Fluids
Fluid Dynamics

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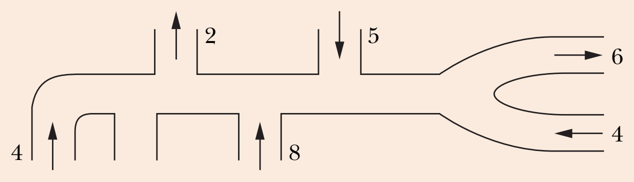
Last time

- Pascal's principle
- measurements of pressure
- introduced fluid dynamics

Warm Up Question

The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section? (Assume that the fluid in the pipe is an ideal fluid.)



A 11 cm^3/s, outward

B 13 cm^3/s, outward

C 3 cm^3/s, inward

D cannot be determined

[1] Halliday, Resnick, Walker, 9th ed, page 373.

Overview

- fluid dynamics
- the continuity equation
- Bernoulli's equation
- Torricelli's law (?)

Fluid Dynamics

We will make some simplifying assumptions:

1. the fluid is **nonviscous**, *ie.* not sticky, it has no internal friction between layers
2. the fluid is **incompressible**, its density is constant
3. the flow is **laminar**, *ie.* the streamlines are constant in time
4. the flow is **irrotational**, there is no curl

In real life no fluids actually have the second property, and almost none have the first.

Flows can have the second two properties, in the right conditions.

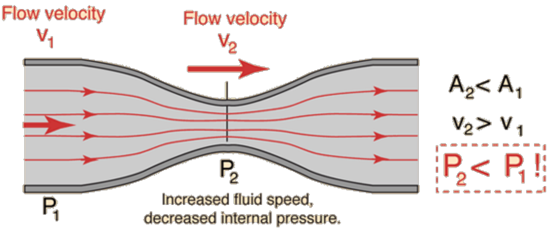
Bernoulli's Principle

A law discovered by the 18th-century Swiss scientist, Daniel Bernoulli.

Bernoulli's Principle

As the speed of a fluid's flow increases, the pressure in the fluid decreases.

This leads to a surprising effect: for liquids flowing in pipes, the pressure *drops* as the pipes get narrower.



Bernoulli's Principle

Why should this principle hold? Where does it come from?

[1] Something similar can be argued for compressible fluids also.

Bernoulli's Principle

Why should this principle hold? Where does it come from?

Actually, it just comes from the conservation of energy, and an assumption that the fluid is **incompressible**.[1]

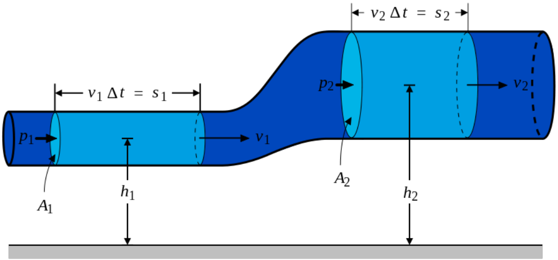
Consider a fixed volume of fluid, V.

In a narrower pipe, this volume flows by a particular point 1 in time Δt.

However, it must push the same volume of fluid past a point 2 in the same time. If the pipe is wider at point 2, it flows more slowly.

[1] Something similar can be argued for compressible fluids also.

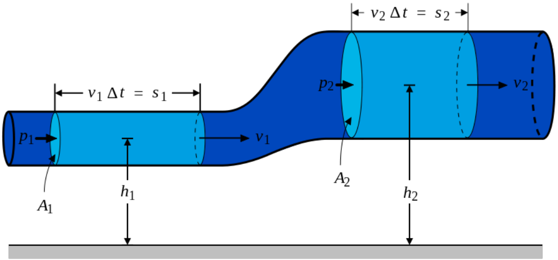
Bernoulli's Principle



$V = A_1 v_1 \Delta t$

also, $V = A_2 v_2 \Delta t$

Bernoulli's Principle



$V = A_1 v_1 \Delta t$

also, $V = A_2 v_2 \Delta t$

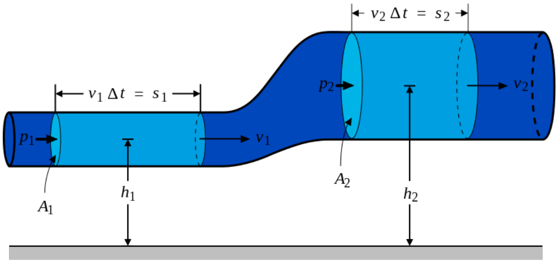
This means

$$A_1 v_1 = A_2 v_2$$

The "Continuity equation".

Bernoulli's Equation

Bernoulli's equation is just the conservation of energy for this fluid. The system here is all of the fluid in the pipe shown.

