda = 1\,m}$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$f = \frac{10\pi}{2\pi}$

frequency, $\underline{f = 5\,Hz}$

$v = f\lambda$

$= (5\,s^{-1})(1\,m)$

$\underline{v = 5\,m/s}$

or $v = \frac{\omega}{k} = \frac{(10\pi\,s^{-1})}{(2\pi\,m^{-1})}$

speed, $\underline{v = 5\,m/s}$

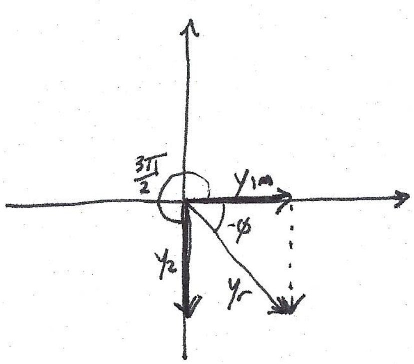
b) $y_r = y_1 + y_2$

Convert y_2 to a sine: $\cos\theta = \sin(\theta + \frac{\pi}{2})$ (identity)

$y_2 = (8.00\,m)\sin(2\pi x - 10\pi t + \pi + \frac{\pi}{2})$

$= (8.00\,m)\sin(2\pi x - 10\pi t + \frac{3\pi}{2})$

phasor diagram:



because $\vec{y}_1$ and $\vec{y}_2$ are at right angles:

$|\vec{y}_r| = \sqrt{|\vec{y}_{1}|^2 + |\vec{y}_{2}|^2}$

$= \sqrt{6^2 + 8^2}$

$\underline{= 10.0\,m}$

$\tan\phi = \frac{-|\vec{y}_2|}{|\vec{y}_1|}$

$\phi = \tan^{-1}\left(-\frac{8}{6}\right)$

$\underline{= -0.927\,rad}$

$\underline{= -0.295\pi\,rad}$

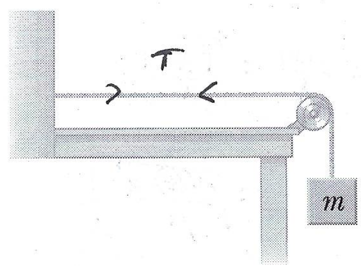
$\underline{= -53.1^\circ}$

$\underline{y_r = y_1 + y_2 = (10.0\,m)\sin((2\pi\,m^{-1})x - (10\pi\,s^{-1})t - 0.927)}$

4. A standing wave in a rope is described by the wave function

$$y = 0.20 \sin(0.75\pi x) \cos(18\pi t)$$

where x and y are in meters and t is in seconds. The rope has a linear mass density of 10.0 g/m. The tension in the rope is provided by a container of mass m suspended as shown in the figure.



(a) What is the mass m? [5 pts]

(b) If the string is vibrating in its fundamental mode, what is the length of the horizontal portion of the string? [4 pts]

(c) Now sand is poured at a constant rate of 200 g/s into the suspended container, initially of mass m, for 3.00 s. Suppose the string vibrates in its fundamental mode throughout this process. Find the number of oscillations it completes during the 3.00-s interval. [7 pts]

a) system: block in equil.

$F_{net} = 0$ $\qquad v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow T = mg \leftarrow T = \mu v^2$

$\mu v^2 = mg$

$m = \frac{\mu v^2}{g} \leftarrow v = \frac{\omega}{k}$

$m = \frac{\mu \omega^2}{g k^2}$

$= \frac{(0.010 \text{ kg/m})(18\pi)^2}{(9.8 \text{ m/s}^2)(0.75\pi)^2}$

$\underline{m = 0.588 \text{ kg}}$

b) For nodes at each end: $f_n = \frac{nv}{2L}$

$f_1 = \frac{v}{2L}$

$\frac{\omega}{\cancel{2}\pi} = \frac{v}{\cancel{2}L}$

$L = \frac{\pi v}{\omega}$ $\qquad v = \frac{\omega}{k}$

$= \frac{\pi}{k}$

$= \frac{\pi}{0.75\pi}$

$= \frac{4}{3}$ m

$\underline{L = 1.33 \text{ m}}$

c) next page

–Extra Workspace–

c) Let the number of oscillations be N – this is the area under a frequency - time curve

Let $r = 200\,g/s$

frequency as a function of time

$$N = \int_0^3 f(t)\,dt \qquad f(t) = \frac{V(t)}{\lambda} = \sqrt{\frac{T(t)}{\mu}}\frac{1}{\lambda}$$

Container remains in equilibrium

$\Rightarrow F_{net} = 0$

$T = m'g$

$$= \sqrt{\frac{m'(t)\,g}{\mu}}\frac{1}{\lambda}$$

$m'(t) = m + rt$

$\lambda = 2L$ (fundamental mode)

$$= \sqrt{\frac{(m+rt)g}{\mu}}\frac{1}{2L}$$

$$= \int_0^3 \frac{g^{\frac{1}{2}}}{2L\mu^{\frac{1}{2}}}(m+rt)^{\frac{1}{2}}\,dt$$

$$= \frac{g^{\frac{1}{2}}}{2L\mu^{\frac{1}{2}}}\left[\frac{2}{3r}(m+rt)^{\frac{3}{2}}\right]_0^3$$

$$= \frac{1}{3Lr}\sqrt{\frac{g}{\mu}}\left[(m+3r)^{\frac{3}{2}} - m^{\frac{3}{2}}\right]$$

$= \underline{33.0}$ oscillations