1. **[12pts]** A sample of a solid substance has a volume V_0 and a density ρ_0 at a temperature T_0. The specific heat capacity of the sample is c and the coefficient of volume expansion is β. The solid sample is heated at in air at constant pressure P_0. If the temperature of the substance is increased by an amount ΔT and it remains a solid,

(a) find and expression for the new density of the substance. [4 pts]

(b) what is the work done on the sample as it is heated? [4 pts]

(c) what is the change in the internal energy of the sample? [4 pts]

a) $\Delta V = \beta V_0 \Delta T$ and $\rho_0 = \frac{m}{V_0}$

$$\rho' = \frac{m}{V_0 + \Delta V} \quad \text{(mass is constant)}$$

$$= \frac{m}{V_0(1+\beta \Delta T)}$$

$$\rho = \frac{\rho_0}{(1+\beta \Delta T)}$$

b) $$W = -\int_{V_0}^{V_0+\Delta V} P_0 \, dV$$

$$= -P_0 (V_0 + \Delta V - V_0)$$

$$W = -P_0 \beta V_0 \Delta T$$

c) $\Delta E_{int} = Q + W$ $\qquad Q = mc\Delta T$

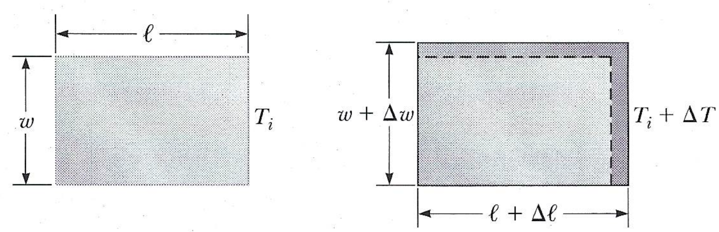
$$= mc\Delta T - P_0 V_0 \beta \Delta T$$

$$\Delta E_{int} = \rho_0 V_0 c \Delta T - P_0 V_0 \beta \Delta T$$

1. The rectangular plate shown has an area A_i equal to ℓw. For this plate α is the average coefficient of linear expansion.

   (a) If the temperature increases by ΔT, show that the increase in area is $\Delta A = 2\alpha A_i \Delta T$. [5 pts]

   (b) What approximation does this expression assume? [1 pt]



a) For linear expansion:

$\Delta L = \alpha L_i \Delta T$ so $\Delta l = \alpha l \Delta T$, $\Delta w = \alpha w \Delta T$

Area increase:

$A_i = lw$

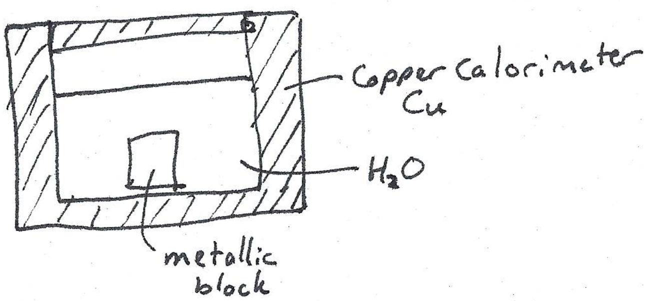
$$\Delta A = A_f - A_i$$
$$= (l + \Delta l)(w + \Delta w) - lw$$
$$= lw + l\Delta w + w\Delta l + \Delta l \Delta l - lw$$
$$= l(\alpha w \Delta T) + w(\alpha l \Delta T) + (\alpha l \Delta T)(\alpha w \Delta T)$$
$$= 2\alpha (lw)\Delta T + (lw)\alpha^2 \Delta T^2$$
$$= 2\alpha A_i \Delta T + \alpha^2 \Delta T^2 A_i$$

assuming $(\alpha \Delta T)^2 \ll 1$ we can neglect the second term.

$$\Delta A = 2\alpha A_i \Delta T$$

b) $(\alpha \Delta T)^2 \ll 1$ (which means we assume $\Delta l \Delta w$ is much less than $w\Delta l + l\Delta w$)

2. A copper calorimeter with a mass of 250 g contains 500 g of water. The calorimeter and water are in thermal equilibrium at 15.0°C. A metallic block is placed into the water, which has a mass of 150.0 g and is originally at a temperature of 125°C. The entire system stabilizes at a final temperature of 20.0°C. The specific heat of copper is 387 J kg^{-1} °C^{-1}. Determine the specific heat of the unknown sample. [6 pts]



$$-Q_{block} = Q_{H_2O + Cu}$$

$$-Q_{block} = Q_{H_2O} + Q_{Cu}$$

$$C_{H_2O} = 1\ cal/g°C = 4.186\ J/g°C = 4186\ J/kg°C$$

$$-m_b C_b \Delta T_b = m_{H_2O} C_{H_2O} \Delta T_{H_2O} + m_{Cu} C_{Cu} \Delta T_{Cu}$$

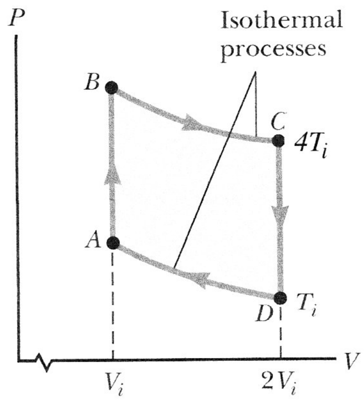
$$\Delta T_{H_2O} = \Delta T_{Cu}$$

$$C_b = \frac{(m_{H_2O} C_{H_2O} + m_{Cu} C_{Cu}) \Delta T_{H_2O}}{-m_b \Delta T_b}$$

$$= \frac{[(0.50\ kg)(4186\ J/kg°C) + (0.250\ kg)(387\ J/kg°C)](20-15)°C}{-(0.150\ kg)(20-125)°C}$$

$$C_b = 695\ J/kg°C$$

4. Consider n moles of an ideal monatomic gas being taken once through the Stirling engine cycle shown, consisting of two isothermal processes at temperatures $4T_i$ and T_i and two constant-volume processes.



(a) Find an expression for the net work done ~~by the engine~~ on the gas in a single complete cycle. [5 pts]

(b) What is the net heat transferred into the gas during a single complete cycle? [2 pts]

a)

$W_{eng} = -W$ ← work done <u>on</u> gas

$= -\left(-\int P\,dV\right)$

$= \int P\,dV$

$W_{eng} = -W_{AB}^{\nearrow 0} - W_{BC} - W_{CD}^{\nearrow 0} - W_{DA}$ 0 const. volume.

$= \int_{BC} P\,dV + \int_{DA} P\,dV$ $PV = nRT$, $P = \frac{nRT}{V}$

$= \int_{BC} \frac{nR(4T_i)}{V}\,dV + \int_{DA} \frac{nRT_i}{V}\,dV$

isothermal:

$= 4nRT_i \ln\left(\frac{2V_i}{V_i}\right) + nRT_i \ln\left(\frac{V_i}{2V_i}\right)$

$= 4nRT_i \ln(2) - nRT \ln(2)$

$W_{eng} = 3nRT_i \ln 2$

<u>$W = -3nRT_i \ln 2$</u>

b) Cycle: $\Delta E_{int} = 0$

$\Delta E_{int} = Q + W$

$0 = Q - W_{eng}$

$Q = W_{eng}$

<u>$Q = 3nRT_i \ln 2$</u> ↙ energy for work done by engine came from heat supplied to engine.