Thermodynamics
Molecular Model of a Gas
Molar Heat Capacities

Lana Sheridan

De Anza College

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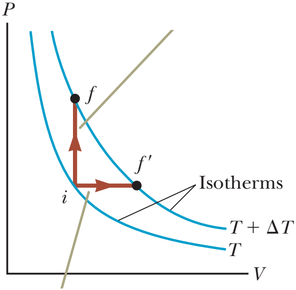
Last time

- modeling an ideal gas at the microscopic level
- rms speed of molecules
- equipartition of energy

Warm Up Questions



Quick Quiz 21.2[1] (i) How does the internal energy of an ideal gas change as it follows path $i \rightarrow f$?

(A) E_{int} increases.

(B) E_{int} decreases.

(C) E_{int} stays the same.

(D) There is not enough information to determine how E_{int} changes.

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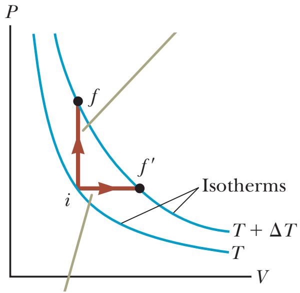
[1] Serway & Jewett, page 631.

Warm Up Questions



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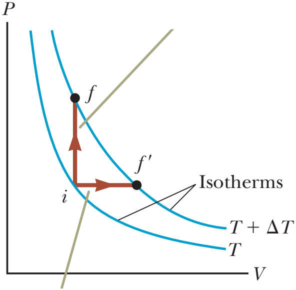

Quick Quiz 21.2[1] (ii) From the same choices, how does the internal energy of an ideal gas change as it follows path $f \rightarrow f'$ along the isotherm labeled $T + \Delta T$?

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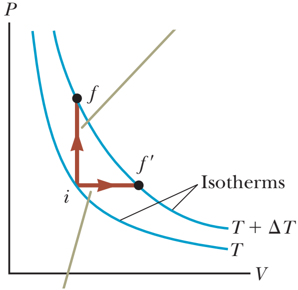
Warm Up Questions



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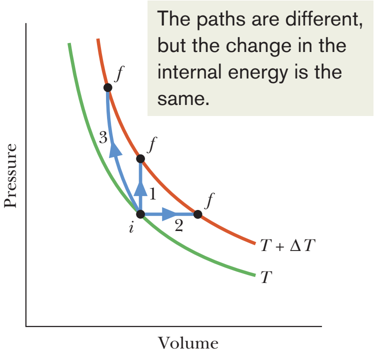
Overview

- molar heat capacities
- values of heat capacities for monatomic ideal gases

Another Look at Heat Capacity

Paths with the same ΔT, ΔE_{int}:



We have related internal energy to temperature through KE (monatomic gas):

$$E_{\text{int}} = K_{\text{tot,trans}} = \frac{3}{2}nRT$$

and from the first law of thermodynamics:

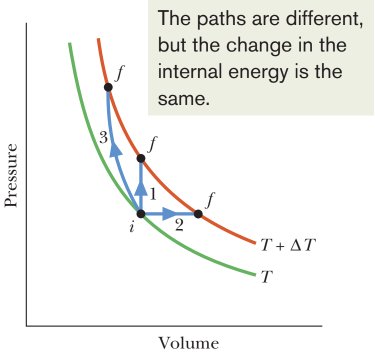
$$\Delta E_{\text{int}} = Q + W$$

Since W is different for the different processes shown, so is Q.

Another Look at Heat Capacity

We already studied specific heat, $c = \frac{Q}{m\,\Delta T}$, particularly for solids and liquids.

Now we must revisit this concept, because for gases there are many ways to change the temperature of a gas by ΔT, with different ways requiring different amounts of heat, Q.



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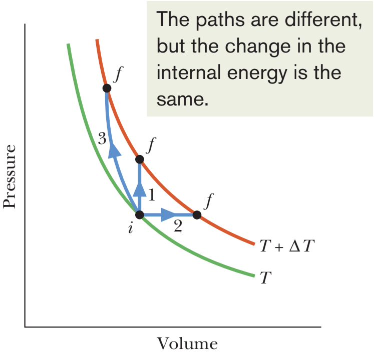
[1] Diagram from Halliday, Resnick, Walker, 9th ed, page 520.

Another Look at Heat Capacity



The paths are different, but the change in the internal energy is the same.