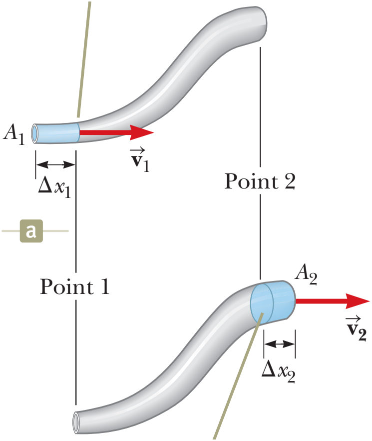


Both light blue cylinders of fluid have the same volume, V, and same mass m.

We imagine that in a time Δt, volume V of fluid enters the left end of the pipe, and another V exits the right.

Bernoulli's Equation

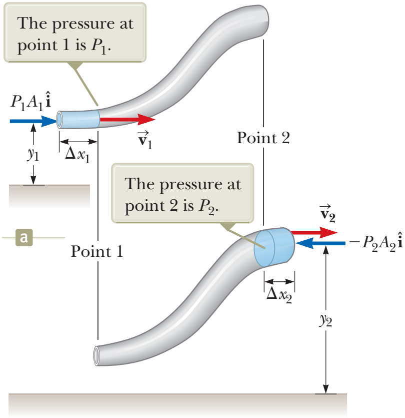
It makes sense that the energy of the fluid might change: the fluid is moved along, and some is lifted up.



How does it change? Depends on the work done:

$$W = \Delta K + \Delta U$$

Bernoulli's Equation



The work done is the sum of the work done on each end of the fluid by more fluid that is on either side of it:

$$\begin{aligned} W &= F_1 \Delta x_1 - F_2 \Delta x_2 \\ &= P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \end{aligned}$$

(The "environment fluid" just to the right of the system fluid does negative work on the system as it must be pushed aside by the system fluid.)

[1] Diagram from Serway & Jewett.

Bernoulli's Equation

Notice that $V = A_1 \Delta x_1 = A_2 \Delta x_2$

$$\begin{aligned} W &= P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \\ &= (P_1 - P_2) V \end{aligned}$$

Conservation of energy:

$$W = \Delta K + \Delta U$$

Bernoulli's Equation

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Conservation of energy:

$$W = \Delta K + \Delta U$$

$$(P_1 - P_2) V = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

Bernoulli's Equation

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Conservation of energy:

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$$(P_1 - P_2) V = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

Dividing by V:

$$\begin{aligned} P_1 - P_2 &= \frac{1}{2} \rho v_2^2 + \rho g (h_2 - h_1) \\ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 &= P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \end{aligned}$$

Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

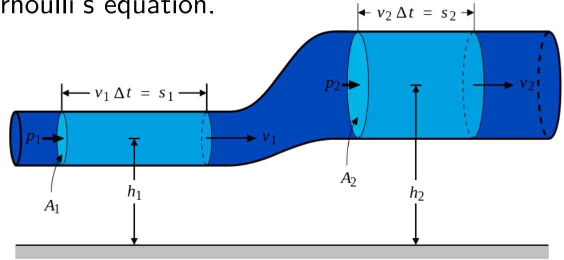
This expression is true for *any* two points along a streamline.

Therefore,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

is constant along a streamline in the fluid.

This is Bernoulli's equation.



Bernoulli's Equation

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

Even though we derived this expression for the case of an incompressible fluid, this is also true (to first order) for compressible fluids, like air and other gases.

The constraint is that the densities should not vary too much from the ambient density ρ.

Bernoulli's Principle from Bernoulli's Equation

For two different points in the fluid, we have:

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2$$

Bernoulli's Principle from Bernoulli's Equation

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$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2$$

Suppose the height of the fluid does not change, so $h_1 = h_2$:

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

Bernoulli's Principle from Bernoulli's Equation

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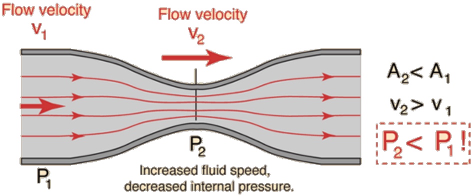
Suppose the height of the fluid does not change, so $h_1 = h_2$:

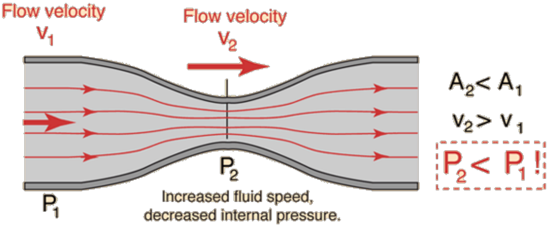
$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

If $v_2 > v_1$ then $P_2 < P_1$.



Bernoulli's Principle

However, from the continuity equation $A_1 v_1 = A_2 v_2$ we can see that if A_2 is smaller than A_1, v_2 is bigger than v_1.



So the pressure really does fall as the pipe contracts!

Summary

Bernoulli's Principle

As the speed of a fluid's flow increases, the pressure in the fluid decreases.

