6. [10pts] The distance between an object and its upright image is 18.0 cm. If the magnification is 0.500,

(a) what kind of mirror is being used to form the image? [1 pt]

191

(b) what is the focal length of the mirror? [9 pts]

a) image is upright and smaller => convex mirror.





$$\frac{1}{2} = M = -\frac{9}{p} - \frac{1}{p}$$

(1)p

$$i = 1 - \frac{q}{P} = \frac{d}{P}$$

$$i = \frac{1}{P} + \frac{1}{q} = \frac{1}{F}$$

$$i = \frac{pq}{P+q}$$

$$p = \frac{d}{(1+M)}$$

$$= \frac{18.0}{(1+\frac{1}{2})}$$

$$p = 12 \text{ cm}$$

$$q = p - d$$

$$q = -6 \text{ cm}$$

$$i = \frac{1}{P} + \frac{1}{q} = \frac{1}{F}$$

$$f = \frac{pq}{P+q}$$

$$= \frac{(12 \text{ cm})(-6 \text{ cm})}{(12 \text{ cm} - 6 \text{ cm})}$$

$$\frac{f = -12 \text{ cm}}{(-ve \text{ sign i} \text{ iddicates } \text{ convex minor})}$$

2. A material having an index of refraction n is surrounded by vacuum and is in the shape of a quarter circle of radius R. A light ray parallel to the base of the material is incident from the left at a distance L above the base and emerges from the material at the angle θ .



- (a) Show that $\sin \alpha = \frac{L}{R}$.
- (b) Show that $\gamma = \alpha \beta$.
- (c) Show that

1

$$\theta = \sin^{-1}\left(\frac{L}{R^2}\left(\sqrt{n^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right)$$

Hint: you might like to consider these two trig identities: $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta, \quad \cos \theta = \sqrt{1 - \sin^2 \theta}.$



surface at point P. Therefore the two angles labelled r are alternate interior angles and have the same measure. α and $\gamma + \beta$ are vertical angles, so $\alpha = \gamma + \beta \Longrightarrow \gamma = \alpha - \beta$.

c)
$$h_1 \sin \theta_1 = h_2 \sin \theta_1$$

 $\sin \alpha = n \sin \beta$
 $n \sin \theta_1 = \frac{1}{R^2} \sin \theta_1$
 $n \sin \theta_1 = \frac{1}{R^2} \sin \theta_1$
 $n \sin \theta_1 = \frac{1}{R^2} \sin \theta_1$
 $n \sin (\alpha - \beta) = \sin \theta$
 $n \sin \alpha \cos \beta - n \cos \alpha \sin \beta = \sin \theta$
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 $n \sin \theta = \sin \theta$
 $n \sin$

3. The magnification of the image formed by a refracting surface is given by

$$M = -\frac{n_1 q}{n_2 p} \,.$$

A paperweight is made of a solid glass hemisphere with index of refraction 1.50. The radius of the circular cross section is 4.00 cm. The hemisphere is placed on its flat surface, with the center directly over a 2.50-mm-long line drawn on a sheet of paper. What is the length of this line as seen by someone looking vertically down on the hemisphere?

Since the center of curvature of the surface is on the side the light comes from, R < 0 giving R = -4.00 cm. For the line, p = 4.00 cm; then,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes

$$\frac{1.00}{q} = \frac{1.00 - 1.50}{-4.00 \text{ cm}} - \frac{1.50}{4.00 \text{ cm}}$$

or

q = -4.00 cm

Thus, the magnification $M = \frac{h'}{h} = -\left(\frac{n_1}{n_2}\right)\frac{q}{p}$ gives

$$h' = -\left(\frac{n_1 q}{n_2 p}\right)h = -\frac{1.50(-4.00 \text{ cm})}{1.00(4.00 \text{ cm})}(2.50 \text{ mm}) = \boxed{3.75 \text{ mm}}$$