# Waves
# Quick Review of Oscillations and SHM
# Introducing Waves
# Pulse Propagation

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# Last time

- heat engines
- wrapped up thermodynamics

# Overview

- oscillations and quantities
- simple harmonic motion (SHM)
- spring systems
- introducing waves
- kinds of waves
- pulse propagation
- wave speed on a string (?)

# Oscillations and Periodic Motion

Many physical systems exhibit cycles of repetitive behavior.

After some time, they return to their initial configuration.

Examples:

- clocks
- rolling wheels
- a pendulum
- bobs on springs

# Oscillations

**oscillation**

motion that repeats over a period of time

**amplitude**

the magnitude of the vibration; how far does the object move from its average (equilibrium) position.

**period, $T$**

the time for one complete oscillation.

After 1 period, the motion repeats itself.

# Oscillations

**frequency**

The number of complete oscillations in some amount of time. Usually, oscillations per second.

$$f = \frac{1}{T}$$

Units of frequency: Hertz. $1\ \text{Hz} = 1\ \text{s}^{-1}$

If one oscillation takes a quarter of a second (0.25 s), then there are 4 oscillations per second. The frequency is $4\ \text{s}^{-1} = 4\ \text{Hz}$.

# Oscillations

**angular frequency**

the rate of change of the phase of a sinusoidal oscillation or wave function

$$\omega = \frac{2\pi}{T} = 2\pi f$$

# Simple Harmonic Motion

The oscillations of bobs on springs and pendula are very regular and simple to describe.

It is called simple harmonic motion.

**simple harmonic motion (SHM)**

any motion in which the acceleration is proportional to the displacement from equilibrium, but opposite in direction

The force causing the acceleration is called the “restoring force”.

# SHM and Springs

How can we find an equation of motion for a mass on a spring?

Newton's second law:

$$\mathbf{F}_{\text{net}} = \mathbf{F}_s = m\mathbf{a}$$

# SHM and Springs

How can we find an equation of motion for a mass on a spring?

Newton's second law:

$$\mathbf{F}_{\text{net}} = \mathbf{F}_s = m\mathbf{a}$$

Using the definition of acceleration: $a = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{k}{m}x$$

Define

$$\omega = \sqrt{\frac{k}{m}}$$

and we can write this equation as:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

# SHM and Springs

To solve:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

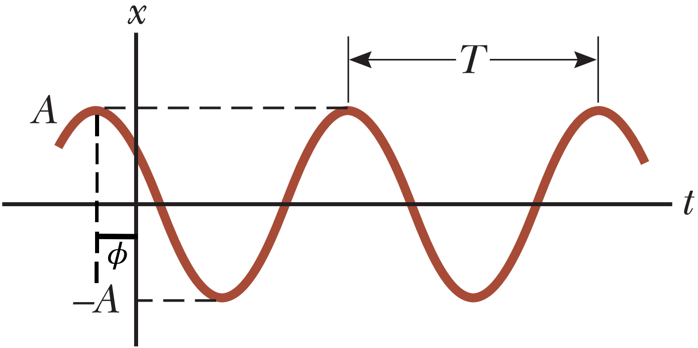
notice that it is a second order linear differential equation.

Any solution can be written in the form:

$$x = A\cos(\omega t + \phi)$$

# Waveform

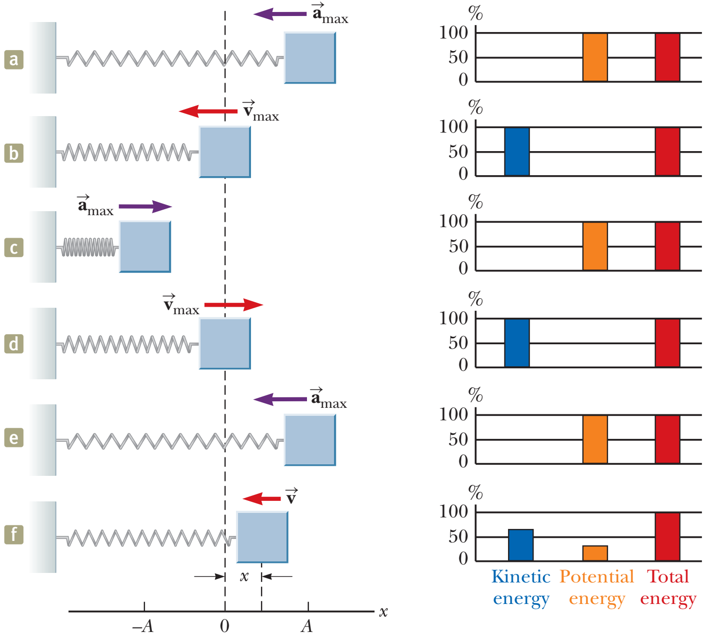
$$x = A\cos(\omega t + \phi)$$




$$f = \frac{1}{T}$$

[1] Figure from Serway & Jewett, 9th ed, pg 453.