The Continuity equation:

$$A_1 v_1 = A_2 v_2$$

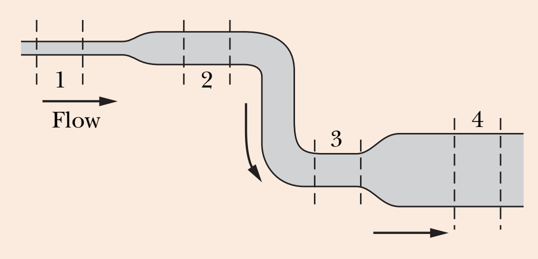
Bernoulli's Equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

is constant along a streamline in the fluid.

Question

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to **the volume flow rate through them**, greatest first.



A 4, 3, 2, 1

B 1, (2 and 3), 4

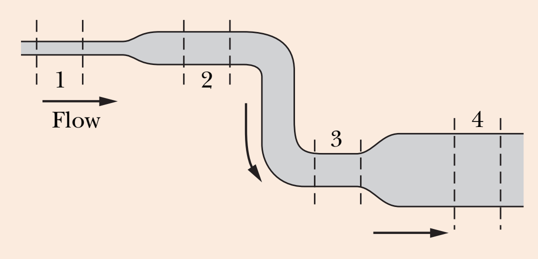
C 4, (2 and 3), 1

D All the same

[1] Halliday, Resnick, Walker, 9th ed, page 375.

Question

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to **the flow speed v through them**, greatest first.



A 4, 3, 2, 1

B 1, (2 and 3), 4

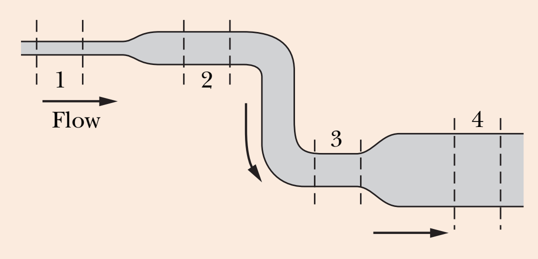
C 4, (2 and 3), 1

D All the same

[1] Halliday, Resnick, Walker, 9th ed, page 375.

Question

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to **the water pressure P within them**, greatest first.



A 4, 3, 2, 1

B 1, (2 and 3), 4

C 4, (2 and 3), 1

D All the same

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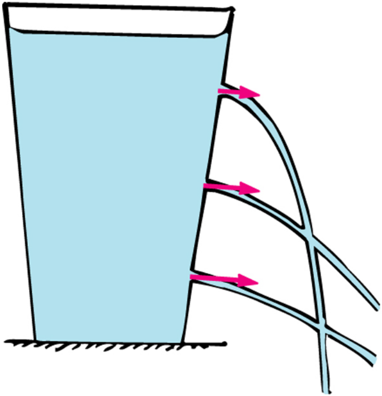
Torricelli's Law from Bernoulli's Equation

Bernoulli's equation can also be used to predict the velocity of streams of water from holes in a container at different depths.



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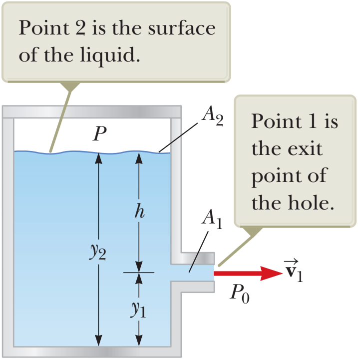
Torricelli's Law from Bernoulli's Equation

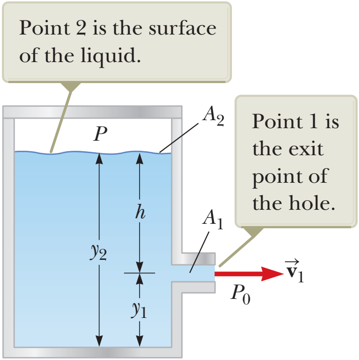
The liquid at point 2 is at rest, at a height y_2 and pressure P.

At point 1 is leaves with a velocity v_1, at a height y_1 and pressure P_0.



$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_0 = \frac{1}{2}\cancelto{0}{\rho v_2^2} + \rho g y_2 + P$$

Torricelli's Law from Bernoulli's Equation

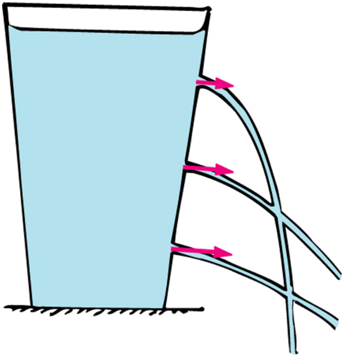


$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_0 = \rho g y_2 + P$$

Rearranging, and using $y = h_2 - h_1$,

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

Torricelli's Law from Bernoulli's Equation



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Notice that if the container is open to the air $(P = P_0)$, then the speed of each jet is

$$v = \sqrt{2gh}$$

where h is the depth of the hole below the surface.

Summary

- fluid dynamics
- the continuity equation
- Bernoulli's equation
- Torricelli's law (?)

Test Wednesday, April 22, in class.