For processes 1, 2, and 3:

$$Q_1 = c_1 m \Delta T$$

$$Q_2 = c_2 m \Delta T$$

$$Q_3 = c_3 m \Delta T$$

Each process has a different value of c!

Molar Specific Heat of an Ideal Gas

For solids and liquids, heat capacity, C, and specific heat capacity, c, are defined to be for **constant pressure** processes.

This is purely because it is very difficult to stop a solid or liquid from expanding and maintain it at a constant volume!
$(\Delta V = \beta V_i \Delta T)$

However, it is possible to measure the heat capacities of solids and liquids at constant volume instead of constant pressure.

When that is done, small differences in the values of the heat capacity are obtained.

Molar Specific Heat of an Ideal Gas

In gases, the variation of the heat capacity obtained for different paths is quite big.

Define:

Molar heat capacity at constant volume, C_V

Along an isovolumetric process (constant volume):

$$Q = nC_V \Delta T$$

Molar heat capacity at constant pressure, C_P

Along an isobaric process (constant pressure):

$$Q = nC_P \Delta T$$

(These are both *intensive* quantities, like specific heat.)

Molar Specific Heat of an Ideal Gas

We have defined **molar** heat capacities (cap. per mole) here and not specific heat capacities (cap. per mass).

Why?

Molar Specific Heat of an Ideal Gas

We have defined **molar** heat capacities (cap. per mole) here and not specific heat capacities (cap. per mass).

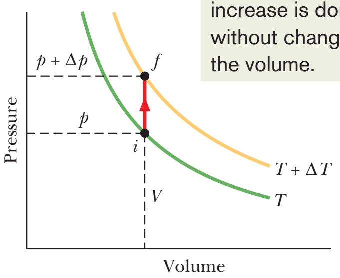
Why?

Using “per mole” as the reference for heat capacity allows us to talk about many different gases with the same relationships, since we will always be talking about the same number of molecules.

It is just more convenient.

Heat Capacity for Constant Volume Processes

In a constant volume process, no work is done: $\Delta E_{\text{int}} = Q$



The temperature increase is done without changing the volume.

Therefore,

$$\Delta E_{\text{int}} = nC_V\,\Delta T$$

and

$$C_V = \frac{1}{n}\left(\frac{\partial E_{\text{int}}}{\partial T}\right)_V$$

(the subscipt V means const volume)

Heat Capacity for Constant Volume Processes

$$C_V = \frac{1}{n}\left(\frac{\partial E_{\text{int}}}{\partial T}\right)_V \qquad \text{(the subscipt } V \text{ means const volume)}$$

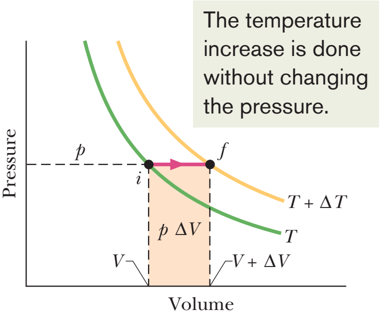
Putting in our value for internal energy (monatomic gas):

$$\begin{aligned} C_V &= \frac{1}{n}\left(\frac{\partial}{\partial T}\left(\frac{3}{2}nRT\right)\right)_V \\ C_V &= \frac{3}{2}R \end{aligned}$$

This is the value for C_V for *all monatomic gases*.

Heat Capacity for Constant Pressure Processes

In a constant pressure process, the work done on the gas is:
$W = -P\,\Delta V$



The temperature increase is done without changing the pressure.

From the ideal gas equation:

$$P\,\Delta V = nR\,\Delta T$$

So,

$$W = -nR\,\Delta T$$

Heat Capacity for Constant Pressure Processes

First law:

$$\Delta E_{\text{int}} = Q + W$$

rearranging:

$$\begin{aligned} \Delta E_{\text{int}} - Q &= W \\ nC_V\,\Delta T - nC_P\,\Delta T &= -nR\,\Delta T \end{aligned}$$

Heat Capacity for Constant Pressure Processes

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because $Q = nC_P\,\Delta T$. Dividing by $-n\,\Delta T$:

$$C_P - C_V = R$$

For a **monatomic** gas:

$$C_P = \frac{5}{2}R$$

Summary

- molar heat capacities
- values of heat capacities for monatomic ideal gases