# Energy in SHM



# Waves

Very often an oscillation or one-time disturbance can be detected far away.

Plucking one end of a stretched string will eventually result in the far end of the string vibrating.

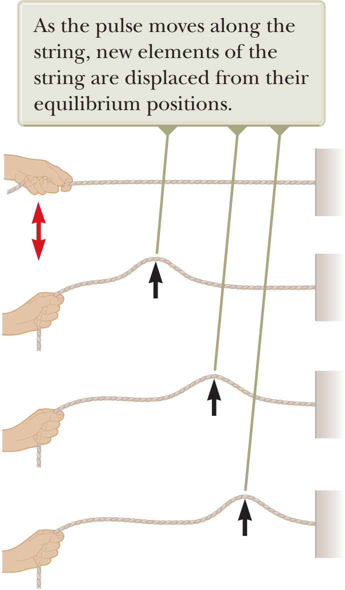
The string is a medium along which the vibration travels.

It carries energy from on part of the string to another.

**Wave**

a disturbance or oscillation that transfers energy through matter or space.

# Wave Pulses


As the pulse moves along the string, new elements of the string are displaced from their equilibrium positions.

# Wave Motion

**Wave**

a disturbance or oscillation that transfers energy through matter or space.

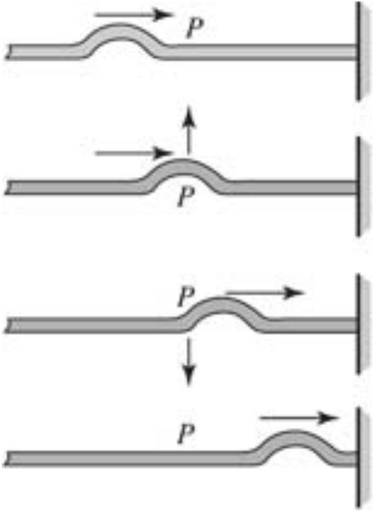
The waveform moves along the medium and energy is carried with it.

The particles in the medium *do not* move along with the wave.

The particles in the medium are briefly shifted from their equilibrium positions, and then return to them.

# Wave pulses

A point $P$ in the middle of the string moves up and down, just as the hand did.



# Kinds of Waves

**medium**

a material substance that carries waves. The constituent particles are temporarily displaced as the wave passes, but they return to their original position.

Kinds of waves:

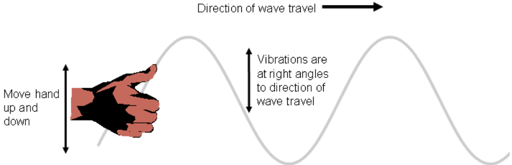
- mechanical waves – waves that travel on a medium, *eg.* sound waves, waves on string, water waves
- electromagnetic waves – light, in all its various wavelengths, *eg.* x-rays, uv, infrared, radio waves
- matter waves – wait for Phys4D!

# Kinds of Waves

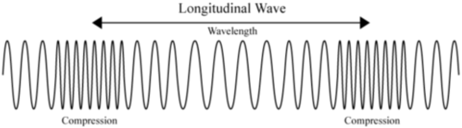
Kinds of waves:

- transverse – displacement perpendicular to direction of wave travel
- longitudinal – displacement parallel to direction of wave travel

Transverse




Longitudinal



# Transverse vs. Longitudinal

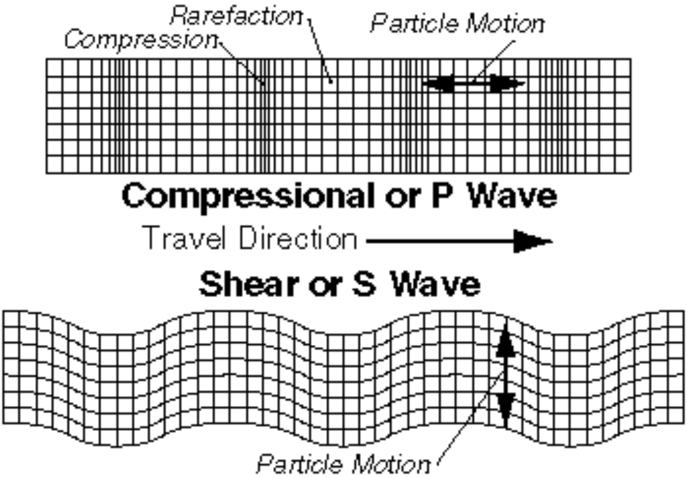
Examples of transverse waves:

- vibrations on a guitar string
- ripples in water
- light
- S-waves in an earthquake (more destructive)

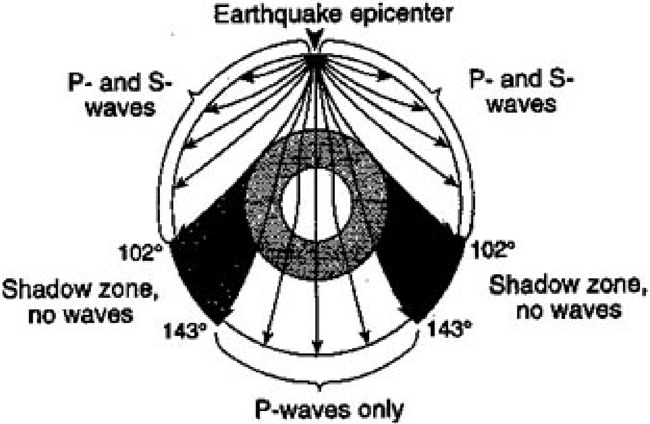
Examples of longitudinal waves:

- sound
- P-waves in an earthquake (initial shockwave, faster moving)

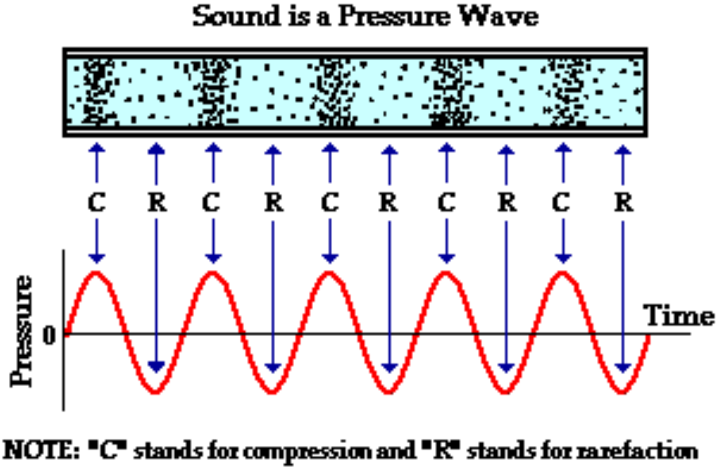
# Earthquakes



# Earthquakes



# Sound waves



Sound is a Pressure Wave
C
R
C
R
C
R
C
R
C
R
Pressure
0
Time
NOTE: "C" stands for compression and "R" stands for rarefaction

# Pulse Propagation

A wave pulse (in a plane) at a moment in time can be described in terms of $x$ and $y$ coordinates, giving $y(x)$.

Suppose that the pulse will move with speed $v$ and be displaced, say in the positive $x$ direction, while maintaining its shape.

That means we can also give $y$ as a function of time, $y(x, t)$.

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Consider a moving reference frame, $S'$, with the pulse at rest, $y'(x') = f(x')$, no time dependence. Galilean transformation:

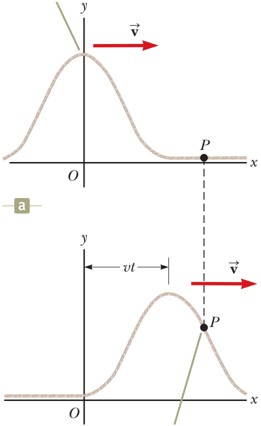
$$x' = x - vt$$

# Pulse Propagation

$$x' = x - vt$$

Then in the rest-frame of the string

$$y(x,t) = f(x') = f(x - vt)$$



# Pulse Propagation

The shape of the pulse is given by $f(x)$ and can be arbitrary.

Whatever the form of $f$, if the pulse moves in the $+x$ direction:

$$y(x, t) = f(x - vt)$$

If the pulse moves in the $-x$ direction:

$$y(x, t) = f(x + vt)$$

# Wave Pulse Example 16.1

A pulse moving to the right along the x axis is represented by the wave function

$$y(x,t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where $x$ and $y$ are measured in centimeters and $t$ is measured in seconds.

What is the wave speed?

Find expressions for the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

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[1] Serway & Jewett, page 486.

[2] This function is an unnormalized Cauchy distribution, or as physicists say "it has a Lorentzian profile".

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$$y(x,t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where $x$ and $y$ are measured in centimeters and $t$ is measured in seconds.

What is the wave speed? **3.0 cm/s**

Find expressions for the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

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[1] Serway & Jewett, page 486.

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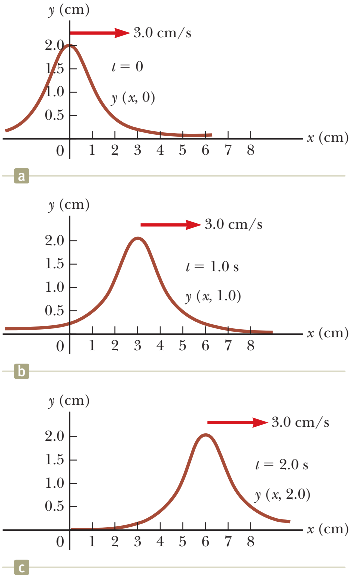
# Wave Pulse Example 16.1

$$t = 0, \quad y(x,0) = \frac{2}{x^2 + 1}$$

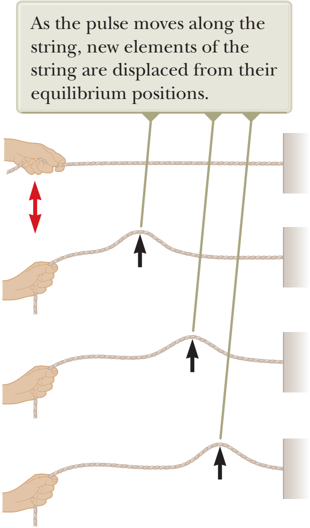
$$t = 1, \quad y(x,1) = \frac{2}{(x - 3.0)^2 + 1}$$

$$t = 2, \quad y(x,2) = \frac{2}{(x - 6.0)^2 + 1}$$



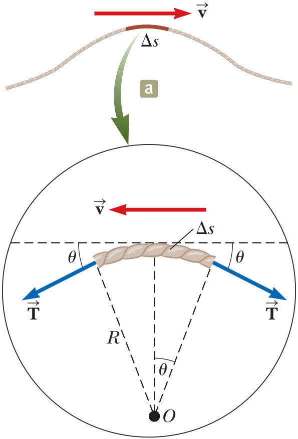
# Wave Speed on a String

How fast does a disturbance propagate on a string under tension?



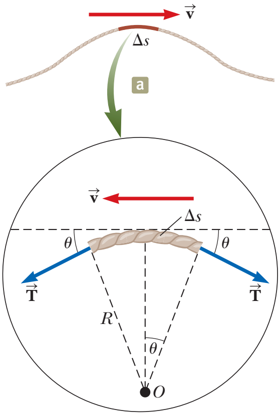
# Wave Speed on a String

Imagine traveling with the pulse at speed $v$ to the right. Each small section of the rope travels to the left along a circular arc from your point of view.




We will find find how fast a point on the string *moves backwards relative to the wave pulse*.

# Wave Speed on a String




We can use the force diagram to find the force on a *small* length of string $\Delta s$:

$$F_{\text{net}} = 2T \sin \theta \approx 2T\theta \tag{1}$$

# Wave Speed on a String

Consider the centripetal force on the piece of string.

If $R$ is the radius of curvature and $m$ is the mass of the small piece of string:

$$F_{\text{net}} = \frac{mv^2}{R}$$

# Wave Speed on a String

Consider the centripetal force on the piece of string.

If $R$ is the radius of curvature and $m$ is the mass of the small piece of string:

$$F_{\text{net}} = \frac{mv^2}{R}$$

Suppose the string has mass density $\mu$ (units: kg m$^{-1}$)

$$m = \mu\,\Delta s = \mu R(2\theta)$$

Put this into our expression for centripetal force:

$$F_{\text{net}} = \frac{2\mu R\theta v^2}{R}$$

# Wave Speed on a String

Put this into our expression for centripetal force:

$$F_{\text{net}} = 2\mu\theta v^2$$

And using eq. (1), $F_{\text{net}} = 2T\theta$:

$$2T\theta = 2\mu\theta v^2$$

The wave speed is:

$$v = \sqrt{\frac{T}{\mu}}$$

For a given string under a given tension, all waves travel with the same speed!

# Summary

- oscillations
- simple harmonic motion (SHM)
- spring systems
- intro to waves
- pulse propagation
- wave speed on a string (?)

**Homework** Serway & Jewett:

- (if you like, for review) Ch 15, onward from page 472. OQs: 13; CQs: 5, 7; Probs: 1, 3, 9, 35, 41, 86